Localization and Intensity Tracking of a Diffusing Point Source Using a Sensor Network

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There is a wide class of events whose spatio-temporal distribution can be described by the diffusion equation:

$$\frac{\partial C(x,y,t)}{\partial t} = \alpha \nabla^2 C(x,y,t) + f(x,y,t),$$

Here, ∇^2 is the Laplacian differential operator, α is the medium diffusion coefficient and $f(x,y,t)$ is the source term. This diffusion equation models diverse physical phenomena such as the conduction of heat, the dispersion of plumes in air, the dispersion of odors through a medium and the migration of living organisms. Depending on the context, $C(x,y,t)$ is the temperature, the pollutant concentration, or the population at location (x,y) at time t, and it is completely determined from the diffusion equation once the initial and boundary conditions are specified.

We consider a network of spatially distributed sensors deployed to track the intensity of a diffusing source, governed by the above diffusion equation, whose location is fixed but unknown. This problem arises in a number of contexts, e.g., in air quality monitoring where information about a polluting source, such as its location and average intensity, are of interest. Each sensor makes local observations and shares it with a fusion center, where the goal is to obtain an accurate estimate of the location of the diffusing source and to track its intensity over time.

We propose a recursive algorithm that can be used at the fusion center, which takes in the latest concentration measurements as input and produces a refined location estimate and an estimate for the latest source intensity as the output. Using Galerkin's method we first develop a state space approximation for the system. We then use a modifield recursive prediction error algorithm to localize and track the source. We prove that the recursive estimator for the source location is asymptotically consistent. We also verify the performance of our algorithm through simulations.