

# DESIGN OF SENSOR SYSTEMS WITH FUSION FOR DETECTION APPLICATIONS

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# System-Theoretic Framework for Sensor Networks

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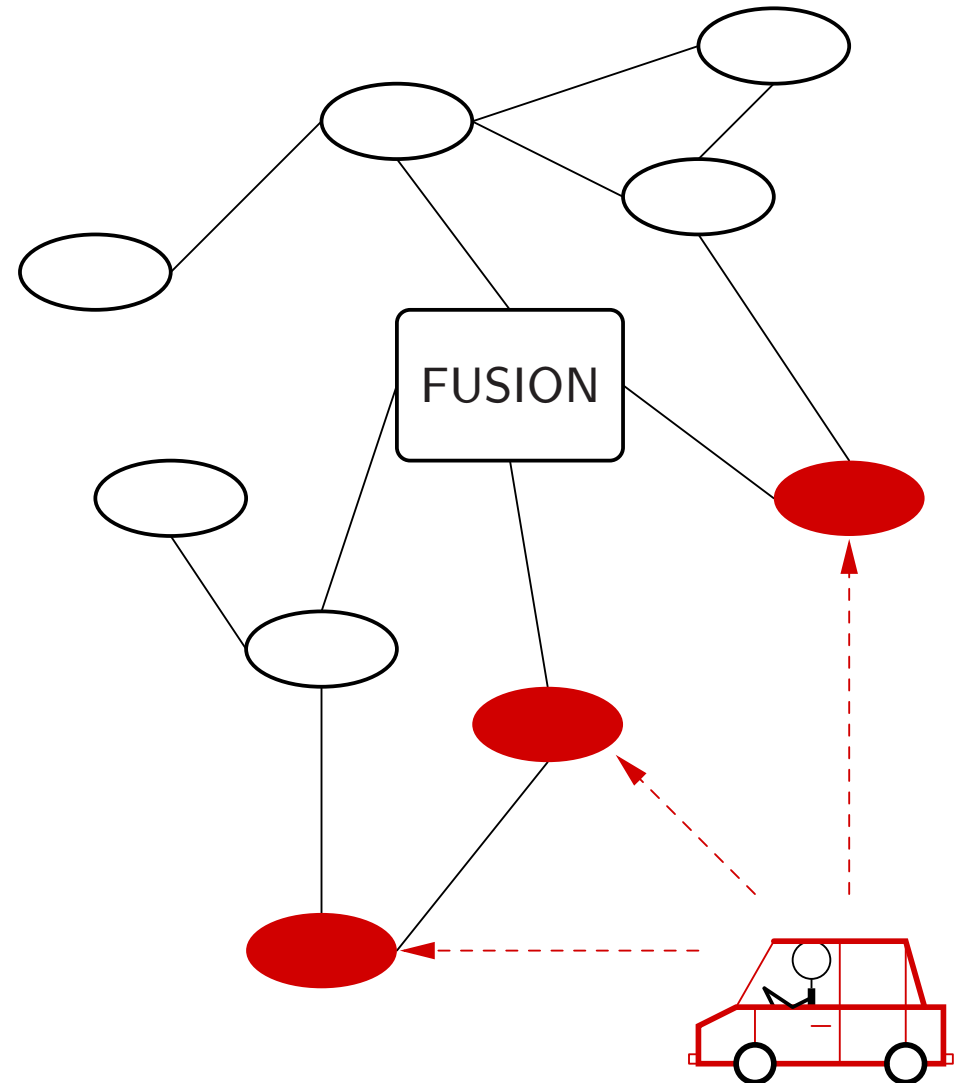
- Research to date on sensors networks largely been focused on:
  - techniques for building sensors
  - protocols for networking them
- Sensor networks are **NOT** simply ad hoc wireless networks with more stringent energy constraints
- Need to incorporate sensing (actuation) into the design and optimization of these networks for specific applications
- Need a core **system-theoretic** framework to provide guidelines

# Detection Applications of Wireless Sensing Systems

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- Health care
- Manufacturing
- Monitoring civil infrastructures
- Quality control
- Security
- Surveillance
- Tracking endangered species
- Traffic monitoring

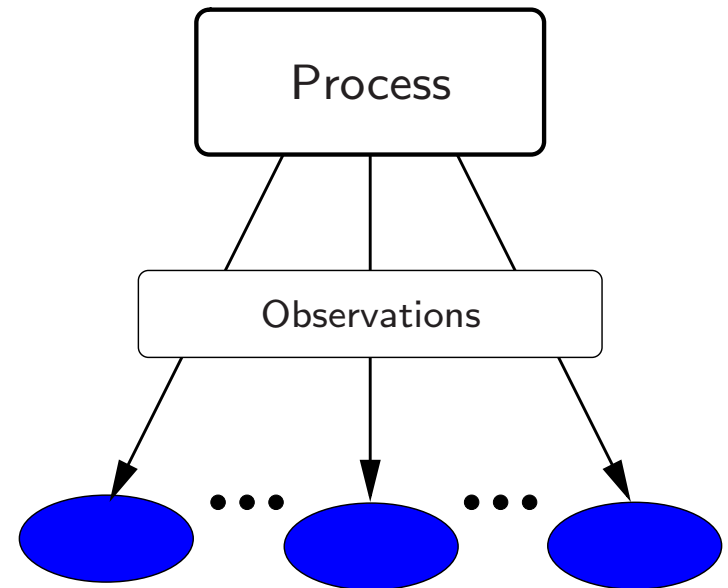
Detection is often **primary step** in sensing process



# Types of Detection Problems

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- Static decision making
  - one block of observations per sensor
  - possible motivation: energy savings from taking occasional “snapshots” of environment
- Process monitoring
  - detecting when observed process goes out of bound
- Dynamic decision making
  - sequential detection – determining block length on-line
  - quickest change detection
- Binary versus M-ary



# System Performance

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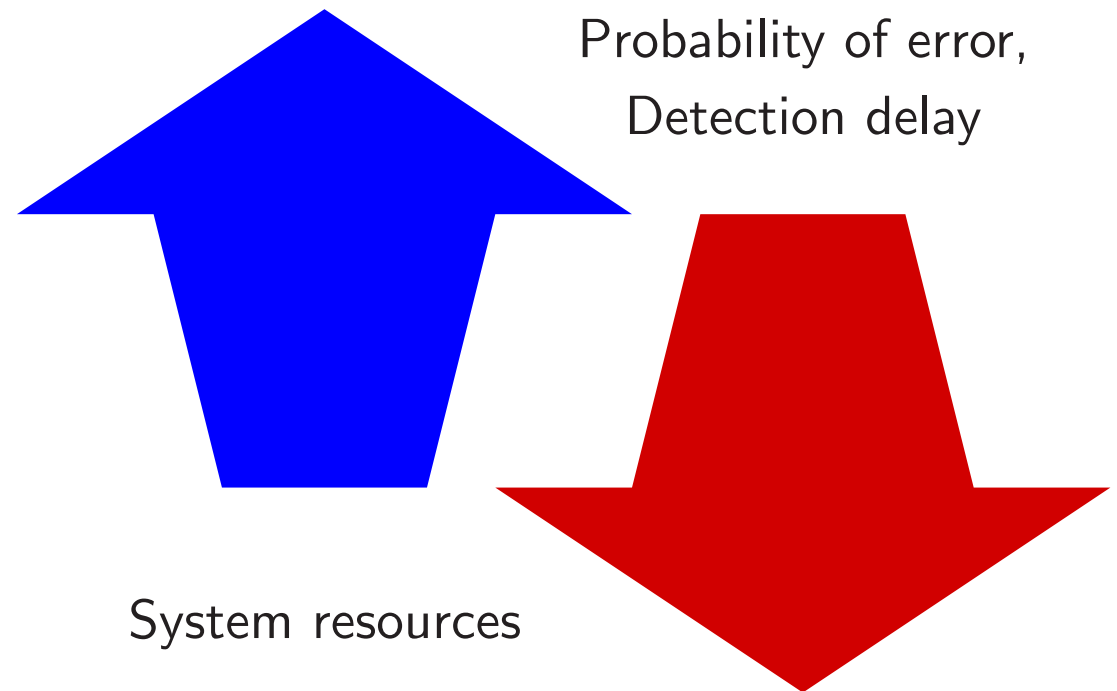
## Performance metrics

- probability of error
- detection delay

## Resource constraints

- bandwidth
- energy
- complexity

## Natural tradeoff problem



# Basic Problem Formulation

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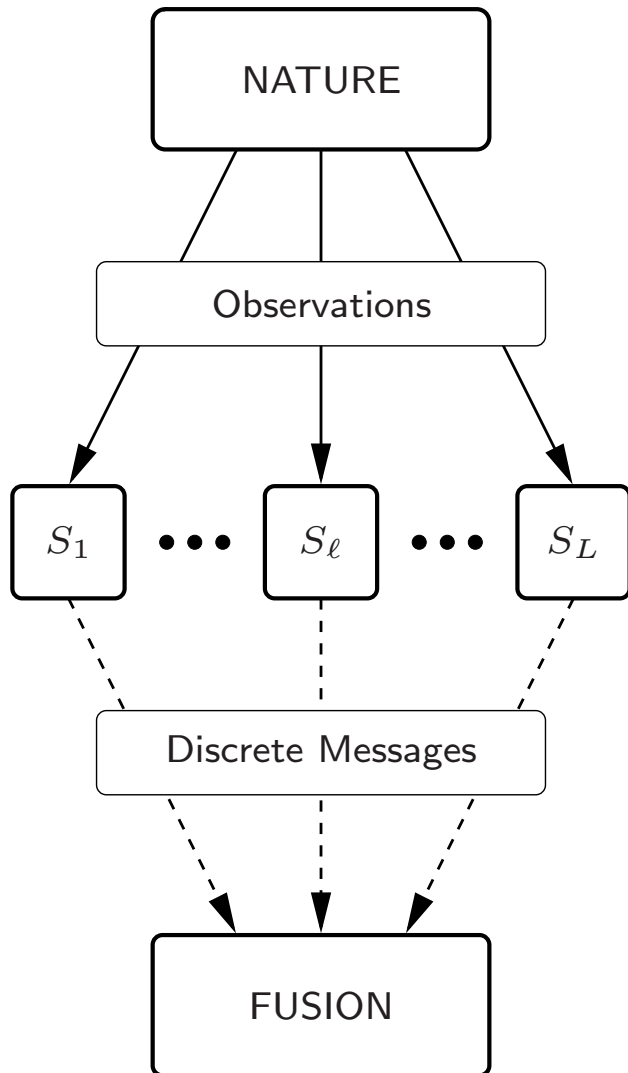
- Binary hypothesis –  $H_0$  and  $H_1$
- Fusion architecture – sensors send a function of their observations to fusion center where decision is made
- Static detection – each sensor has single observation
- Bayesian setting – minimize  $P_e$  at fusion center

$$P_e = \alpha P(H_0) + \beta P(H_1)$$

- Neyman-Pearson setting – minimize  $\beta$  subject to  $\alpha \leq \alpha_0$

# Classical Decentralized Detection Framework

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Resource constraints captured by imposing **finite alphabet** constraints on sensor outputs

$$\gamma_\ell(Y_\ell) \in \{1, 2, \dots, D_\ell\}$$

**Perfect reception** of sensor outputs at fusion center

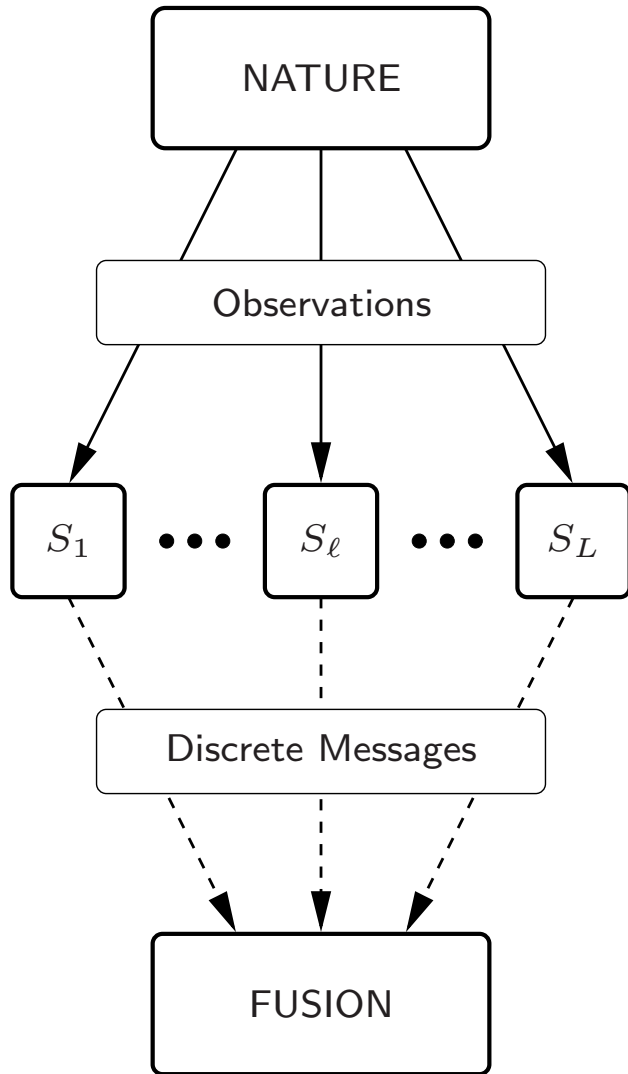
Jointly optimize sensor mappings and fusion rule

Optimization problem is in general hard, but tractable under **conditional independence** assumption

Solution does **not scale** with number of sensors

# Classical Framework

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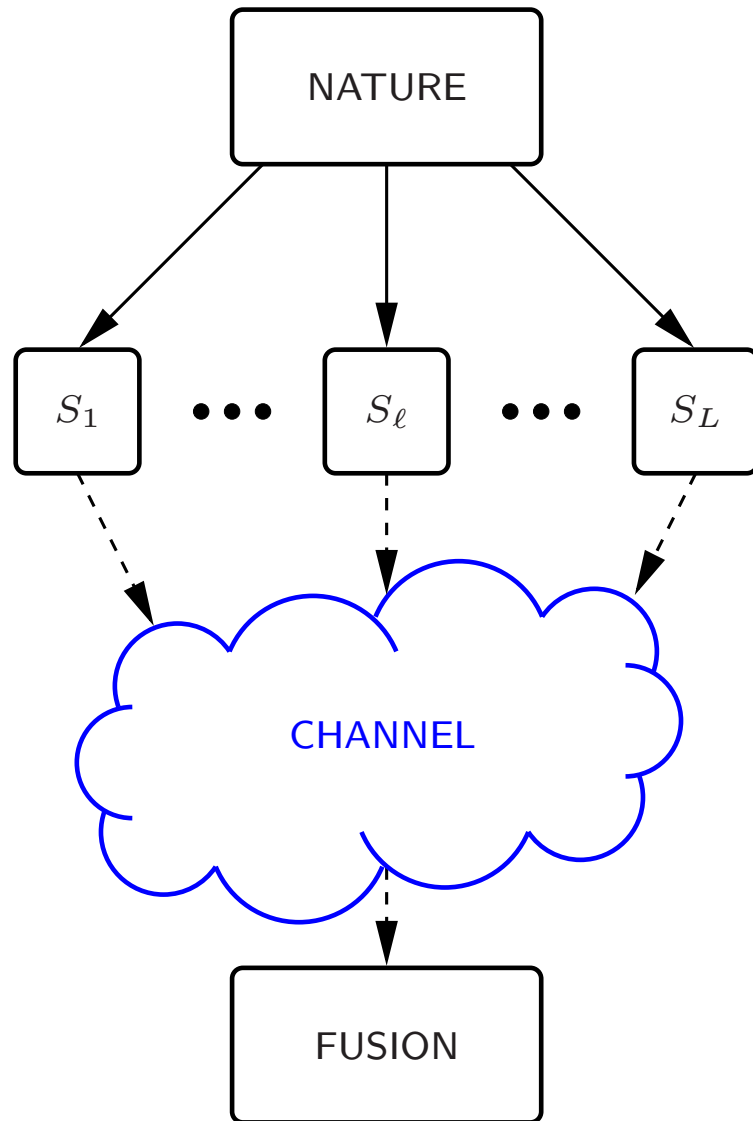


Drawbacks of classical framework in context of modern sensor systems

- assumes idealized channel and overlooks physical layer
- resource constraints not captured adequately for efficient design
- focus on optimal decision rules; should be on **system design** strategies

# Alternative Framework

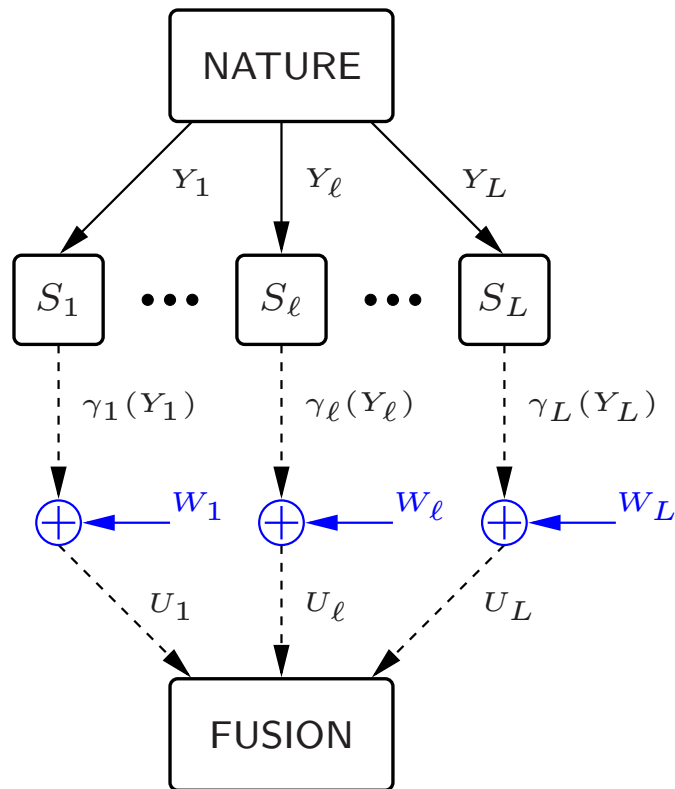
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## Features of alternative framework

- resource constraints represented more generally in “channel” from sensors to fusion center
- asymptotics – large number of sensors
- focus on good solutions that scale well
- number/density of nodes as parameter to be optimized
- conditionally dependent observations

# Example 1: Noisy Sensor-Fusion Channels



$Y_\ell$  observation

$\gamma_\ell$  transmission map

$W_\ell$  additive noise

$$U_\ell = \gamma_\ell(Y_\ell) + W_\ell$$

$A$  total power constraint

$$\sum_{\ell=1}^L f(\gamma_\ell) \leq A$$

where  $f(\gamma_\ell)$  is the expected power consumed by node  $\ell$

$L$  number of sensor nodes  
not fixed a priori

## Noisy Channels – Problem Definition

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An **admissible strategy**  $\mathcal{G}$  is a vector function  $(\gamma_1, \dots, \gamma_L)$  such that

$$F(\mathcal{G}) = \sum_{\ell=1}^L f(\gamma_\ell) \leq A$$

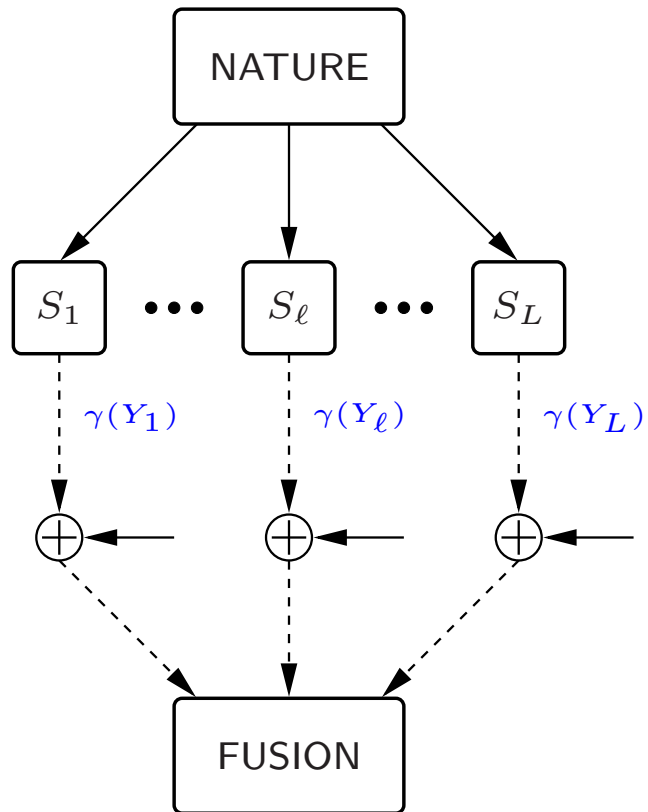
where  $f(\gamma_\ell) > 0$  represents the expected power consumed by node  $\ell$

**Problem** Find collection of strategies  $\{\mathcal{G}_A\}$ , one for each  $A$ , that maximizes error exponent

$$- \lim_{A \rightarrow \infty} \frac{\log P_e(\mathcal{G}_A)}{A}$$

where  $\mathcal{G}_A$  denotes strategy such that  $F(\mathcal{G}_A) \leq A$

# Normalized Chernoff Information



Large deviations

Using **identical** sensor nodes is asymptotically optimal with **symmetric** observations

Sensors should be designed to maximize the **normalized Chernoff information** at input to fusion center

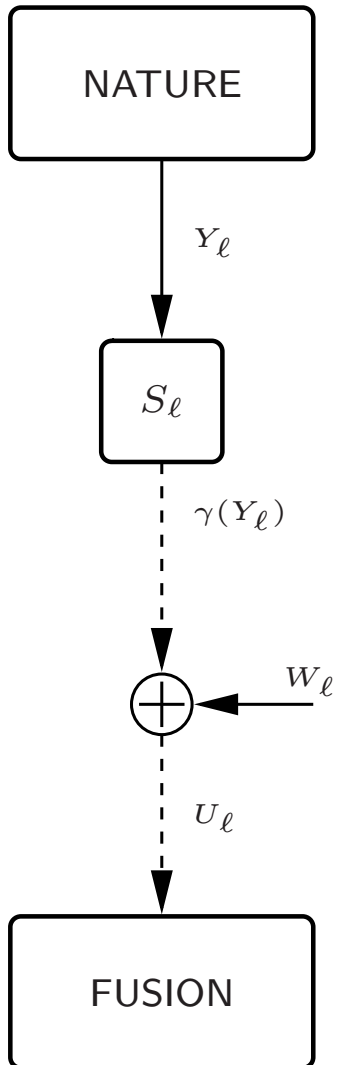
$$-\frac{1}{f(\gamma)} \min_{\lambda \in [0,1]} \log \mathbb{E}_{\mathcal{Q}_{0,\gamma}} \left[ \left( \frac{d\mathcal{Q}_{1,\gamma}}{d\mathcal{Q}_{0,\gamma}} \right)^\lambda \right]$$

**Decoupled design!**

For the **Neyman-Pearson** problem, metric based on normalized **Kullback-Leibler** distance

# Designing Sensor Map – Gaussian Example

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$Y_\ell$  Gaussian observation

$$p_{Y|H}(y|H_0) \sim \mathcal{N}(-m_y, \sigma_y^2)$$

$$p_{Y|H}(y|H_1) \sim \mathcal{N}(m_y, \sigma_y^2)$$

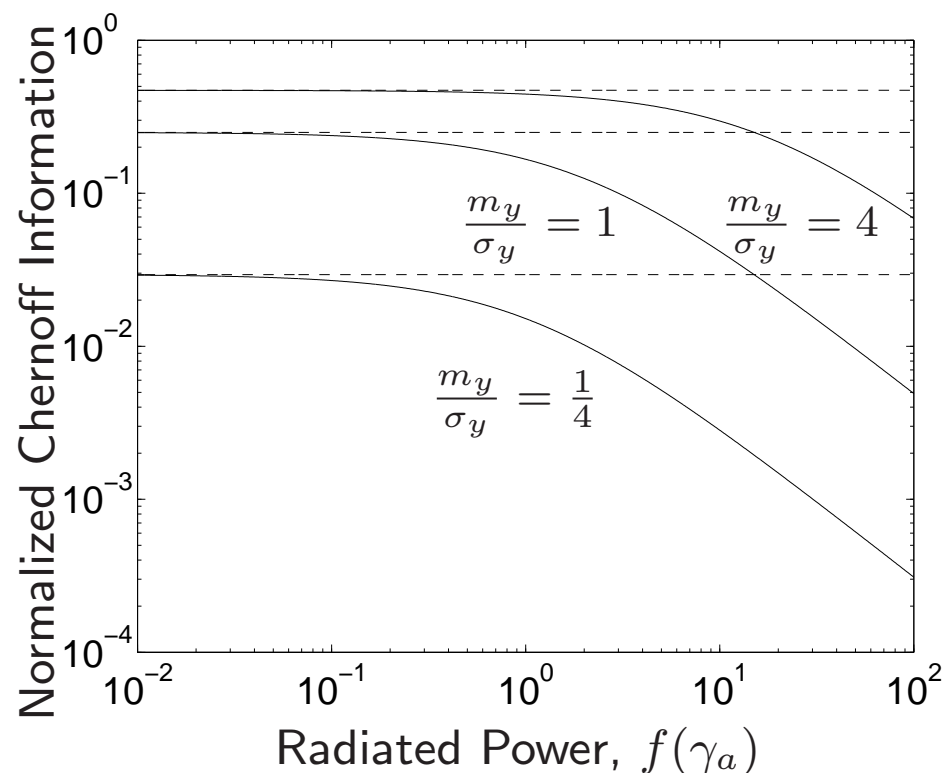
$\gamma$  transmission map

$W_\ell$  additive Gaussian noise  $\sim \mathcal{N}(0, \sigma_w^2)$

$$U_\ell = \gamma(Y_\ell) + W_\ell$$

Use normalized Chernoff information to compare candidate  $\gamma$ 's

# Analog Relay Amplifier



Analog relay amplifier

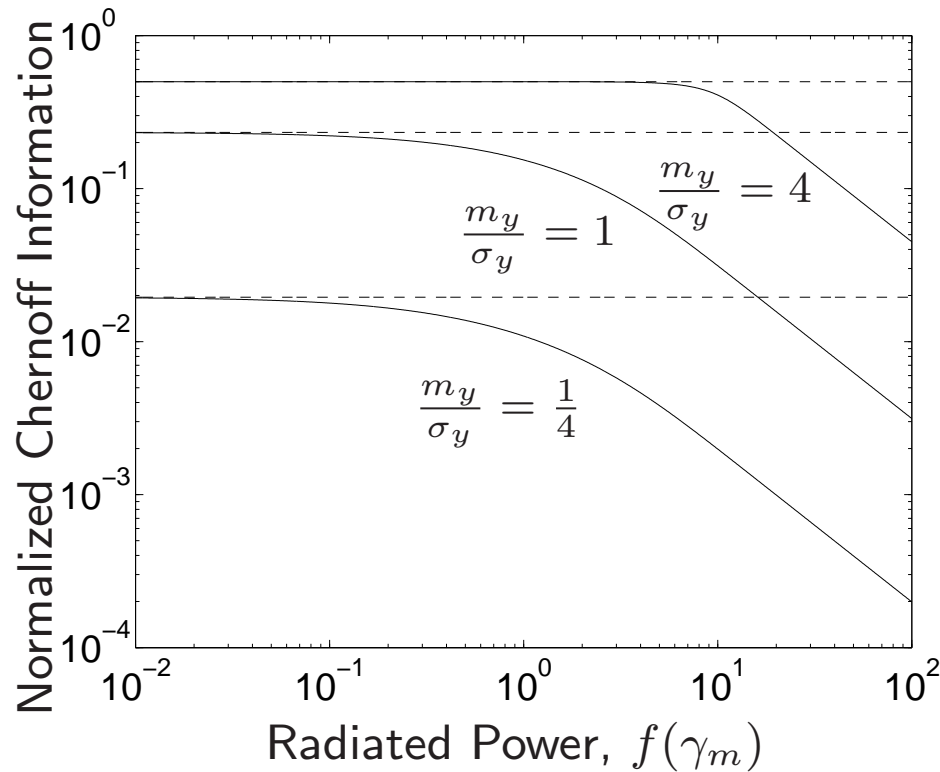
$$\gamma_a(y) = ay$$

$$f(\gamma_a) = a^2 m_y^2 + a^2 \sigma_y^2$$

The Normalized Chernoff information corresponding to analog relay amplifier  $\gamma_a$  is **nonincreasing** in  $f(\gamma_a)$

$$\frac{m_y^2}{2(m_y^2 + \sigma_y^2)(a^2 \sigma_y^2 + \sigma_w^2)} \xrightarrow{a \downarrow 0} \frac{m_y^2}{2\sigma_w^2(m_y^2 + \sigma_y^2)}$$

# Binary Threshold Node



Binary threshold node

$$\gamma_m(y) = \begin{cases} m : y \geq 0 \\ -m : y < 0 \end{cases}$$

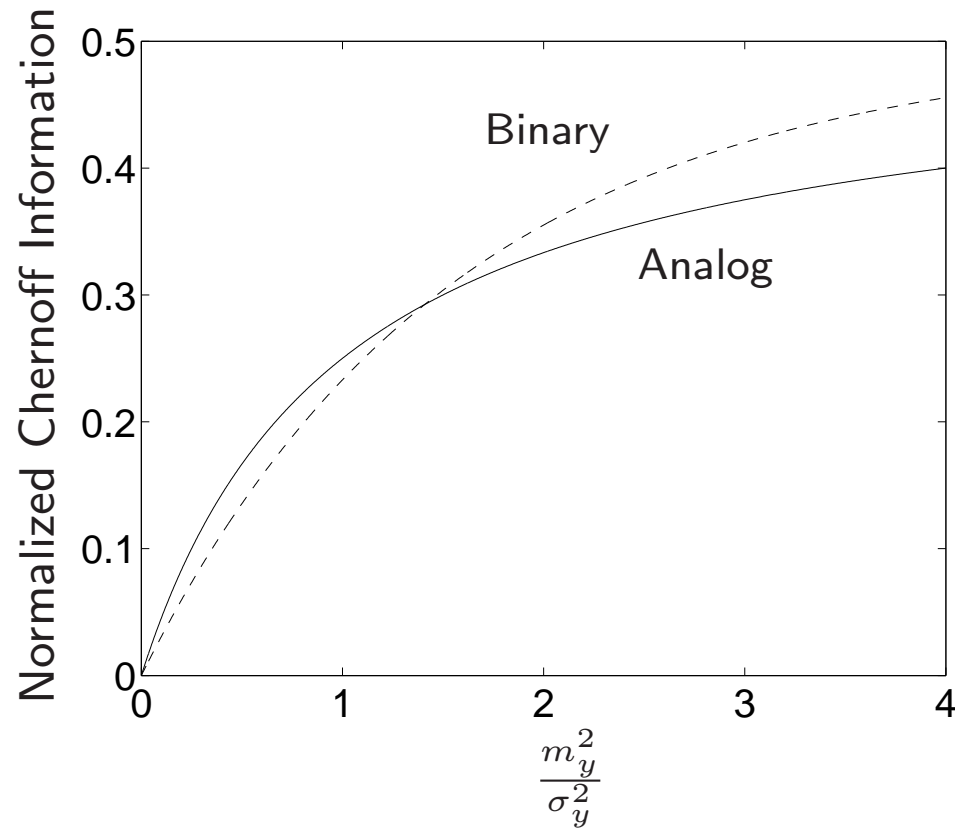
$$f(\gamma_m) = m^2$$

The Normalized Chernoff information corresponding to binary threshold node  $\gamma_m$  is **nonincreasing** in  $f(\gamma_a)$

$$-\frac{1}{m^2} \log \int_{-\infty}^{\infty} \sqrt{\mathcal{Q}_{0,\gamma_m}(u) \mathcal{Q}_{1,\gamma_m}(u)} du \xrightarrow{m \downarrow 0} \frac{1}{2\sigma_w^2} \left[ Q\left(-\frac{m_y}{\sigma_y}\right) - Q\left(\frac{m_y}{\sigma_y}\right) \right]^2$$

# Analog versus Binary Nodes

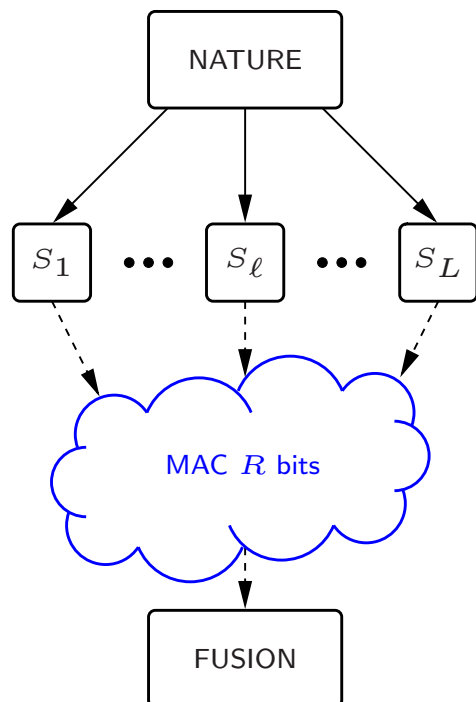
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- Unreliable observations: analog relay amplifiers perform better
- Reliable observations: binary threshold nodes perform better

## Example 2: MAC Fusion with Sum Rate Constraint

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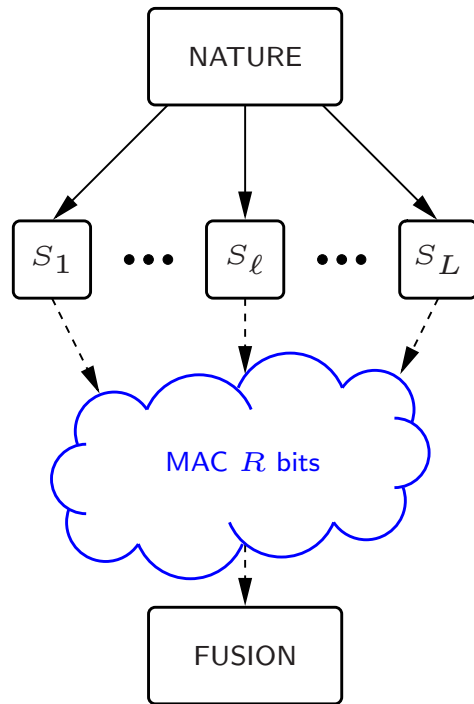
- Sum power constraint replaced by **sum bit rate constraint**
- MAC channel can carry  $R$  bits from sensors to fusion center
- $f(\gamma) = \lceil \log_2 D_\gamma \rceil$
- Asymptotics as  $R \rightarrow \infty$

Sensor map should be chosen to maximize

$$-\frac{1}{\lceil \log_2 (D_\gamma) \rceil} \min_{\lambda \in [0,1]} \left\{ \log E_{\mathcal{Q}_{0,\gamma}} \left[ \left( \frac{d\mathcal{Q}_{1,\gamma}}{d\mathcal{Q}_{0,\gamma}} \right)^\lambda \right] \right\}.$$

# Optimality of Binary Sensors

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If there exists a binary function  $\gamma_b$  such that

$$C(\gamma_b(Y)) \geq \frac{C(Y)}{2}$$

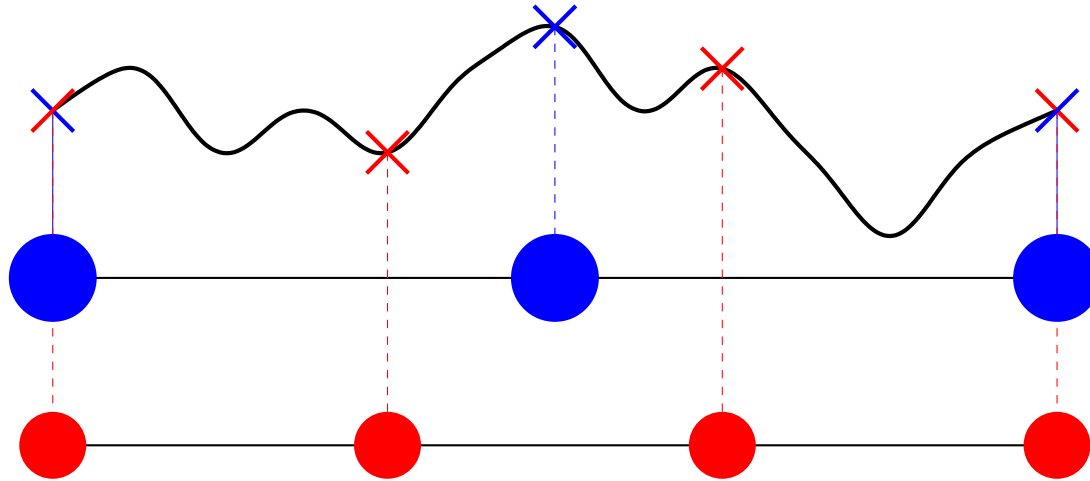
then having  $R$  sensor nodes, each sending **one bit** of information, is asymptotically optimal

This condition can be verified for many observation models of interest

Having more sensor nodes outperforms the benefits of getting detailed information from each node

## Correlated Sensors – Density Analysis

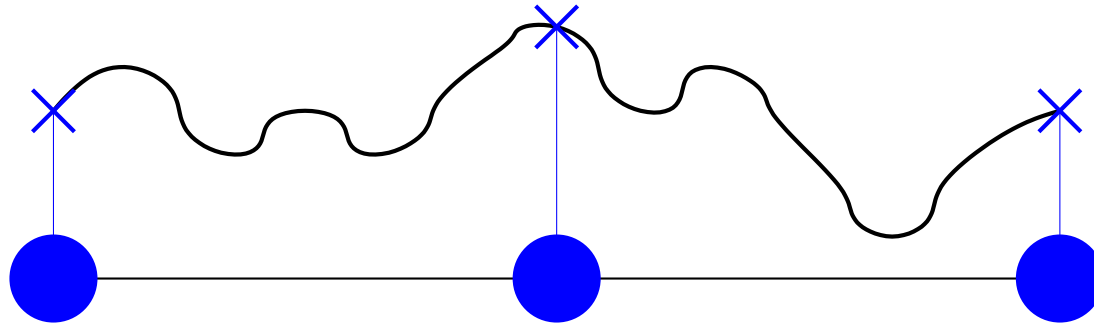
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- Sensor nodes on line, sampling **correlated** spatial stochastic process
- **Power density constraint:** power per unit distance fixed, i.e., power per node decreases linearly with node density
- Correlation between node observations increases with node density
- Q: How dense should the network be?

# Problem Formulation

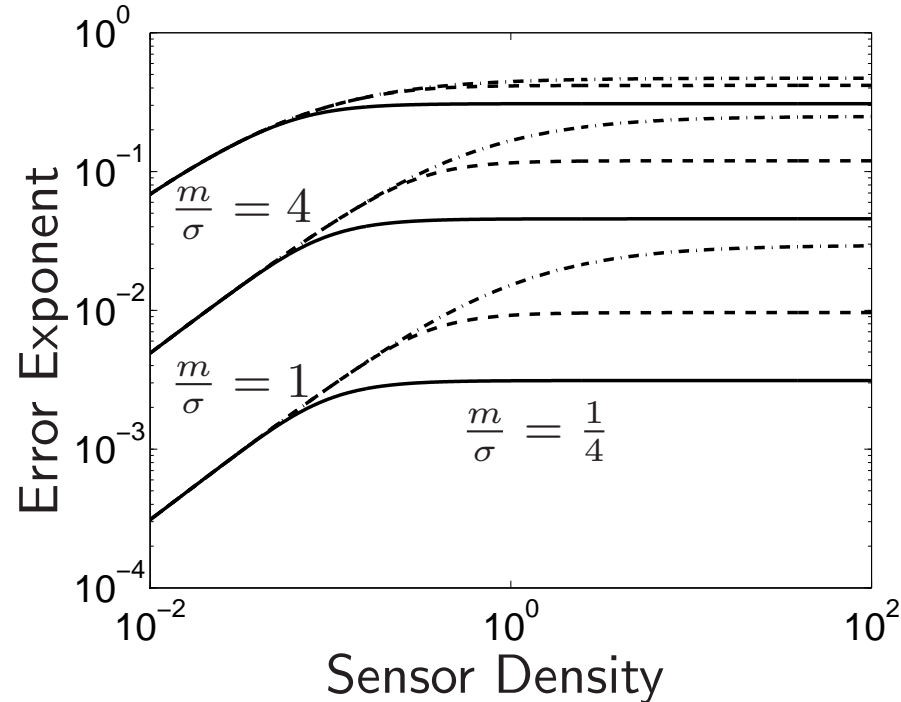
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- Observed spatial process is one of two possible **stationary** signals  $\mathbf{X}_0(s)$  and  $\mathbf{X}_1(s)$
- Signal is corrupted by **stationary** observation noise process  $\mathbf{V}(s)$
- Two possible models for correlation at sensors
  - **Model I:** Deterministic signal with Gauss-Markov noise
  - **Model II:** Zero mean Gauss-Markov signal with white noise

# Deterministic Signal with Gauss-Markov Noise

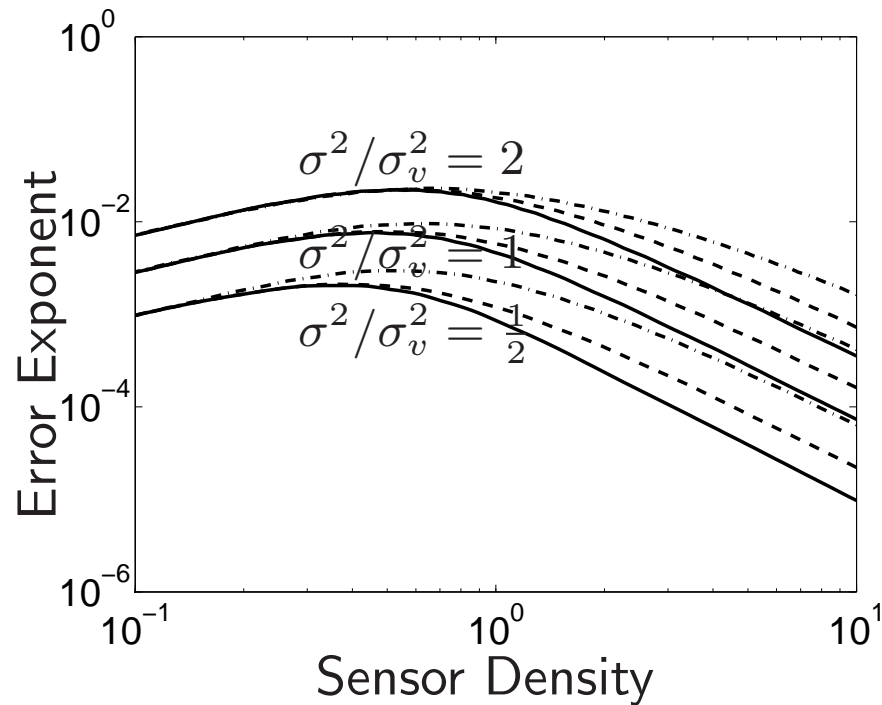
- Corr. Coeff.  $\rho \in \{0, 0.4, 0.8\}$ , power per unit distance  $c = 1$ ,  $\sigma_w^2 = 1$
- The fusion center does **not** need to know  $\rho$  for optimum asymptotic performance!



- Higher densities are always better!

# Zero-Mean Gauss-Markov Signal with White Noise

- Corr. Coeff.  $\rho \in \{0.2, 0.5, 0.8\}$ , power per unit distance  $c = 1$ ,  $\sigma_w^2 = 1$
- The fusion center needs to know  $\rho$  for optimum asymptotic performance



- Optimum value for density!

# Design and Optimization Strategies

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- Optimum solution may be too complicated and not robust – focus on good suboptimum solutions that are robust and scalable
- Soften optimization metrics – error exponents, deflection, etc.
- Decouple optimization problem – choose sensor operations to optimize local metrics
- Allow number (density) of sensors to be system design parameter that needs to be optimized before deployment
- Fully exploit modes of operation of sensor to minimize resource consumption for while meeting application performance criteria
- Design communication protocol with detection application in mind – e.g., some bits are more important than others

## Some Questions and Challenges

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- What if the observations statistics are not known precisely?
- How should sensor outputs be communicated on channel? Is it necessary to convert to bits/packets?
- What is the role of coding?
- How much do we gain by allowing sensors to talk to each other in the fusion configuration?
- What is the right architecture for the network? Decentralized with fusion or distributed?
- What feedback should we provide to sensor designers?

## References

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