Dynamic Generation of Operationally Acceptable Reroutes

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A method is proposed for generating operationally acceptable reroutes for air traffic management when a weather event is encountered. A decision support system that dynamically creates flight specific reroutes would aid traffic managers in efficiently maneuvering flights, but only if the reroutes provided were acceptable alternatives. As such, this research proposes a methodology that captures, through the network definition and metric evaluations, properties of reroutes that are both flexible and operationally acceptable. The methodological approach is reviewed and details are provided on the modeling formulation and solution algorithm employed. The results of the implementation of a simple example problem are investigated to identify how the different metrics defining reroute operational acceptability impact reroute quality.

I. Introduction

With the anticipated increase in airspace demand, managing congestion, especially during weather events, requires improved methods for assisting decision makers and increasing operational efficiency. Hazardous weather requires traffic managers to reroute flights passing through the weather, while balancing demand through sectors with reduced capacity or increased traffic volumes (resulting from other flights deviated from their original routes). Today’s methods for rerouting traffic are mostly manual: air traffic managers employ their expertise to handle a single flight, or entire flows of traffic are rerouted using National Playbook routes. These methods have been historically employed due to the complexity of defining an operationally-acceptable route in real time; however the reroute alternatives provided are limited. As the need to maximize all available airspace capacity is imperative, it is necessary to widen the set of operationally acceptable reroutes provided to decision makers or decision support systems.

Research in decision support systems for both the strategic and tactical timeframes models the impact on congestion due to weather and investigates how and when to act to maintain safe airspace throughput. When rerouting options are limited to traditional route alternatives such as National Playbook routes, and dynamic rerouting is not employed, the solutions derived provide a limited set of resolution options as compared to real tactical operations.

Including the capability to dynamically generate reroutes provides a larger solution space; however the additional computation expense can be significant. Even in a non real-time decision support system, such as NASA’s Airspace Concept Evaluation System, the dynamic rerouting performed must be simplified for computation considerations. The Constrained Airspace Rerouting Planner discussed in Reference 4 considers trajectories and weather constraints evolving in time and generates the shortest deviation reroute around the weather. However, considering only distance during the evaluation may produce a reroute that does not satisfy other traffic management concerns such as sector or traffic flow coordination.

Alternatively, Reference 5 develops a fast-time simulation that provides reroutes derived from a grid network overlaying the original flow. A reroute is defined as the shortest path through the network that avoids areas restricted by weather; however again the operational acceptability of these proposed reroutes remains in question. Using a fully populated grid network is a common modeling technique for rerouting as it is fast to implement and solve. In Reference 6, a similar modeling technique is used to generate a reroute for a general aviation (GA) aircraft in free flight. The assumption of free flight works well for GA flights, but does not necessarily extend for commercial flights under Air Traffic Control (ATC).

Reference 7 considers both a grid network and a waypoint-based network when defining reroutes for aircraft flows approaching the arrival terminal. The nodes in a waypoint-based network consist of existing waypoints, or
slight permutations. The reroutes are constructed using the Flow-Based-Route Planner\textsuperscript{8} with constraints governing interactions with other flows and therefore accurately simulate the restrictions of the transition approach environment. A similar approach was used in Reference 9 where the development of weather-specific Coded Departure Routes (CDRs) for pre-departure flights was investigated. Using the Flow-Based-Route-Planner, new CDRs were defined that better matched current weather predictions, but were still limited by the restrictive constraints placed on the network reroutes to ensure feasibility.

An alternative approach to defining a rerouting network is to represent only existing connections, as performed in terrestrial rerouting research. Reference 10 examines the problem of terrestrial vehicle rerouting where there is a prediction of future congestion along the original route. As terrestrial rerouting networks use only existing roadways, any reroutes found are guaranteed to be operationally feasible in the sense that the reroutes exclusively utilize permissible pathways to the destination.

As is seen from previous research into this area, one of the major difficulties in dynamic rerouting is capturing conventional guidelines for quality route development while recognizing that rerouting around a weather event may require maneuvers that deviate from these guidelines. Specifying end-to-end route options provides acceptable, but limited reroute alternatives for a given flight. Alternatively, defining a grid network produces a larger number of potentially operationally-unacceptable reroutes. Any decision support system developed for dynamic rerouting must bridge this divide by defining operationally-acceptable reroutes in real time to effectively aid decision makers in managing air traffic.

In this research we investigate the generation of operationally-acceptable reroutes by segmenting historically flown routes into fix-pair segments\textsuperscript{11}. Thus, all nodes in the network consist of existing fixes, and all arcs in the network consist of previously-flown connections between these fixes. Therefore, each individual arc in the network has some level, depending on usage, of operational acceptability. As reroutes are constructed from these arcs, additional metrics of operational acceptability evaluate the reroute in its entirety, and the set of reroutes that best meet these criteria are presented as potential alternatives to decision makers.

The purpose of this work is to define a methodology for constructing reroutes and evaluate the quality of these reroutes by the metrics of operational acceptability considered in order to explore how different considerations impact the solutions returned. Section II provides an overview of the greater research initiative that this work is a part of and discusses the specific definitions of operational acceptability considered. Section III provides a description of the problem formulation and models developed. The algorithm and metrics used to generate and evaluate reroutes are presented in Section IV. Section V presents an example rerouting problem and analyzes the impact of different modeling parameters on the operational acceptability of the reroutes generated. Section VI presents the conclusions drawn from this research and the ongoing work in this area.

II. Defining Operationally Acceptable Reroutes

The goal of this research is to define dynamic, flight specific reroutes for pre-departure or en-route flights that provide operationally-acceptable alternatives to decision makers. Defining operational acceptability however can be difficult as it requires extracting the essence of quality route design, as understood by subject matter experts (SMEs) into quantifiable and generic evaluation metrics or constraints. Issues of route geometry, the interaction of the route with current sector geometries and with other traffic makes this problem difficult to tackle.

This research implements a bi-level approach for defining operationally acceptable reroutes, as shown in Figure 1. The extraction of SME knowledge into metrics defining operational acceptability is described on the left side of Figure 1. The right side of Figure 1 shows the process of constructing reroutes, beginning with the network model that is developed using only historic fix-pair segments to produce reroutes. The set of reroutes generated are further evaluated by the operational acceptability metrics developed, and the best of these reroute alternatives are provided to decision makers. A set of reroute options, as opposed to a single best reroute, are returned as inevitably there will be air traffic requirements specific to a given situation that cannot be captured by the general operational acceptability metrics. Finally, iterations with SMEs will be performed during the prototyping stage of the research to ensure that the operational acceptability metrics defined produce useful reroute alternatives to decision makers.

The collaboration with SMEs which formed the basis of the operational acceptability metrics considered in this research was conducted as part of the larger research initiative comprising this work and described in greater detail by Reference 11. The remainder of this section provides an overview of these operational acceptability metrics.
A. Route Distance
The most frequently considered metric of reroute quality is the distance of the reroute as compared to the original route. As both the original route and the reroute alternatives are defined by a list of fixes along the route, the distance of a route is defined as the sum of the distances between each consecutive pair of fixes. The distance between a fix pair can be interpreted as the ground track distance, assuming both the wind and aircraft velocity profiles are provided. However, for the purposes of this research, we assume a simple great circle path between consecutive fix pairs in the route, and therefore define the reroute distance as the sum of the great circle arc distances between consecutive fixes. The operational acceptability of a reroute, however, lies in how the reroute changes the planned operations. As such, the reroute distance metric scales the distance of the reroute by the original route distance.

B. Flow Factor
The flow conformance of a route is a measure of how consistent the route is to historical routing. Although all arcs defined in the network were historically-flown, these arcs may not be historically used by flights traveling between a particular pair of regions. To compute this conformance measure, each arc in the network is assigned a flow factor. The flow factor, as defined in Reference 11, is computed as follows.

Approximately 4000 airports were grouped into 35 geographically distinct regions, and for each of these region pairs, the historical usage of each fix-pair segment was analyzed. The flow factor assigned to each fix-pair segment is not a count of usage, but a relative comparison of usage ranging between high usage (flow factor approaching zero) and almost no usage (flow factor approaching one). All fix-pair segments not used between a region pair are assigned a flow factor of one. The region pair for a given flight is determined by the flight’s departure and arrival airports, which in turn determine the flow factors of the fix-pair segments.

C. Lateral Deviation
The lateral deviation of a reroute is a measure of how differently the reroute will impact sectors as compared to the original route. Mathematically, we define the maximum lateral deviation of a reroute as the maximum distance of all fixes along the reroute at their closest approach to the original route. The methodology behind this computation can best be illustrated by a simple example as shown in Figure 2.

Figure 2 shows two routes that differ by only one fix, where the original route is in blue and the proposed reroute is in red. As such, the maximum lateral deviation of the reroute occurs at this dissimilar fix. However, the lateral deviation is defined as the closest approach. Therefore, we consider five measurements, DL1 to DL5. The first
three distances are the distances from each original fix to this new fix. The remaining two distances are orthogonal to the original two fix-pair segments on the original route. As we can see, the final distance ($D_{L5}$) can be ignored because the orthogonal connector does not intersect the original fix-pair segment. Of the four remaining distances, the minimum distance ($D_{L2}$) provides the closest approach to this new fix and therefore defines the maximum lateral deviation of the reroute.

As a metric of operational acceptability, the maximum lateral deviation of a route is further scaled using a piece-wise linear regression, described in Reference 11, to a zero-one scale. A score approaching zero is assigned to reroutes with small lateral deviations and a score approaching one is assigned to reroutes with large lateral deviations.

![Diagram of lateral deviation calculation]

III. Problem Formulation and Network Model Development

The dynamic rerouting problem considered in this research begins with the identification of a flight either pre-departure or en-route to the destination airport that must deviate from its route because of a weather event. To promote the generation of operationally-acceptable reroutes, the network is derived from a database of historically-flown fix segments. As such, the nodes of the network comprise the set of fixes identified in this historical fix-pair segment database and the connections between these nodes, or directed arcs, are taken from the connections that have previously been flown. This section explains in greater detail the mathematical equations and modeling assumptions required to formulate this problem.

A. The Rerouting Problem

The flight requiring a reroute is identified in the model by its route string, which consists of a set of fixes from the departure airport to the destination airport. As the flight can be anywhere between the departure airport and the arrival airport when a reroute is initiated, the deviation point and the rejoin point of the original route must be specified. The deviation point is the fix along the original route where the reroute can begin, which is any fix including the departure airport (when the flight is pre-departure), that occurs before the weather intersects the original route. Similarly the rejoin point is the final fix along the original route where the reroute reconnects to the original route and can be any fix along the original route after the weather event, including the destination airport.

B. Defining the Flight Specific Network

The fix segment database is defined for the entire National Airspace System (NAS); however for a given flight, only a subset of these segments is useful in defining the network. Therefore, the search area of the network is scoped as follows.
Given the route string of a flight, the deviation point, and the return point, a unique network is defined as an ellipse containing the allowable set of nodes and arcs. The parameters defining the ellipse, namely the origin, semi-major axis \((a)\) and semi-minor axis \((b)\) are defined as follows.

Given the deviation point of the route \((R_d)\) and the rejoin point of the route \((R_r)\), additional search buffers are appended to the great circle connecting these points to ensure that all reasonable connections are identified. The buffer length at the deviation point \((b_d)\) is defined to be 100 nm if the deviation point is the first point on the original route, corresponding to the departure airport; otherwise the buffer is 25 nm. This distinction recognizes that if the deviation point is the departure airport, a greater search area that encompasses multiple Standard Instrument Departures (SIDs) is necessary to evaluate alternative routes; however if the deviation point is a fix en-route, the search area for feasible connections, especially opposite the direction of travel, is more limited. The buffer length at the rejoin point \((b_r)\) is 100nm to provide a large area encompassing possible reconnections to the original route or destination airport.

Figure 3 provides a depiction of how the ellipse is constructed. The origin of the ellipse is defined as the midpoint of the line connecting the great circle formed between the buffer points \(b_d\) and \(b_r\). The semi-major axis of the ellipse \((a)\) is the distance between the origin of the ellipse and either buffer point. The semi-minor axis \((b)\) is defined as the maximum value of either half the semi-major axis or 100 nm greater than the lateral deviation of the original route from the great circle connecting the origin and destination airports. Here, lateral deviation refers to the furthest distance traveled along a line orthogonal to the great circle between the origin and destination airports, to a fix on the original route.

C. Defining Arc Costs

For each arc defined in the network, the cost represents the value of selecting that particular arc over another when constructing the set of best paths through the network. As the cost of the arc must be defined as an inherent property of the arc itself and independent of the path, it is desirable to identify factors that provide a measure of the
quality of selection. In this research, two factors have been identified as arc parameters that promote the definition of operationally acceptable paths: distance and flow factor.

The distance of an arc is defined as the great circle distance from the starting fix or node to the end fix or node. As stated previously, the distance computation is simplified to be independent of wind, but this is a modeling simplification and not a limitation of the methodology. As such, we define the distance of an arc between node $i$ and node $j$ as

$$d_{i,j} = 2 \cdot \sin^{-1} \left[ \sin \left( \frac{\theta_i - \theta_j}{2} \right) \right] + \cos(\theta_i) \cdot \cos(\theta_j) \cdot \sin \left( \frac{\varphi_i - \varphi_j}{2} \right)$$ (1)

where $\theta_i$ and $\varphi_i$ are the latitude and longitude of node $i$ respectively and $\theta_j$ and $\varphi_j$ are the latitude and longitude of node $j$, respectively.

The flow factor of an arc represents how often that arc is used when traveling from the general region of the departure airport to the general region of the arrival airport, as discussed in Section II.

In order to combine these two quantities into a single arc cost, the two terms must be scaled to be of a similar order of magnitude. As such, the distance of the arc is normalized by the distance of the original route ($d^R$), resulting in the distance cost of the arc from node $i$ to node $j$, as shown in Equation 2.

$$c_{i,j}^d = \frac{d_{i,j}}{d^R}$$ (2)

The flow cost of the arc ($c_{i,j}^f$) is defined as the flow factor of the arc from node $i$ to node $j$ between the departure airport region and the arrival airport region ($f_{i,j}^{DA}$) multiplied by the normalized distance of the arc ($c_{i,j}^d$), as shown in Equation 3.

$$c_{i,j}^f = f_{i,j}^{DA} \cdot \frac{d_{i,j}}{d^R}$$ (3)

The flow factor is scaled in this manner to emphasize that the longer the arc, the more important it is for the arc to have a low flow factor.

The total cost of an arc from node $i$ to node $j$ is then represented as

$$c_{i,j} = k_d \cdot c_{i,j}^d + k_f \cdot c_{i,j}^f$$ (4)

where $k_d$ is the weighting factor on the distance cost of an arc and $k_f$ is the weighting factor on the flow factor cost of an arc.

D. Accounting for Weather

One major cause of flights deviating from their intended route is the current or predicted presence of weather along the original route. Although it is possible to include a fully simulated weather event moving through the region in question, the “weather model” included in this paper is simply a restricted area region defined by a latitude and longitude grid. Specifically the weather region is defined to be centered at a point in space, and for our purposes, corresponds to a point along the original route. The size of the weather is determined by two parameters: the change in latitude ($\Delta \theta^w$) and the change in longitude ($\Delta \varphi^w$). The center point of the weather region ($\theta^w, \varphi^w$) can also be offset in either or both the latitude ($\delta \theta^w$) or the longitude ($\delta \varphi^w$) to analyze how a particular solution changes in response to a slightly different weather outcome.

The impact of the weather region on the network model and resulting solutions can be viewed in two ways. One method would be to allow reroutes to travel through the weather, but penalize the choice. In this case, the cost of any arc that penetrates the weather region would be augmented by a high penalty cost. The resulting paths that include arcs traveling through the weather could then be evaluated by decision makers or decision system in the context of the greater air traffic management situation to determine if the route is safe and if the other desirable qualities of the route outweigh the detraction of traveling through weather.

Alternatively, weather can be viewed as a forbidden region in the network where no nodes and no arcs exist. This definition extends to arcs that originate and terminate outside the restricted region, but pass through it. Modeling weather in this manner ensures that all options generated provide paths that deviate around the weather and alleviate the problem of traveling along the original route, and is the method implemented in this research.
IV. Generating Operationally Acceptable Solutions

This research defines a set of reroute alternatives, as opposed to a single option for each flight. Specifically, a set of alternatives that best meet different goals of operational acceptability, as discussed in Section II are returned. This allows decision makers to evaluate different route alternatives in the context of the larger traffic management situation and determine the best reroute for a flight.

In this section, we first describe the generation of the reroute alternatives using a k-shortest path approach. We then discuss how the aspects of reroute operational acceptability presented in Section II provide quantitative metrics to measure the overall quality of the route alternatives and inform decision makers. The overall flow of the algorithm is shown in Figure 4.

Figure 4. Flow Diagram for Computation of Operationally-Acceptable Reroutes

A. Generating k-shortest paths

The generation of a single or multiple paths through a network can be performed by implementing a variety of solution methods. Dijkstra’s Algorithm is the classic means of defining a path through the network, as shown in Reference 5. Dynamic Programming, which is a related technique, is used by Reference 13 for path construction. An A* search method is implemented to construct the paths defined by the Flow-Based-Route-Planner, described in Reference 8, and utilized in Reference 7 and Reference 9. Modifications to the A* search that include a heuristic estimation function to speed the shortest path search are employed by Reference 6 and Reference 14. Reference 10 uses a heuristic method known as multi-agent systems to search and update path performance in dynamic networks.

In this research we construct our set of reroute options using a k-shortest paths algorithm, as described by Reference 15, which employs a Dijkstra’s algorithm for constructing the shortest paths returned. Note that Reference 15 implements HEAP structures for improved efficiency in solving the k-shortest path problem, but this additional step was omitted in our implementation.

Given the source node $s$ and sink node $t$, where $s$ and $t$ correspond to the route deviation point ($R^d$) and route rejoin point ($R^r$), respectively, we seek the k-shortest paths connecting these nodes. To compute the set of paths, we select a node $i$ from the set of nodes yet to be examined ($S$) and find the shortest path from $s$ to $i$, which is defined
as $p^i_j$. As the definition of a path is a set of nodes we can therefore refer to the $j$-th node of $p^i_j$ as $p^i_j(j)$, where $p^i_j(1) = s$ and $p^i_j([p^i_j(j)]) = i$.

The efficiency in the k-shortest path approach lies in recognizing that for every node $m$, where $m \in p^i_j$, Bellman’s principle of optimality\textsuperscript{12} states that the shortest path from $s$ to $m$ is defined as

$$p^m_s = \{p^i_s(1), ..., p^i_s(i)\}$$

where $m = p^i_s(j)$. We then proceed to find the shortest path from every node $m$ in $p^i_j$ to the sink node $t$ ($p^m_t$). By adjoining the path $p^m_s$ and $p^m_t$ (and removing the repeated node $m$), we obtain the shortest path from node $s$ to node $t$ through node $m$. Node $m$ is then removed from $S$ and this process repeats until all nodes have been examined and $S = \emptyset$. The shortest k paths are returned.

B. Evaluating Operational Acceptability

Once the k shortest paths are defined, the operational acceptability metrics discussed in Section II can evaluate each reroute to determine its quality. In order to compare the different aspects of reroute quality, a multi-metric path objective function is defined that incorporates the three operational acceptability metrics, namely distance, flow factor, and lateral deviation.

The distance metric ($D^p$) is defined as the accumulated scaled arc distance from the deviation point to the rejoin point, as shown in Equation 6.

$$D^p = \sum_{(i,j) \in p} c^d_{i,j}$$

Here, the arc distance cost $c^d_{i,j}$ is as defined in Section II and $p$ is the reroute defined by the set of arcs. The flow factor metric ($F^p$) is the accumulated distance-weighted flow factor ($c^f_{i,j}$) defined in Section III and is provided in Equation 7.

$$F^p = \sum_{(i,j) \in p} c^f_{i,j}$$

The lateral deviation ($L^p$) of the reroute is the third operationally acceptable metric considered. The computation of the reroute lateral deviation and translation into the zero-one scale is as described in Section II and Reference 11.

Combining these three performance metrics into an overall objective function for the reroute operational acceptability yields the expression in Equation 8

$$C^p = w_d \ast D^p + w_f \ast F^p + w_l \ast L^p$$

where $w_d$, $w_f$, and $w_l$ are the relative weighting factors for reroute distance, weighted average flow factor, and scaled lateral deviation, respectively.

V. Results

The research presented in this paper examines how varying both the arc weighting factors and the operational acceptability weighting factors impacts solution quality. The goal of these trials is to inform parameter selection when utilizing this methodology as dynamic rerouting decision support system. This section presents the preliminary results for a single reroute and analyzes how changes in the weighting factors of the arc cost and the operational acceptability objective function affect the alternative reroutes provided to decision makers.

A. Example Problem Definition

The example problem considered is the rerouting of a flight traveling from O’Hare International Airport (ORD) to Houston International Airport (IAH). The example simulates the problem of weather obstructing the third fix in the original route, as shown in Figure 4. The weather is simulated as a 100 nm by 100 nm restricted airspace region. For this example, we consider the aircraft to already be en-route, somewhere between the first fix (ORD) and the second fix (FAM). As such, the deviation point of the route is at the second fix and we define the rejoin point as the destination airport (IAH). We then construct the network using all historic fix segments (shown in blue) within the ellipse search area.
Figure 5. Example Problem Route with Weather and Network Search Area

B. Weighting Factor Values

Sections III and IV define two sets of weighting factors: arc cost weighting factors and operational acceptability weighting factors. The arc cost presented in Equation 4 balances the distance cost of the arc with the weighted distance flow factor of the arc. By varying the arc cost weighting factors, we can potentially change the k-shortest path reroutes returned. We consider seven values for the arc cost weighting factors as shown in Table 1.

<table>
<thead>
<tr>
<th>Description</th>
<th>$k_d$</th>
<th>$k_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance Only Weighted</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Distance Heavily Weighted</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Distance Moderately Weighted</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Distance and Flow Factor Equally Weighted</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Flow Factor Moderately Weighted</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Flow Factor Heavily Weighted</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Flow Factor Only Weighted</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The second weighting factors introduced were in the operational acceptability objective function defined in Equation 8. The reroute operational acceptability weighting factors define the relative level of importance of operational acceptability metrics, namely normalized distance, normalized flow factor and normalized lateral...
deviation. Each reroute produced by the k-shortest path implementation is evaluated by Equation 8 with one of the weighting factor cases and associated weighting factor values presented in Table 2.

Table 2. Operationally-Acceptable Weighting Factor Values

<table>
<thead>
<tr>
<th>Description</th>
<th>( w_d )</th>
<th>( w_f )</th>
<th>( w_l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance Only</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Flow Factor Only</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Lateral Deviation Only</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Distance and Flow Factor</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Distance and Lateral Deviation</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Flow Factor and Lateral Deviation</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Distance, Flow Factor, and Lateral Deviation</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Heavier Weighting on Distance</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Heavier Weighting on Flow Factor</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Heavier Weighting on Lateral Deviation</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Heavier Weighting on Distance and Flow Factor</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Heavier Weighting on Distance and Lateral Deviation</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Heavier Weighting on Flow Factor and Lateral Deviation</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

C. Impact of Weighting Factor Combinations on Reroutes

For each set of arc weighting factors, we generate the first 200 shortest paths. These paths are then further culled to remove any path with a cycle, or repetition of nodes, as these are clearly unacceptable. The remaining reroutes are then evaluated against the operational acceptability objective function using each of the operational acceptability weighting factor cases defined in Table 2. The five reroutes that best meet the each weighted objective function are returned.

The first observation obtained from this analysis was that there was a high degree of repetition between the set of five best reroute alternatives provided for a given arc cost weighting. Especially in the cases where all three metrics were evaluated in the objective function, the set of alternatives was often the same and variations were either from the inclusion of a single alternative or simply a reordering of the alternatives. However, a change in order of the five best reroutes returned would not impact the information provided to the decision maker.

The next observation obtained from this analysis was that for the arc weighting factor cases where arc flow factor was weighted more significantly than the arc distance, similar reroute options were returned, regardless of how great the relative importance between the two factors. As such, using arc weighting factors that prioritized distance in the arc cost produced more distinct sets of alternatives. The remainder of this section discusses some of the results obtained to illustrate these observations.

1. The Five Shortest Paths

The five shortest paths are the lowest cost paths through the network, defined solely by the arc cost in Equation 4, and the specific set of arc weighting factors considered. Examining the first five reroute options returned provides insight into how arc costs correlate with operational acceptability. For the seven different arc weighting factor combinations considered in Table 1, we obtain only four different sets of reroute alternatives. Each unique set of reroute alternatives is labeled, as described in Table 3.

<table>
<thead>
<tr>
<th>Shortest Path Reroute Cases</th>
<th>Arc Weighting Factor Cases (Table 1)</th>
<th>Route Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Distance Only</td>
<td>Blue</td>
</tr>
<tr>
<td>Case 2</td>
<td>Distance Heavily Weighted</td>
<td>Green</td>
</tr>
<tr>
<td>Case 3</td>
<td>Distance Moderately or Equally Weighted</td>
<td>Red</td>
</tr>
<tr>
<td>Case 4</td>
<td>Flow Factor Moderately, Heavily or Only Weighted</td>
<td>Cyan</td>
</tr>
</tbody>
</table>

The four cases described in Table 3 each consist of five reroutes, as shown in Figure 6. By examining Case 1 we see that all of the reroutes use the same arc to deviate from the original route, but from there, multiple paths are taken to the rejoin point. In Case 2, the situation changes, as the reroutes use multiple options to travel from the deviation point, but only a single option to connect to the rejoin point. The reroutes shown in Cases 3 and 4 all
travel from the deviation point on one arc, albeit a different one from Case 1, but travel the same single arc to the 
rejoin point. Examining all cases in Figure 6, we notice that the reroutes of a given case are often differentiated by 
only a single node. As such, a single reroute case, or set of arc weighting factors, will provide alternatives with only 
minor variations. However, this may be useful for decision makers in situations where outside factors would make a 
particular alternative undesirable.

For each reroute case described in Table 3, the best reroute from each set of five is further evaluated on the three 
metrics of operational acceptability, as shown in Figure 7. Examining Figure 7 reveals, as expected, an increase in 
normalized route distance and a decrease normalized flow factor as the set of arc weighting factors increases the 
relative importance of arc flow factor over arc distance. Furthermore, we note that the five shortest paths defined by 
a distance only arc cost are the five shortest distance paths through the network. Similarly, the five shortest paths 
defined by a flow factor only arc cost are the five most flow conforming reroutes through the network. The impact 
on normalized lateral deviation provides insight into how the arc weighting factors influence an alternate metric that 
is not a component of arc cost. From Figure 7 we see that Case 2, which corresponds to heavily weighted arc 
distance, provides the best lateral deviation of the four reroutes. As such, Case 2 provides arc weighting factors that 
best achieve a compromise between the operational acceptability factors.

![Figure 6. Shortest Path Reroute Cases](image)

**a) Case 1**

**b) Case 2**

**c) Case 3**

**d) Case 4**
2. **Operational Acceptability Objective of Distance Only**

   When reroute alternatives are selected using only normalized distance as the objective function, the five shortest paths found in the network are returned. When arc distance is the only component of the arc cost, these five paths correspond to the shortest reroute alternatives from the deviation to the rejoin point of the original route. However, as flow factor is weighted into the arc cost, the k-shortest path algorithm will provide reroutes based on the cost as calculated using the specific weighting factors considered. As such, evaluating the normalized distance metric for each set of reroute alternatives shows the impact of the arc weighting factors on reroute distance.

   Table 4. Reroute Options for Objective Function of Only Normalized Distance

<table>
<thead>
<tr>
<th>Normalized Distance Only Reroute Cases</th>
<th>Arc Weighting Factor Cases (Table 1)</th>
<th>Route Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Distance Only</td>
<td>Blue</td>
</tr>
<tr>
<td>Case 2</td>
<td>Distance Heavily Weighted</td>
<td>Green</td>
</tr>
<tr>
<td>Case 3</td>
<td>Distance Moderately Weighted</td>
<td>Red</td>
</tr>
<tr>
<td>Case 4</td>
<td>Distance and Arc Flow Factor Equally Weighted</td>
<td>Cyan</td>
</tr>
<tr>
<td>Case 5</td>
<td>Arc Flow Factor Moderately Weighted</td>
<td>Magenta</td>
</tr>
<tr>
<td>Case 6</td>
<td>Arc Flow Factor Heavily or Only Weighted</td>
<td>Yellow</td>
</tr>
</tbody>
</table>

   The seven arc weighting factor combinations, produce six distinct sets of reroute alternatives when normalized distance is the only metric in the objective function. The repeated set of five best reroute alternatives occurs when arc flow factor is either heavily or only weighted in the arc cost. Table 4 provides the distinct cases and the corresponding weighting factors attributed to them, for the reroute alternatives shown in Figure 8.

   Examining Figure 8 reveals that there is a significant distinction between the reroutes obtained using arc weighting factors where distance is prioritized and the reroutes produced using arc weighting factors where flow factor is prioritized. Specifically, for the reroutes in Case 1 and Case 2, we see the same single arc is used to travel from the deviation point, and a variety of options to connect to the rejoin point. Case 3 and Case 4 expand the number of options used to travel from the deviation point and maintain the number of connections to the rejoin point. Case 5 and Case 6 retain the flexibility of multiple connection options from the deviation point, but all reroutes use a single connection to the rejoin point.
Figure 9 shows how the best reroute from each case performs against each of the operational acceptability metrics. Here, we note that Figure 9 provides only five unique cases as Case 3 and Case 4 have the same best reroute alternative. Examining Figure 9 reveals that as the arc weighting factors prioritize flow factor relative to distance, the normalized distance increases and the normalized flow factor decreases. In addition, the lateral...
deviation is the lowest for the reroute option of Case 3 and Case 4. As such, we find that the best compromise between all operational acceptability metrics is provided by Cases 3 and 4, where arc distance is moderately or equally weighted with arc flow factor.

3. Operational Acceptability Objective of Flow Factor Only

When reroute alternatives are selected using only normalized flow factor as the objective function, the five most flow conformant paths found in the network are returned. When arc flow factor is the only component of the arc cost, these five paths correspond to the five most utilized reroute alternatives connecting the deviation and rejoin point of the original route. However, as distance is weighted into the arc cost, the k-shortest path algorithm will provide reroutes based on the cost as calculated using the specific weighting factors considered. As such, evaluating the normalized flow factor metric for each set of reroute alternatives shows the impact of the arc weighting factors on reroute flow conformance.

The seven arc weighting factor combinations, produce four distinct sets of reroute alternatives when normalized flow factor is the only metric in the objective function. Table 5 describes the weighting factor combinations for each case. Here, we see that for all arc weighting factor cases, where arc flow factor is prioritized over arc distance, the same set of reroute alternatives is returned. Figure 10 provides a visualization of the reroutes in each case.

![Figure 9. Metric Comparison for Best Distance Only Reroutes Under Varying Arc Weighting Factor](image)

Table 5. Reroute Options for Objective Function of Only Normalized Flow Factor

<table>
<thead>
<tr>
<th>Flow Factor Only Reroute Cases</th>
<th>Arc Weighting Factor Cases (Table 1)</th>
<th>Reroute Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Distance Only</td>
<td>Blue</td>
</tr>
<tr>
<td>Case 2</td>
<td>Distance Heavily Weighted</td>
<td>Green</td>
</tr>
<tr>
<td>Case 3</td>
<td>Distance Moderately or Equally Weighted</td>
<td>Red</td>
</tr>
<tr>
<td>Case 4</td>
<td>Flow Factor Moderately, Heavily or Only Weighted</td>
<td>Cyan</td>
</tr>
</tbody>
</table>

Examining Figure 10 reveals that every case except Case 1 (distance only) uses the same single arc to travel from the deviation point and every case uses the same single arc to connect to the rejoin point. Thus, as arc flow factor is increased, the only change in reroute options is how we travel between these connecting arcs. Furthermore, Case 3 highlights the need for additional metrics of operational acceptability. Examining one of the reroute options in Case 3, we see a large angle turn is planned in order to maintain routes using low flow factor arc; however this turn is not operationally-acceptable and should be excluded from options presented in to decision makers.

Figure 11 shows how each of the three unique best reroute options (Case 3 and Case 4 have the same best reroute) measure against the operational acceptability metrics. Examining Figure 11 reveals that Case 2 provides the
Figure 10. Normalized Flow Factor Only Reroute Options

Figure 11. Metric Comparison for Best Flow Factor Only Reroutes Under Varying Arc Weighting Factor
The best compromise between the three operational acceptability metrics. The normalized distance of the Case 2 reroute is only slightly higher than that of Case 1, which only considers arc distance in the arc cost. Similarly, the normalized flow factor of Case 2 is only slightly higher than that of the reroute in Cases 3 and 4, which is the minimum normalized flow factor route available through the network.

4. **Operational Acceptability Objective of Lateral Deviation Only**

The previous two operational acceptability weighting factor analyses considered distance and flow factor, which are the two components of the arc costs. Normalized lateral deviation, however, is an operational acceptability metric that is not represented in the arc cost. By examining the reroute alternatives selected using only this metric in the objective function, we can analyze the impact of varying the arc weighting factors on an unrelated metric of operational acceptability.

Table 6. Reroute Options for Objective of Normalized Lateral Deviation Only

<table>
<thead>
<tr>
<th>Lateral Deviation Only Reroute Cases</th>
<th>Arc Weighting Factor Cases (Table 1)</th>
<th>Reroute Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Distance Heavily or Only Weighted</td>
<td>Blue</td>
</tr>
<tr>
<td>Case 2</td>
<td>Distance Moderately or Equally Weighted</td>
<td>Green</td>
</tr>
<tr>
<td>Case 3</td>
<td>Flow Factor Moderately or Heavily Weighted</td>
<td>Red</td>
</tr>
<tr>
<td>Case 4</td>
<td>Flow Factor Only Weighted</td>
<td>Cyan</td>
</tr>
</tbody>
</table>

Figure 12. Normalized Lateral Deviation Only Reroute Options
There are four distinct sets of reroute options produced when only lateral deviation is considered in the objective, which are described in Table 6. The set of reroutes generated for each of these cases are shown in Figure 12. Again, we see that the Case 1 reroutes all employ the same arc to travel from the deviation point, but utilize multiple pathways to reach the rejoin point. In addition, we see that as the arc flow factor weight is increased, we obtain multiple pathways from the deviation point, but only a single pathway to the rejoin point. Finally, with the flow factor only arc weighting, we obtain a single deviation point arc and single rejoin point arc. Again, we notice that as flow factor is prioritized, the number of reroutes showing a high turn angle increases. Examining Figure 12 reveals that most reroute alternatives for Case 2 through Case 4 are very similar, having only slight variations between each other. If we examine how the best reroute for each case measures against the operational acceptability metrics, as shown in Figure 13, we see that there are only three distinct reroutes to compare.

Examining Figure 13 reveals that all solutions produce the same best lateral deviation. As the arc weighting factors vary between arc distance only and arc flow factor only, there is the expected change in the corresponding operational acceptability factors. We see here that for all arc weighting factors where arc flow factor is prioritized (Cases 3 and 4), we have a large increase in the normalized distance without much improvement in normalized flow factor, as compared to Case 2. Similarly, the decrease in normalized flow factor from Case 1 to Case 2 is significant and the corresponding increase in normalized distance is small. As such, all operational acceptability metrics seem to be best satisfied with the reroute option of Case 2.

5. Operational Acceptability Objective Evaluating All Metrics

<table>
<thead>
<tr>
<th>All Metrics Equally Weighted Reroute Cases</th>
<th>Arc Weighting Factor Cases (Table 1)</th>
<th>Reroute Colors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Distance Only Weighted</td>
<td>Blue</td>
</tr>
<tr>
<td>Case 2</td>
<td>Distance Heavily Weighted</td>
<td>Green</td>
</tr>
<tr>
<td>Case 3</td>
<td>Distance Moderately or Equally Weighted</td>
<td>Red</td>
</tr>
<tr>
<td>Case 4</td>
<td>Flow Factor Moderately Weighted</td>
<td>Cyan</td>
</tr>
<tr>
<td>Case 5</td>
<td>Flow Factor Heavily or Only Weighted</td>
<td>Magenta</td>
</tr>
</tbody>
</table>
Having examined the impact of the operational acceptability metrics individually, we now consider the objective function using an equal weighting of all three metrics. For this set of operational acceptability weighting factors, there are five unique sets of reroutes, which are described in Table 7. Figure 14 presents the reroute options for each of these cases.

Figure 14. Equally Weighted Metric Reroute Options
Examining Figure 14 shows reroute cases that are similar to the reroutes produced by the lateral deviation only objective, shown in Figure 12. However, unlike the previous results, we notice that all cases show multiple pathways from the deviation point. This is a desirable quality of the reroute options as flexibility is necessary near the weather. However, some of the reroutes maintain the undesirable quality of high turn angles, especially when arc flow factor is prioritized in the arc cost. As the desirability to conform to the flow overrides distance considerations, routes may feature segments that are not necessarily in the overall flight direction, causing large angle turns. Again, these results suggest the need for an additional operational acceptability metric.

Having examined the impact of the operational acceptability metrics individually, we now consider the objective function using an equal weighting of all three metrics. For this set of operational acceptability weighting factors, there are five unique sets of reroutes, which are described in Table 7. Figure 14 presents the reroute options for each of these cases. Examining Figure 14 shows that the reroute options for Case 4 and Case 5 produce very similar set of alternatives, differentiated by only one reroute option.

The previous analyses for the three individual operational acceptability weighting factors provided a lower bound on the best metric values. As such, we can now measure how the reroutes evaluated in Figure 14 compare to the best metric values possible. The best normalized distance value was obtained using a distance only arc cost and evaluating only distance in the operational acceptability objective function (Figure 9). The best normalized flow factor value was obtained using a flow factor only arc cost and evaluating on normalized flow factor in the operational acceptability objective function (Figure 11). The best normalized lateral deviation found (as there is no guarantee it is the minimum), was obtained using a lateral deviation only operational acceptability objective function with any arc cost weighting (Figure 13). Figure 16, shows the reroute options provided in Figure 15 as a percentage over the best possible values obtained.

![Chart showing normalized route distance, normalized flow factor, and normalized lateral deviation values for different cases.](image)

**Figure 15.** Metric Comparison for Best Lateral Deviation Only Reroutes Under Varying Arc Weighting Factor

Examining Figure 16 reveals that only normalized distance and flow factor are presented, as the lateral deviation for each of the reroutes in Figure 15 is the same as the minimum lateral deviation. Thus, examining the trade-off between distance and flow factor as percentage of minimum value, we clearly see that the best compromise is obtained by Case 3, which corresponds to an arc weighting factor of distance moderately or equally weighted.
VI. Conclusions

The research presented in this paper describes a methodology for designing operationally acceptable flight specific reroutes. Dynamically generating reroutes is challenging as it requires capturing aspects of quality route design that are generalizable to any flight. As such, the methodology proposed in this research begins with consultations with SMEs to define metrics of reroute operational acceptability. In addition, the network model is developed using only operationally acceptable segments of historic routes. Finally, acknowledging that there will be aspects of a given traffic scenario that cannot be captured by the metrics developed, the methodology is proposed as a decision support tool for traffic managers or decision makers, where multiple reroute options for each flight will be generated for further evaluation and selection.

The purpose of this paper was to analyze how varying the relative importance of both the arc cost components and the operational acceptability metrics impact solution quality. The goal of this analysis was to inform parameter selection. From the single example considered, we saw that the reroutes were often insensitive to variations in the arc weighting factors if arc flow factor was prioritized over arc distance by any ratio. In addition, the analyses showed that moderately prioritizing arc distance in the arc cost produced results that performed better in the three operational acceptability metrics.

Although, only a single reroute example was considered in this paper, the results pointed towards areas of future research. A more detailed analysis of arc weighting factors near the moderately weighted distance case is desirable to determine if there is a more suitable ratio. In addition, further examples would need to be evaluated to determine if this is simply a property of the network constructed around this specific route, or if this is parameter selection is extensible to multiple problems.

The next step of this research would be to consult with SMEs and decision makers to confirm that the reroutes generated are in fact operationally acceptable and useful in a decision support framework. Completing the feedback with SMEs would help to identify any other metrics necessary to evaluate the reroutes constructed.

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References