Unambiguous Tracker for GPS Binary-Offset-Carrier Signals

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BIOGRAPHY

Ronald Fante is a Fellow of The MITRE Corporation and holds a Ph.D. from Princeton University. He has published 150 journal articles and one textbook. He has been elected a Fellow of the IEEE, the Optical Society of America, and the Institute of Physics. He has received the IEEE Third Millennium Medal, shared the IEEE Schelkunoff Prize (twice), the USAF Marcus O’Day Prize, and the MITRE Best Paper Award (twice). He has served as IEEE Distinguished Lecturer and was Editor in Chief of the IEEE Transactions on Antennas and Propagation.

ABSTRACT

Binary-offset-carrier waveforms have autocorrelation functions containing multiple peaks, thus leading to potential tracking ambiguities. We have developed a discriminator that is monotonic over the entire range of delays where the correlation function has significant support; thus eliminating ambiguities.

The price paid is the need for additional gates, some loss in sensitivity, and some additional computations.

INTRODUCTION

It is now well known that the binary offset-carrier (BOC) waveform proposed [1] for the future Global Positioning System (GPS) has a correlation function containing multiple peaks with magnitudes that are nearly equal to the magnitude of the central peak, as is evident from Figure 1. This means that a signal tracker can lock onto the wrong peak, thus producing a tracking error. Fine and Wilson [2] have devised a novel approach known as “bump-jump” to remove this ambiguity. Their approach compares the outputs in very-early and very-late gates with the prompt gate output to determine if the prompt gate is tracking the largest peak; if not, the tracker jumps to the largest of the other two gates and repeats the procedures until the prompt gate has the largest output. In this report, we will present an alternative approach that is in some sense a variant of the bump-jump method. Our approach uses samples of the correlation function at 2N different times, and then forms an appropriate discriminator that removes all ambiguities. In particular, we are able to derive a discriminator that is monotonic over the entire range of delays where the correlation function has significant support. That is, that discriminator is monotonic over an interval of approximately 336ns.

THEORY OF OPERATION

The classic early-late gate discriminator that is used for conventional correlation functions has the error function

$$E_n(\tau) = |R(\tau + D/2)|^n - |R(\tau - D/2)|^n$$

where $n$ is usually either 1 or 2, $D$ is the spacing between the early and late gates, the prompt gate has $D = 0$ and evaluates $|R(\tau)|^n$ and $R(\tau)$ is the cross correlation function. A typical plot of $E_2(\tau)$ in the noise-free limit is shown in Figure 2 for the correlation function shown in Figure 1.
The discriminator in Equation (1) is sensitive to signal amplitudes, and in order to remove this sensitivity the normalized discriminator

\[ S_n(\tau) = \frac{\left| R(\tau + \frac{D}{2}) \right|^n - \left| R(\tau - \frac{D}{2}) \right|^n}{\left| R(\tau + \frac{D}{2}) \right|^n + \left| R(\tau - \frac{D}{2}) \right|^n} \] (2)

is often used, but this again has multiple ambiguities for the BOC.

\[ \sum_{m=1}^{2N} a(m) \left| R \left[ \left( m - N - \frac{1}{2} \right) D + \tau \right] \right|^n \]

and Equation (2) is replaced by

\[ \sum_{m=1}^{2N} a(m) \left| R \left[ \left( m - N - \frac{1}{2} \right) D + \tau \right] \right|^n \]

Our goal is to choose the coefficients \( a(m) \) so as to obtain an error function that is linear with the form

\[ E_n(\tau) = \alpha \tau \text{ or } S_n(\tau) = \alpha \tau \] for \(-T_i < \tau < T_i\),

where \( \alpha \) is an arbitrary constant and \( 2T_i \) is the region of support of the correlation function \( R(\tau) \). From Figure 1 we see that \( T_i \approx 3.5T \).

Let us first concentrate on the \( E_n(\tau) \) error function. We can derive the desired coefficients by minimizing the mean square error between \( E_n(\tau) \) and \( \alpha \tau \). That is, we minimize

\[ \delta = \frac{1}{2T_i} \int_{-T_i}^{T_i} d\tau \left| E_n(\tau) - \alpha \tau \right|^2 \]

where we can choose \( \alpha \) such that

\[ \sum_{m=1}^{2N} a(m)^2 = 1 \]

Let us define the vectors

\[ a^T = [a(1) a(2) \ldots a(2N)] \]

and

\[ R_n^T = \left[ \left| R \left( \left( N - \frac{1}{2} \right) D + \tau \right) \right|^n \ldots \left| R \left( \left( N - \frac{1}{2} \right) D + \tau \right) \right|^n \right] \]

Then, it is readily shown that \( \delta \) is minimized if the coefficient vector \( a \) satisfies

\[ a = \frac{1}{\gamma} \Phi^{-1} g \] (9)

where

\[ \Phi = \frac{1}{2T_i} \int_{-T_i}^{T_i} R_n R_n^T d\tau \]

\[ g = \frac{1}{2T_i} \int_{-T_i}^{T_i} \tau R_n(\tau) d\tau \]

and

\[ \gamma^{-2} = \left( \Phi^{-1} g \right)^T \left( \Phi^{-1} g \right) \] . (12)

Using the coefficients obtained in Equation (9), one can readily calculate the error function \( E_n(\tau) \) for any \( n, N, \) and \( D \).

In Figure 3, we show the \( E_2 \) error function using 16 taps for three different tap spacings. A similar response was achieved using 14 taps, but 12 taps were not sufficient to achieve the goal of a linear, monotonic response for \( |\tau| < 3.5T \).

We would like to achieve a monotonic response with fewer than 14 or 16 taps. Therefore, we have also considered the \( S_n \) discriminator defined in Equation (4).
Unfortunately, there is no simple method to determine the coefficients, \( a(n) \), that produce the minimum mean square error between \( S_n(\tau) \) and \( \alpha \tau \), so we choose the \( a(m) \) coefficients as

\[
a(m) = 1 + \beta (m - N)/N, \quad m = 1 \text{ to } N
\]

\[
= -1 + \beta (m - N + 1)/N, \quad m = N + 1 \text{ to } 2N \tag{13}
\]

where \( \beta \) is an arbitrary constant. The values of \( a(m) \) in Equation (13) are asymmetric and give \( S_n(0) = 0 \), as required. In Figure 4, we show the \( S_2 \) error function for \( \beta = -2 \) when 10 and 12 taps are used. Note that using 12 taps for and \( \beta = -2 \) gives a monotonic error function over the entire range \( |\tau| \leq 3.5T \) where \( R(\tau) \) has significant support (A similar conclusion is true for \( \beta = -1 \). Thus, the \( S_2 \) discriminator can achieve our goal using only 12 taps.

Thus, we have shown that a 12 tap \( S_2 \) discriminator or a 14 tap \( E_2 \) discriminator can achieve our goals in the absence of noise. In the next section, we consider how noise affects the estimate of delay.

**SENSITIVITY LOSS**

Although we have derived a monotonic error function, we now demonstrate that we pay a price in loss of sensitivity, relative to the classical two-tap discriminator. If we include the effects of noise, it is possible to show [3] that the standard deviation \( \sigma \) of the time-of-arrival error is

\[
\sigma = \frac{KT}{(S/N)^{1/2}} \tag{14}
\]

where S/N is the signal-to-noise ratio at the correlator output, \( T=48.88\text{ns} \) and values of \( K \) are presented in Table 1.

<table>
<thead>
<tr>
<th>Type</th>
<th>Number of Taps</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_2 )</td>
<td>2</td>
<td>0.35</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>12</td>
<td>1.32</td>
</tr>
<tr>
<td>( E_2 )</td>
<td>14</td>
<td>3.5</td>
</tr>
</tbody>
</table>

From the results in Table 1, we see that the 12 tap \( S_2 \) discriminator is 3.77 times less sensitive than the 2 tap \( S_2 \) discriminator, and the 14 tap \( E_2 \) discriminator is even less sensitive. This is the price paid for the monotonic behavior of the 12 tap \( S_2 \) and 14 tap \( E_2 \) discriminators. This suggests we could combine the 12 tap \( S_2 \) discriminator (or the 14 tap \( E_2 \)) with a 2 tap \( S_2 \) (or \( E_2 \)) discriminator to get the best of both worlds: unambiguous behavior plus good sensitivity at \( \tau = 0 \) (at the prompt gate). For example, we could use the 12 tap \( S_2 \) discriminator to get us to the correct correlation peak and the 2 tap \( E_2 \) or \( S_2 \) discriminator for the “fine tuning”. For this procedure the gates at \( t/T = \pm 0.175 \) are the conventional early and late gates, whereas the
gates at \( t/T = \pm 0.525, \pm 0.825, \pm 1.225, \pm 1.575 \) and \( \pm 1.925 \) are multiple very early and very late gates.

It should be noted that the relatively large value of \( K \) in Table 1 for 14 tap \( E_2 \) discriminator suggests that a better design procedure (than requiring that \( E_2(\tau) = \alpha \tau \) ) would be to require in Equation (5) that \( E_2(\tau) = \alpha_1 |\tau| \) for \( |\tau| < T \) and \( E_2(\tau) = \alpha_2 |\tau| \) for \( |\tau| > T \), where \( \alpha_1 > \alpha_2 \). The results of this design are shown in Figure 5. This design reduces the sensitivity factor \( K \) from 3.5 to 2.5.

**Figure 5.** Comparison of Linear and Piecewise Linear \( E_2 \) Errors

We next need to determine the signal-to-noise ratio one can expect, and if it is sufficient to avoid the aforementioned errors. The carrier-to-noise ratio in dB-Hz is given by

\[
\frac{C}{N_o} = P_s + G - 10 \log_{10}(kT_o) - F - L \quad (15)
\]

where \( P_s = \) received power \((dBW)\), \( G = \) receive antenna gain in satellite direction, \( k = \) Boltzmann’s constant, \( T = 293^\circ \)K, \( F = \) receiver noise figure and \( L = \) loss. Suppose the minimum value of received power for the BOC at L1 is \(-157dBW\). If the receiver antenna has a cosine power pattern, then its average gain over the region from 15° above the horizon to zenith is approximately 1dBi. If the receiver noise figure is 4dB and the losses are 2dB, then from Equation (15) we see that the minimum value of \( C/N_o = 42 \) dB-Hz.

The signal-to-noise ratio after one integrate-and-dump cycle is \( CT_o/N_o \), where \( T_o = \) integration interval.

The phase lock loop integrates \( M = (B_L T_o)^{-1} \) of these cycles, so the output signal-to-noise ratio is

\[
\frac{S}{N} = \frac{CT_o}{N_o} \frac{1}{T_o B_L} = \frac{C}{B_L N_o} \quad (16)
\]

where \( B_L \) loop bandwidth

If Equation (16) is used in (14), we obtain the results in Table 2 for the tracking accuracy for some typical loop bandwidths when \( C/N_0 = 42 \) dB.

**Table 2.** Tracking Accuracy of 12-TAP \( S_2 \) and 14 Tap \( E_2 \) Discriminators

<table>
<thead>
<tr>
<th>( B_L ) (Hz)</th>
<th>( \sigma_\tau ) (ns) for ( S_2 )</th>
<th>( \sigma_\tau ) (ns) for linear ( E_2 )</th>
<th>( \sigma_\tau ) (ns) for piecewise - linear ( E_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.46</td>
<td>1.2</td>
<td>0.85</td>
</tr>
<tr>
<td>3</td>
<td>0.89</td>
<td>2.3</td>
<td>1.6</td>
</tr>
<tr>
<td>5</td>
<td>1.15</td>
<td>3</td>
<td>2.1</td>
</tr>
<tr>
<td>8</td>
<td>1.45</td>
<td>3.5</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Thus, the 12-tap \( S_2 \) tracker or the 14-tap \( E_2 \) tracker can typically achieve accuracies on the order of 1 to 3 feet. If smaller accuracies are required then a 2-tap adjunct discriminator must be used.

**SENSITIVITY TO CODE DISTORTION**

We have evaluated the effect of uncorrected dispersive errors on the performance of the \( E_2 \) and \( S_2 \) trackers developed here. The details are omitted, but the results are summarized in Table 3. Based on the results in Table 3, we require that the group delay in the receiver be equalized, so that the maximum group delay errors are less than 3 ns across the operating band.

We also examined the effect of ionospheric dispersion on the trackers. The ionosphere has a negligible impact.
Table 3. Effect of Dispersive Errors on $E_2$ Error Function

<table>
<thead>
<tr>
<th>Type</th>
<th>Effect on Slope</th>
<th>Effect on Null Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude – Linear</td>
<td>Increase</td>
<td>None</td>
</tr>
<tr>
<td>Amplitude – Quadratic</td>
<td>Decrease</td>
<td>None</td>
</tr>
<tr>
<td>Amplitude – Cubic</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Amplitude – Sinusoidal Ripple</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Group Delay – Linear</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Group Delay – Quadratic</td>
<td>None</td>
<td>May be Significant unless Error $&lt;3$ns</td>
</tr>
<tr>
<td>Group Delay – Cubic</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Group Delay – Sinusoidal Ripple</td>
<td>None</td>
<td>May be Significant unless Error $&lt;3$ns</td>
</tr>
</tbody>
</table>

**SUMMARY**

We have demonstrated that by a judicious choice of the number of taps, tap spacing and tap coefficients we can synthesize a tracker that is monotonic over the entire range of delays where the BOC autocorrelation function has significant support. This allows one to track BOC Autocorrelation maximum to an accuracy of order of 0.5 to 3 nanoseconds (depending on loop bandwidth) and without fear of the tracker moving in the wrong direction or stalling in a local minimum. If finer accuracy is needed, 2-taps at $t/T = \pm \delta$, where $\delta \leq 0.25$ can be added.

**LIST OF REFERENCES**

