Localization and Intensity Tracking of Diffusing Sources

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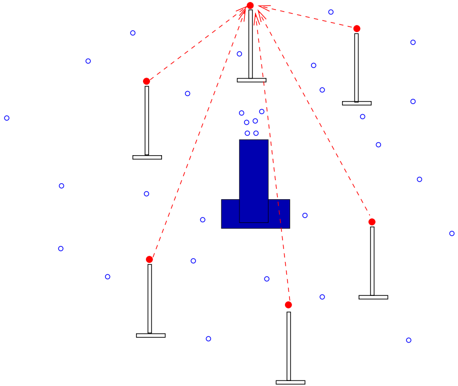
Engineering Problem



Determine location and time-varying rate of diffusing source from concentration measurements made at discrete times and locations

Event Propogation Model and Sensing Model

- Spatio-temporal concentration distribution, $C(x,t)$, is determined by diffusion equation

$$\frac{\partial C(x,t)}{\partial t} = \alpha \nabla^2 C(x,t) + f(x,t) + N(x,t)$$

- Dirichlet boundary conditions
- $N(x,t)$ models ambient (white) noise and α is diffusion coefficient
- $f(x,t)$ models source
  - Point diffusing source at θ
  - Source intensity $\{I_k\}$ piece-wise constant over unit time intervals
  - Across k, $\{I_k\}$ is Gauss-Markov process

$$f(x,t) = \sum_{k=1}^{\infty} I_k \; p(t-k+1)\delta(x-\theta)$$

- Sensors measurements: $Y(x_i,k) = C(x_i,k) + N(0,\sigma_m^2)$
- Sensors communicate measurements to fusion center
- Mathematical Problem: Determine $\{I_k\}$ and θ from $\{Y(x_i,k)\}$ recursively

Diffusion Equation—Solution strategy

- Galerkin's approximation: Approximate solution by finite series that satisfies boundary and initial conditions

$$\tilde{C}(x,t) = \sum_{l=1}^{l_0} Z_{l,t}\ \phi_k(x)$$

- Equivalent state-space representation

$$\begin{bmatrix} \mathbf{Z}_{k+1} \\ I_{k+1} \end{bmatrix} = \begin{bmatrix} A & B(\theta) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{Z}_k \\ I_k \end{bmatrix} + \begin{bmatrix} \mathbf{W}_a(k;\theta) \\ W_s(k) \end{bmatrix}$$

$$\mathbf{Y}_{k+1} = D\mathbf{Z}_{k+1} + \mathbf{W}_m(k+1)$$

Recursive Prediction Error Formulation

- Predictor for $\mathbf{Y}_{k+1}$ from observations made until time k

$$\begin{bmatrix} \hat{\mathbf{Z}}_{k+1|k} \\ \hat{I}_{k+1|k} \end{bmatrix} = \begin{bmatrix} A & B(\theta) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{Z}}_{k|k-1} \\ \hat{I}_{k|k-1} \end{bmatrix} + \mathbf{K}(\theta) \left[\mathbf{Y}_k - \hat{\mathbf{Y}}_k \right]$$

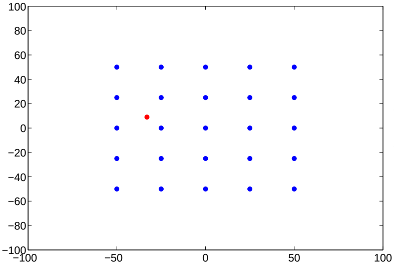
$$\hat{\mathbf{Y}}_{k+1} = D\hat{\mathbf{Z}}_{k+1}$$

- $\mathbf{K}(\theta)$ is steady-state Kalman gain
- Recursive Prediction Error (RPE) estimator [Ljung& Soderstrom '81]
  - Stochastic gradient method to minimize

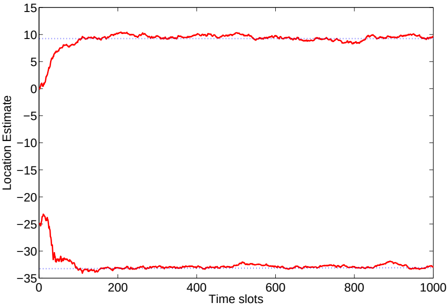
$$V(\theta) = \lim_{K\to\infty} \sum_{k=1}^{K} \frac{\mathrm{E}\left(\left(\mathbf{Y}_{k+1} - \hat{\mathbf{Y}}_{k+1}(\theta, Y^k)\right)^2\right)}{K}$$

- Issue: RPE requires gradient of $\mathbf{K}(\theta)$
- Modify RPE
  - Ignore term involving gradient of $\mathbf{K}(\theta)$ in RPE
  - Equivalent to extended Kalman filter with prior on θ

Simulation Results

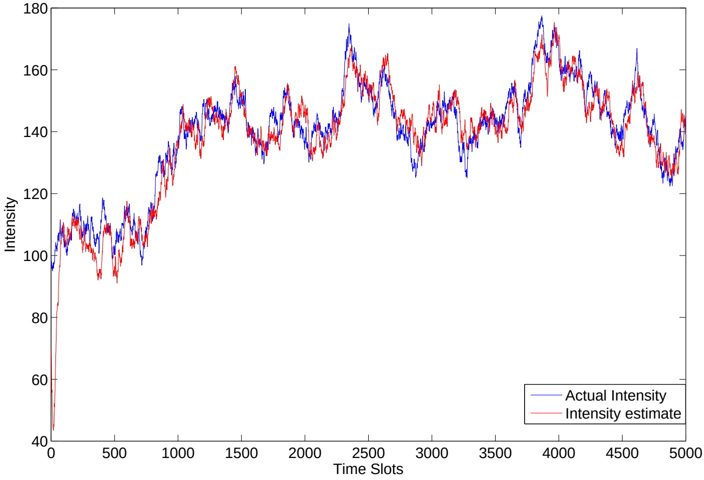


Simulation setup



Localization Results

Simulation Results—Intensity Tracking



Closest sensor is taken to be the initial point for the recursion

Discussion and Future Work

- Modified RPE procedure is also asymptoticaly consistent
- Applications
  - Pollution monitoring
  - Heat source monitoring
  - Brain wave monitoring
- Future Work
  - Change detection to detect and temporally ‘localize’ source
  - Multiple source localization and tracking
  - Distributed implementation
  - Validation on testbed