Maximum Likelihood Approach to Joint Array Detection/Estimation

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The problem of detecting the number of (possibly correlated) narrowband sources of energy and estimating the direction of arrival (DOA) of each detected source using data received by an array of sensors is investigated. A combined detection and estimation approach based on the likelihood function (LF) is used. The approach is motivated by detection theoretic considerations instead of information theoretic criteria and uses maximum likelihood (ML) signal-to-noise ratio (SNR) estimates of hypothesized sources as detection statistics rather than maximizing the LF with a penalty function. Performance comparisons are made to unstructured and structured techniques based on Akaike information theoretic criteria (AIC), minimum description length (MDL), and Bayesian predictive density (BPD) approaches as well as the Minimum Variance Distortionless Response (MVDR) approach. An important feature that distinguishes this approach is the ability to trade off detection and false alarm performance, which is not possible with the other LF-based approaches, while achieving performance levels comparable to or exceeding the LF-based and MVDR approaches.

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I. INTRODUCTION AND OVERVIEW

The problem of detecting and localizing narrowband sources of energy using an array of sensors is an important problem in many fields including radar, sonar, wireless communications, and medical imaging. The objective here is to determine the number of sources and the direction of arrival (DOA) of each detected source.

The problem can be decomposed into a detection-only procedure, where the number of sources, or model order, is estimated, followed by an estimation-only procedure, where the source DOAs are estimated. Unstructured model order estimation techniques [26] such as those based on Akaike information theoretic criteria, (AIC) [4] or minimum description length (MDL) [17], [20] may be used for the detection-only procedure, and direction finding techniques such as maximum likelihood (ML) [9, 10, 14], minimum variance distortionless response (MVDR) [11], multiple signal classification (MUSIC) [19], estimation of signal parameters by rotational invariance technique (ESPRIT) [18], and weighted subspace fitting (WSF) [24] (to name a few, see [22, Ch. 8–9]) may be used for the estimation-only procedure. Most estimation-only procedures require knowledge of the number of sources, and performance can be adversely affected if this knowledge is inaccurate. Alternatively, a combined detection and DOA estimation procedure can be used, such as those based on structured AIC (SAIC) or MDL [25], Bayesian predictive densities (BPDs) [13], or MVDR [11]. The approach presented here falls into this last category of combined detection and DOA estimation procedures. The focus is on the detection aspects of the problem, where the main contribution lies. However, estimation of the source DOAs and powers is an integral part of the procedure.

At the highest level, most of the above approaches have as their basis the maximization of the likelihood function (LF) with respect to parameters given a set of observed data. The attractiveness of an ML approach as a general estimation approach is well known [23]. However, an unconstrained ML approach does not work for estimation of dimensionality parameters. It will always lead to the selection of the highest possible dimension. Excellent discussions of this phenomena for generic model order estimation problems are given in [4], [17], and [20]. In response, each presents a solution in terms of maximizing a score function, which is the sum of a data term and a penalty function. The data term is the unconstrained log likelihood function (LLF) and is common to all. The penalty function includes the number of free adjusted parameters in the model. The purpose of the penalty function is to counterbalance the propensity of an unconstrained ML approach to overestimate the dimensionality. Differing approaches are taken to the penalty function. The criterion for the penalty function in [4] and [17] is based on information theory. The criterion for the penalty function in [20] is based on a Bayesian approach and is asymptotically equivalent to [17].

The unstructured AIC (UAIC) and unstructured MDL (UMDL) detectors for the narrowband source array problem [26] are derived using the results from [3] with the penalty functions of [4], [17], and [20]. They use an unstructured data model (i.e., not specific to the array processing problem) and analyze eigenvalues of the sample covariance matrix for the "best" grouping of signal subspace and noise subspace eigenvalues. Eigenvectors, containing DOA information, are discarded. As a result, the UAIC and UMDL detectors do not fully exploit the physical and statistical structure of the data in the array processing problem. UAIC is known for its aggressive estimates (overestimation of model order) and UMDL is known for its conservative estimates (low detection sensitivity) and asymptotic consistency [26, 34]. Neither has any mechanism to adjust the probability of false alarm (P_f). Many theoretical studies and variations on these techniques for a variety of signal and array models have been proposed, including [1], [2], [27]–[29], [31]–[34].

The SAIC and structured MDL (SMDL) approaches [25] and the BPD approach [13] use a structured data model specific to the correlated signals array processing problem. To that end, a general signal covariance matrix is assumed. The signal covariance matrix and noise power are estimated via a signal and noise subspace decomposition. SAIC, SMDL, and BPD have a common data term (the unconstrained LLF) which must be maximized over DOAs. The difference among SAIC, SMDL, and BPD is the approach to determine the penalty function. SAIC and SMDL follow the approaches of [4], [17], and [20] to determine the penalty function similar to UAIC and UMDL. BPD incorporates a prior probability density function (pdf) in a Bayesian context selected according to BPD principles. SAIC, SMDL, and BPD also have no means for controlling P_f. Variations on these techniques using different signal models and information theoretic criteria can be found in [30] and [16].

While the MDL, AIC, and related approaches have been the most prevalent in the literature, there have been several alternative approaches which allow for control of P_f. A test which relies on the χ^2 property of the MDL and AIC detection statistic in the unstructured model was presented in [21]. Structured model approaches for different signal models appear in [15], [12], and [24].

MVDR [11] relies on picking peaks above a selected threshold in a spatial spectrum generated according to the MVDR criterion. P_f in MVDR can be controlled by selection of the threshold. However,

TABLE I
Table of Approach Acronyms

Approach	Description
BPD	Bayesian Predictive Density [13]
MLEG	Maximum Likelihood Estimation General Signals [here]
MLEI	Maximum Likelihood Estimation Independent Signals [here]
MVDR	Minimum Variance Distortionless Response [11]
SAIC	Structured Akaike Information Theoretic Criteria [25]
SMDL	Structured Minimum Description Length [25]
UAIC	Unstructured Akaike Information Theoretic Criteria [26]
UMDL	Unstructured Minimum Description Length [26]

MVDR suffers with respect to the other techniques in its ability to resolve closely spaced and correlated signals.

In the work presented here we seek a solution to the combined detection and DOA estimation problem which allows for control of P_f via a tuning parameter and thus allows a trade-off with probability of detection (P_d), but which has resolution performance similar to the LF-based approaches. In [7], we developed an LF-based approach for uncorrelated sources which provided control of P_f by setting a lower limit on estimated signal power. We take a different approach here and use ML signal-to-noise ratio (SNR) estimates of hypothesized sources as detection statistics rather than maximizing a penalized LF. A preliminary version of this technique for uncorrelated sources was presented in [8]. We consider both the correlated and uncorrelated source problems. We demonstrate performance of the proposed approach compared with existing techniques with full regard for the importance of false alarms by showing receiver operating characteristic (ROC) curves as well as the standard detection performance curves.

This paper is organized as follows. The mathematical model and background are presented in Section II. The new technique is presented in Section III. In Section IV, signal and noise power ML estimates (MLEs) are developed for the general correlated signals model and the independent signals model. Performance results are given in Section V, and a summary is given in Section VI. For convenience, a list of algorithm abbreviations is provided in Table I.

II. MATHEMATICAL MODEL AND BACKGROUND

The mathematical model follows [22]. The complex narrowband scalar $z_{n,\kappa}$ is observed at each sensor n for N sensors and at snapshot κ for K snapshots. Element n of the complex $N \times 1$ data vector $\mathbf{z}_\kappa$ at snapshot κ is $z_{n,\kappa}$. The observation model for $\mathbf{z}_\kappa$ is

$$\mathbf{z}_\kappa = \mathbf{V}^{(L)}\mathbf{s}_\kappa + \mathbf{w}_\kappa, \qquad \kappa = 1,\ldots,K \tag{1}$$

where $\mathbf{w}_\kappa$ is a complex $N \times 1$ vector of additive noise samples at snapshot κ, $\mathbf{s}_\kappa$ is a complex $L \times 1$ vector of signal samples for L sources (one signal per source) at snapshot κ, and $\mathbf{V}^{(L)}$ is a complex $N \times L$ matrix whose columns are the L complex $N \times 1$ array manifold vectors for the L sources:

$$\mathbf{V}^{(L)} = [\mathbf{v}(\mathbf{u}_1)\cdots\mathbf{v}(\mathbf{u}_L)] \tag{2}$$

where $\mathbf{u}_l$ is the 3×1 DOA unit vector of the lth source. For a standard uniform linear array (sensor spacing is half signal wavelength), $\mathbf{u}_l$ reduces to scalar DOA $u_l = \sin(\theta_l)$, where θ_l is angular source l DOA with respect to the array axis perpendicular.

All signal samples $s_{\kappa,l}$ of $\mathbf{s}_\kappa$ are assumed to be complex zero mean Gaussian random variables. The complex $L \times L$ signal covariance matrix is $\mathbf{S}_\mathbf{s}^{(L)} = \mathrm{E}[\mathbf{s}_\kappa\mathbf{s}_\kappa^\mathrm{H}]$. The diagonal elements of $\mathbf{S}_\mathbf{s}^{(L)}$, σ_l^2 ($l = 1,\ldots,L$), are the real signal powers corresponding to sources $l = 1,\ldots,L$. Two signal models are considered. The first is a general signal model where all elements of $\mathbf{S}_\mathbf{s}^{(L)}$ are estimated. This allows for the correct modeling of actual correlated signals. The second is an independent signal model where the off-diagonal elements of $\mathbf{S}_\mathbf{s}^{(L)}$ are assumed zero and only the diagonal elements (signal powers σ_l^2) are estimated. This allows for more accurate estimation of signal powers σ_l^2 when signals are actually independent.

All noise samples $w_{\kappa,n}$ of $\mathbf{w}_\kappa$ are assumed to be independent complex zero mean Gaussian random variables. The diagonal $N \times N$ noise covariance matrix is $\mathbf{S}_\mathbf{w} = \mathrm{E}[\mathbf{w}_\kappa\mathbf{w}_\kappa^\mathrm{H}] = \sigma_w^2\mathbf{I}$, where scalar σ_w^2 is the noise power at all sensors.

The input, or sensor level, SNR of signal l is σ_l^2/σ_w^2. The output, or array level SNR (ASNR) of signal l is $N\sigma_l^2/\sigma_w^2$, where the full array gain is realized.

The complex $N \times N$ observation data covariance matrix $\mathbf{S}_\mathbf{z}^{(L)}$ is

$$\mathbf{S}_\mathbf{z}^{(L)} = \mathrm{E}[\mathbf{z}_\kappa\mathbf{z}_\kappa^\mathrm{H}] = \mathbf{V}^{(L)}\mathbf{S}_\mathbf{s}^{(L)}\mathbf{V}^{(L)\mathrm{H}} + \sigma_w^2\mathbf{I}. \tag{3}$$

Let $\mathbf{Z}$ be the collection of K observation vectors $\mathbf{z}_\kappa$. The pdf of all observed data $\mathbf{Z}$ is complex zero mean Gaussian,

$$\mathrm{p}_L(\mathbf{Z}) = \frac{1}{\pi^{KN}|\mathbf{S}_\mathbf{z}^{(L)}|^K}\exp\left(-\sum_{\kappa=1}^{K}\mathbf{z}_\kappa^\mathrm{H}(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{z}_\kappa\right). \tag{4}$$

For an ML approach, the pdf $\mathrm{p}_L(\mathbf{Z})$ in (4) is viewed as an LF, denoted as Γ_L, which is maximized with respect to the constant unknown parameters with observations $\mathbf{Z}$ given. The parameter values which achieve this are the MLEs. The maximization of (4) is over the entire source parameter state, which includes the number of sources L, source DOAs $\mathbf{u}_1,\ldots,\mathbf{u}_L$, the signal covariance matrix $\mathbf{S}_\mathbf{s}^{(L)}$, and the noise power σ_w^2.

TABLE II
Runaway Degrees of Freedom Example

Model Order	0	1	2	3
Free Parameters	σ_w^2	σ_w^2, σ_1^2, $\mathbf{u}_1$	σ_w^2, σ_1^2, σ_2^2, $\mathbf{u}_1$, $\mathbf{u}_2$	σ_w^2, σ_1^2, σ_2^2, σ_3^2, $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$
Constrained Parameters	σ_1^2, σ_2^2, σ_3^2	σ_2^2, σ_3^2	σ_3^2	(none)
Trivial Parameters	$\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$	$\mathbf{u}_2$, $\mathbf{u}_3$	$\mathbf{u}_3$	(none)

In implementation, LLF $\gamma_L = \ln(\Gamma_L)$ is used in place of LF Γ_L in (4) for maximization. Neglecting constant terms, γ_L is given by

$$\gamma_L = \ln(\Gamma_L) = -K\ln|\mathbf{S}_\mathbf{z}^{(L)}| - \sum_{\kappa=1}^{K}\mathbf{z}_\kappa^\mathrm{H}(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{z}_\kappa \tag{5}$$

$$= -K\{\ln|\mathbf{S}_\mathbf{z}^{(L)}| + \mathrm{tr}[(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{C}_\mathbf{z}]\} \tag{6}$$

where $\mathbf{C}_\mathbf{z}$ is the complex $N \times N$ sample covariance matrix defined as

$$\mathbf{C}_\mathbf{z} = \frac{1}{K}\sum_{\kappa=1}^{K}\mathbf{z}_\kappa\mathbf{z}_\kappa^\mathrm{H}. \tag{7}$$

With an unconstrained LLF γ_L in (6), an ML approach for model order estimation will not be successful. The reason is illustrated by an example in Table II for a maximum model order $L_{\max} = 3$ with an independent signals assumption. We must test model orders 0, 1, 2, and 3 to see which best fits the data. We can view each model order as containing all the parameters (σ_w^2, σ_1^2, σ_2^2, σ_3^2, $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$). The difference among the model orders is which parameters are free parameters available to maximize LLF γ_L in (6). For example, consider model order 1. Free parameters in the model are σ_w^2, σ_1^2, and $\mathbf{u}_1$. Since there is only one source present, signal powers σ_2^2 and σ_3^2 are constrained to be 0 with DOA parameters $\mathbf{u}_2$ and $\mathbf{u}_3$ as trivial parameters. A particular model order is a superset of all lower model orders in terms of free parameters. Regardless of the actual model order of the observed data, the LF with the greatest number of free parameters will have the largest maximum, resulting in selection of $L_{\max}$ as the model order estimate.

This is the same phenomena investigated in [4], [17], and [20], where the highest dimensionality is always selected as a solution. This phenomena may be characterized as "runaway" degrees of freedom.

For this reason, maximization of LLF γ_L in (6) is unreliable for estimation of model order. As a solution, UAIC, UMDL, SAIC, SMDL, and BPD augment the LLF with a penalty function depending on model order L so that a nontrivial maximum over model order may be found.

The ML approach can be equivalently viewed as a multihypothesis detection problem, where each hypothesis L corresponds to model order L, from zero up to a maximum number $L_{\max}$. For each hypothesized model order L, the remaining parameter estimates (DOAs, signal covariance matrix, noise power) are found which maximize the LLF in (6). Then, the model order hypothesis is chosen which has the maximum LLF. This is equivalent to a generalized likelihood ratio test (GLRT) procedure.

Because the DOA, signal, and noise parameters are unknown and must be estimated, there is no uniformly most powerful test for this problem. Therefore, the GLRT, as well as any other test procedure, has no inherent optimality properties, and the only validation of any procedure is performance results.

As stated, the GLRT suffers from "runaway" degrees of freedom. In [7], a GLRT based technique was developed that used a modified LLF in the maximization by limiting the estimated ASNR of each source to a minimum value. Limiting the estimated ASNR provided a means for controlling false alarms and motivated the development of the technique presented herein.

III. NEW TECHNIQUE

In this paper, we reinterpret model order L as L potential sources present. At each model order L, a detection decision is made whether L sources are actually present. There is no relative comparison of the LLF over model order. The detection criterion is developed heuristically and validated with simulation results.

To perform this detection decision for model order L, the presence of each potential source l ($l = 1,\ldots,L$) is tested individually rather than testing all potential sources as a group. There are two hypotheses for each potential source l. The null hypothesis corresponds to potential source l absent ($\sigma_l^2 = 0$). The alternative hypothesis corresponds to potential source l present ($\sigma_l^2 > 0$). For a single source with all other parameters ($\mathbf{u}_l$s and σ_w^2) assumed known, the uniformly most powerful test of the null hypothesis versus the alternative hypothesis is a one-sided upper tailed test on an estimate of signal l power $\hat{\sigma}_l^2$. Here, we have a more complicated problem for which there is no uniformly most powerful test. Therefore, we choose our test statistic to be the MLE of the source ASNR for each source. The critical region (rejection of the null hypothesis) is set by a threshold based on false alarm requirements.

TABLE III
Algorithm Overview

Select tuning parameter threshold $\mathrm{ASNR}_{\mathrm{nom}}$.
Compute sample covariance matrix $\mathbf{C}_{\mathbf{z}}$.
Initialize model order estimate $\hat{L}$ to 0.
For $L = 1$ to $L_{\max}$
Compute noise power MLE $\hat{\sigma}_w^{2(L)}$ and signal power MLEs $\hat{\sigma}_l^{2(L)}$, $l = 1,\ldots,L$. Embedded in this procedure is estimation of DOAs $\hat{\mathbf{u}}_l^{(L)}$, $l = 1,\ldots,L$.
Compute ASNR estimates $\widehat{\mathrm{ASNR}}_l = N\hat{\sigma}_l^{2(L)}/\hat{\sigma}_w^{2(L)}$ and compare to threshold $\mathrm{ASNR}_{\mathrm{nom}}$.
If all $\widehat{\mathrm{ASNR}}_l > \mathrm{ASNR}_{\mathrm{nom}}$, update model order estimate $\hat{L}$ to L. If test fails, search is stopped.
Next L

This approach can be considered a "smarter" multidimensional MVDR in the sense that we are finding the number of peaks above some ASNR threshold in a multidimensional likelihood surface for each potential model order, rather than the one-dimensional MVDR spatial spectrum.

The algorithm is summarized in Table III as a sequential search over model order. For each model order L ($L = 1,\ldots,L_{\max}$), we find all signal power MLEs $\hat{\sigma}_l^{2(L)}$ ($l = 1,\ldots,L$) and noise power MLE $\hat{\sigma}_w^{2(L)}$. These MLEs depend on DOA MLEs $\hat{\mathbf{u}}_l^{(L)}$. Therefore, all MLEs must be jointly found per model order. Then, the ASNR MLEs $N\hat{\sigma}_l^{2(L)}/\hat{\sigma}_w^{2(L)}$ are used as detection statistics of hypothesized sources. That is, hypothesized source l is accepted as a detected source if its ASNR MLE is above a selected threshold ASNR, and rejected otherwise. The ASNR threshold is specified as the tuning parameter $\mathrm{ASNR}_{\mathrm{nom}}$. Higher tuning parameter $\mathrm{ASNR}_{\mathrm{nom}}$ lowers both P_f and P_d for fixed actual ASNR, and vice-versa. For model order L to be accepted as a possible solution (all hypothesized sources present), the ASNR MLEs of all L hypothesized sources must pass this test. The interpretation of a successful test is that L or more sources are present. The correct model order cannot be determined until this test fails for a model order. That is, the final solution is the largest model order L for which this test is true. If no model order L passes this test, the final solution for model order L is zero.

Implementation involves finding the MLEs of the signal powers and noise power at each model order, which in turn requires ML estimation of the source DOAs. The performance and complexity of the detection algorithm is closely coupled with the implementation of this complicated multidimensional optimization problem. In the next section, we discuss ML estimation for two models: a general (possibly correlated) signal model and an uncorrelated signal model. The implementations are designed to work with the sequential nature of the detection problem and use parameter estimates from the previous model order for initialization.

IV. MLE DERIVATIONS AND IMPLEMENTATION

In this section, we derive the signal and noise power MLEs used in the model order estimation algorithm presented in the previous section. A noise power estimate valid for both the correlated and uncorrelated signal models is derived first, followed by signal power estimates for the two models. An efficient iterative implementation that makes use of the sequential steps in the detection problem is outlined. Additional details may be found in [5] and [6], where detailed derivations of all MLEs along with performance demonstrations are given.

A. Noise Power

We derive the noise power MLE $\hat{\sigma}_w^{2(L)}$ as a function of an arbitrary signal covariance matrix $\mathbf{S}_\mathbf{s}^{(L)}$, which is assumed to be Hermitian and positive semi-definite. Rewriting (3) with an eigenvalue decomposition gives

$$\mathbf{S}_\mathbf{z}^{(L)} = \mathbf{W}(\mathbf{D} + \sigma_w^2\mathbf{I})\mathbf{W}^{\mathrm{H}} \tag{8}$$

where $\mathbf{D}$ is a diagonal matrix of eigenvalues with $d_l \geq 0$ for $l = 1,\ldots,L$ (the "signal subspace" eigenvalues) and $d_n = 0$ for $n = L+1,\ldots,N$. Note that we can write the eigenvector matrix $\mathbf{W} = [\mathbf{W}_\mathbf{s}\ \ \mathbf{W}_\mathbf{n}]$, where $\mathbf{W}_\mathbf{s}$ is the $N \times L$ matrix of signal subspace eigenvectors, and $\mathbf{W}_\mathbf{n}$ is the $N \times (N-L)$ matrix of noise subspace eigenvectors. $\mathbf{W}_\mathbf{s}$ and $\mathbf{D}$ can be obtained from an eigendecomposition of $\mathbf{V}^{(L)}\mathbf{S}_\mathbf{s}^{(L)}\mathbf{V}^{(L)\mathrm{H}}$ (i.e., $\mathbf{W}_\mathbf{s}\mathbf{D}\mathbf{W}_\mathbf{s}^{\mathrm{H}} = \mathbf{V}^{(L)}\mathbf{S}_\mathbf{s}^{(L)}\mathbf{V}^{(L)\mathrm{H}}$). The noise eigenvectors are any set of $N-L$ orthonormal vectors which are orthogonal to the columns of $\mathbf{W}_\mathbf{s}$.

From the eigendecomposition, we have

$$\ln|\mathbf{S}_\mathbf{z}^{(L)}| = \sum_{n=1}^{N} \ln(d_n + \sigma_w^2) \tag{9}$$

and

$$\begin{aligned}\mathrm{tr}[(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{C}_\mathbf{z}] &= \mathrm{tr}[\mathbf{W}(\mathbf{D} + \sigma_w^2\mathbf{I})^{-1}\mathbf{W}^{\mathrm{H}}\mathbf{C}_\mathbf{z}] \\ &= \mathrm{tr}[(\mathbf{D} + \sigma_w^2\mathbf{I})^{-1}\mathbf{W}^{\mathrm{H}}\mathbf{C}_\mathbf{z}\mathbf{W}] \\ &= \mathrm{tr}[(\mathbf{D} + \sigma_w^2\mathbf{I})^{-1}\mathbf{B}] \\ &= \sum_{n=1}^{N} \frac{b_{nn}}{d_n + \sigma_w^2}\end{aligned} \tag{10}$$

where $\mathbf{B} = \mathbf{W}^{\mathrm{H}}\mathbf{C}_\mathbf{z}\mathbf{W}$ and $b_{nn} = [\mathbf{B}]_{nn}$ (diagonal elements of $\mathbf{B}$). Substituting (9) and (10) into (6) gives

$$\gamma_L = -K\sum_{n=1}^{N}\left[\ln(d_n + \sigma_w^2) + \frac{b_{nn}}{d_n + \sigma_w^2}\right]. \tag{11}$$

Taking the partial derivative of (11) with respect to σ_w^2 and setting the result equal to zero, we get

$$\frac{\partial \gamma_L}{\partial \sigma_w^2} = -K\sum_{n=1}^{N}\left(\frac{1}{d_n + \sigma_w^2} - \frac{b_{nn}}{(d_n + \sigma_w^2)^2}\right) = 0. \tag{12}$$

Using the fact that $d_n = 0$ for $n = (L+1),\ldots,N$,

$$\sum_{n=1}^{L}\left(\frac{1}{d_n + \sigma_w^2} - \frac{b_{nn}}{(d_n + \sigma_w^2)^2}\right) + \frac{N-L}{\sigma_w^2} - \frac{1}{\sigma_w^4}\sum_{n=L+1}^{N} b_{nn} = 0. \tag{13}$$

Rearranging gives

$$\sigma_w^2 = \frac{1}{N-L}\left(\sum_{n=L+1}^{N} b_{nn} + (\sigma_w^2)^2 \times \sum_{n=1}^{L}\left(\frac{b_{nn}}{(d_n + \sigma_w^2)^2} - \frac{1}{d_n + \sigma_w^2}\right)\right). \tag{14}$$

Equation (14) is the basis for an iterative update of the noise power. Let $\tilde{\sigma}_w^{2(L)}$ denote a previous estimate of the noise power. The current estimate $\hat{\sigma}_w^{2(L)}$ can be computed from

$$\hat{\sigma}_w^{2(L)} = \frac{1}{N-L}\left(\sum_{n=L+1}^{N} b_{nn} + (\tilde{\sigma}_w^{2(L)})^2 \times \sum_{n=1}^{L}\left(\frac{b_{nn}}{(d_n + \tilde{\sigma}_w^{2(L)})^2} - \frac{1}{d_n + \tilde{\sigma}_w^{2(L)}}\right)\right). \tag{15}$$

An initial estimate for the noise power $\hat{\sigma}_w^{2(L)}$ for model order L which does not depend on any other parameter is

$$\hat{\sigma}_w^{2(L)} = \frac{1}{N-L}\sum_{n=L+1}^{N} \lambda_n \tag{16}$$

where λ_n are the N descending rank ordered eigenvalues of complex $\mathbf{C}_\mathbf{z}$. The noise power estimate $\hat{\sigma}_w^{2(L)}$ in (16) is the MLE originally derived in [3] based on an unstructured signal and noise subspace decomposition and is used in [26]. It is, in fact, the arithmetic average of the noise subspace eigenvalues.

The noise power MLE $\hat{\sigma}_w^{2(L)}$ in (15) is conditioned on model order L, source DOAs $\mathbf{u}_1,\ldots,\mathbf{u}_L$, and signal covariance matrix $\mathbf{S}_\mathbf{s}^{(L)}$. It is the true MLE if and only if all these other parameters are known or are MLEs.

B. Correlated Signal Model, Signal Power

A closed form analytical solution for the MLEs of signal covariance matrix $\mathbf{S}_\mathbf{s}^{(L)}$ and noise power $\sigma_w^{2(L)}$ can be found for a general signal model conditioned on known model order L and source DOAs $\mathbf{u}_1,\ldots,\mathbf{u}_L$. The complex $L \times L$ signal covariance matrix MLE $\hat{\mathbf{S}}_\mathbf{s}^{(L)}$ is [22, (8.316)] and is due to [9] and [14]:

$$\hat{\mathbf{S}}_\mathbf{s}^{(L)} = (\mathbf{V}^{(L)\mathrm{H}}\mathbf{V}^{(L)})^{-1}\mathbf{V}^{(L)\mathrm{H}}(\mathbf{C}_\mathbf{z} - \hat{\sigma}_w^{2(L)}\mathbf{I})\mathbf{V}^{(L)}(\mathbf{V}^{(L)\mathrm{H}}\mathbf{V}^{(L)})^{-1}. \tag{17}$$

Using $\hat{\mathbf{S}}_\mathbf{s}^{(L)}$ in (17), the noise power MLE $\hat{\sigma}_w^{2(L)}$ for model order L has a closed form expression, which is [22, (8.313)]:

$$\hat{\sigma}_w^{2(L)} = \frac{\mathrm{tr}(\mathbf{P}_\mathbf{V}^{\perp}\mathbf{C}_\mathbf{z})}{N-L}. \tag{18}$$

The noise power MLE $\hat{\sigma}_w^{2(L)}$ in (18) is explicitly conditioned on model order L, source DOAs $\mathbf{u}_1,\ldots,\mathbf{u}_L$ and is implicitly conditioned on signal covariance matrix estimate $\hat{\mathbf{S}}_\mathbf{s}^{(L)}$ being the MLE in (17). The signal covariance matrix MLE $\hat{\mathbf{S}}_\mathbf{s}^{(L)}$ in (17) is conditioned on model order L, source DOAs $\mathbf{u}_1,\ldots,\mathbf{u}_L$, and noise power $\hat{\sigma}_w^{2(L)}$ and is the MLE if and only if those parameters are known or are MLEs.

Substitution of (17) and (18) into the LF in (6) leads to

$$\gamma_L = -K\ln(\mathfrak{L}) + c \tag{19}$$

where

$$\mathfrak{L} = \det\left(\mathbf{P_V C_z P_V} + \frac{\mathrm{tr}(\mathbf{P}_\mathbf{V}^\perp \mathbf{C_z})}{N-L}\mathbf{P}_\mathbf{V}^\perp\right) \tag{20}$$

and c is an irrelevant additive constant. LLF γ_L in (19) is the general compressed LF that is used for ML DOA estimation [22]. It is also the common LF-based data term in SAIC and SMDL [25] and BPD [13]. The data term $\mathfrak{L}$ in (20) must be minimized with respect to the L DOAs $\mathbf{u}_1,\ldots,\mathbf{u}_L$ to obtain DOA MLEs for those approaches.

Although the true signal covariance matrix $\mathbf{S}_\mathbf{s}^{(L)}$ must always be positive semi-definite, the MLE $\hat{\mathbf{S}}_\mathbf{s}^{(L)}$ in (17) is only guaranteed to maximize (6) and is not guaranteed to be positive semi-definite [22, p. 988]. These "invalid" MLEs do, in fact, occur in practice. Although DOA estimation performance does not seem to be sensitive to the definiteness of the estimated signal covariance matrix [22, p. 1001], our detection algorithm is affected. Constraining the covariance matrix MLE $\hat{\mathbf{S}}_\mathbf{s}^{(L)}$ to the set of nonnegative definite matrices results in a complicated ML estimation procedure [10]. Simpler ad-hoc approaches have also been proposed [22, p. 1000]. In the implementation described later in this section, we use a simple ad-hoc procedure to guarantee positive semi-definiteness of the estimate in our algorithm. When the estimated signal covariance is not given by (17), the noise power estimate in (18) is no longer the MLE. However, the estimate in (15) is still valid.

C. Independent Signal Model, Signal Power

We derive the MLE of the power of a single signal with all other parameters assumed known. Taking the partial derivative of γ_L in (6) with respect to σ_l^2, we get [22, (8.17)]:

$$\frac{\partial\gamma_L}{\partial\sigma_l^2} = -K\left(\mathrm{tr}\left[(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\frac{\partial\mathbf{S}_\mathbf{z}^{(L)}}{\partial\sigma_l^2}\right] - \mathrm{tr}\left[(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{C_z}(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\frac{\partial\mathbf{S}_\mathbf{z}^{(L)}}{\partial\sigma_l^2}\right]\right). \tag{21}$$

For notational simplicity, let $\mathbf{v}_l = \mathbf{v}(\mathbf{u}_l)$, and define

$$\mathbf{\Sigma}_l = \sum_{l'=1 l'\neq l}^{L}\sigma_{l'}^2\mathbf{v}_{l'}\mathbf{v}_{l'}^\mathrm{H} + \sigma_w^2\mathbf{I}. \tag{22}$$

Then,

$$\mathbf{S}_\mathbf{z}^{(L)} = \sigma_l^2\mathbf{v}_l\mathbf{v}_l^\mathrm{H} + \mathbf{\Sigma}_l \tag{23}$$

and

$$\frac{\partial\mathbf{S}_\mathbf{z}^{(L)}}{\partial\sigma_l^2} = \mathbf{v}_l\mathbf{v}_l^\mathrm{H}. \tag{24}$$

Setting the derivative in (21) equal to zero and substituting from (24) yields

$$\mathrm{tr}[(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{v}_l\mathbf{v}_l^\mathrm{H}] - \mathrm{tr}[(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{C_z}(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{v}_l\mathbf{v}_l^\mathrm{H}] = 0 \tag{25}$$

$$\mathbf{v}_l^\mathrm{H}(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{v}_l - \mathbf{v}_l^\mathrm{H}(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{C_z}(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{v}_l = 0 \tag{26}$$

$$\mathbf{v}_l^\mathrm{H}(\mathbf{S}_\mathbf{z}^{(L)})^{-1}(\mathbf{C_z} - \mathbf{S}_\mathbf{z}^{(L)})(\mathbf{S}_\mathbf{z}^{(L)})^{-1}\mathbf{v}_l = 0. \tag{27}$$

The inverse of $\mathbf{S}_\mathbf{z}^{(L)}$ in (23) is given by

$$(\mathbf{S}_\mathbf{z}^{(L)})^{-1} = \mathbf{\Sigma}_l^{-1} - \mathbf{\Sigma}_l^{-1}\mathbf{v}_l\mathbf{v}_l^\mathrm{H}\mathbf{\Sigma}_l^{-1}\frac{\sigma_l^2}{1+\sigma_l^2\mathbf{v}_l^\mathrm{H}\mathbf{\Sigma}_l^{-1}\mathbf{v}_l} \tag{28}$$

therefore

$$\begin{aligned}\mathbf{S}_\mathbf{z}^{(L)-1}\mathbf{v}_l &= \mathbf{\Sigma}_l^{-1}\mathbf{v}_l\left(1 - \frac{\sigma_l^2\mathbf{v}_l^\mathrm{H}\mathbf{\Sigma}_l^{-1}\mathbf{v}_l}{1+\sigma_l^2\mathbf{v}_l^\mathrm{H}\mathbf{\Sigma}_l^{-1}\mathbf{v}_l}\right)\\ &= \mathbf{\Sigma}_l^{-1}\mathbf{v}_l\frac{1}{1+\sigma_l^2\mathbf{v}_l^\mathrm{H}\mathbf{\Sigma}_l^{-1}\mathbf{v}_l}.\end{aligned} \tag{29}$$

Substituting (23) and (29) into (27) yields

$$\frac{\mathbf{v}_l^\mathrm{H}\mathbf{\Sigma}_l^{-1}(\mathbf{C_z} - \sigma_l^2\mathbf{v}_l\mathbf{v}_l^\mathrm{H} - \mathbf{\Sigma}_l)\mathbf{\Sigma}_l^{-1}\mathbf{v}_l}{(1+\sigma_l^2\mathbf{v}_l^\mathrm{H}\mathbf{\Sigma}_l^{-1}\mathbf{v}_l)^2} = 0. \tag{30}$$

Solving for σ_l^2 in (30), we obtain

$$\hat{\sigma}_l^{2(L)} = \frac{\mathbf{v}_l^\mathrm{H}\mathbf{\Sigma}_l^{-1}(\mathbf{C_z} - \mathbf{\Sigma}_l)\mathbf{\Sigma}_l^{-1}\mathbf{v}_l}{(\mathbf{v}_l^\mathrm{H}\mathbf{\Sigma}_l^{-1}\mathbf{v}_l)^2}. \tag{31}$$

Signal power MLE $\hat{\sigma}_l^{2(L)}$ in (31) is conditioned on model order L, source DOAs $\mathbf{u}_1,\ldots,\mathbf{u}_L$, noise power σ_w^2, and all other signal powers $\sigma_{l'}^{2(L)}$ ($l' \neq l$). It is the true MLE if and only if all these other parameters are known or are MLEs. Similar to the signal covariance matrix MLE in (17), $\hat{\sigma}_l^2$ in (31) is not guaranteed to be nonnegative, and an adjustment must be made in the implementation to ensure this property.

D. Implementation

Let MLEG denote the general signal model MLE-based detection algorithm and MLEI denote the independent signal model algorithm. The algorithms are implemented sequentially over model order L, as summarized in Table III. An exhaustive multidimensional search over DOA per model order is computationally infeasible. Therefore, DOA estimates from the previous model order are leveraged in the DOA search over the current model order.

For MLEG, three steps are performed at each model order L in order to achieve the goal of a positive definite signal covariance matrix MLE $\hat{\mathbf{S}}_{\mathbf{s}}^{(L)}$. First, an initial one-dimensional search for the "new" source DOA is performed. The noise power MLE $\hat{\sigma}_w^{2(L)}$ is computed from (18), and signal covariance matrix MLE $\hat{\mathbf{S}}_{\mathbf{s}}^{(L)}$ is computed from (17) for each candidate "new" DOA holding the $L-1$ previously estimated DOAs constant. Positive definiteness is enforced via an eigenvalue decomposition in this step and in the subsequent two steps. Specifically, all eigenvalues are limited to a selected small minimum value, which is 0.0001 here. The modified signal covariance matrix MLE $\hat{\mathbf{S}}_{\mathbf{s}}^{(L)}$ is used to compute the LLF γ_L in (6). The second step is to perform an iterative procedure for signal covariance matrix MLE $\hat{\mathbf{S}}_{\mathbf{s}}^{(L)}$ using (17) and noise power MLE $\hat{\sigma}_w^{2(L)}$ using (15) for each maxima of the initial search with source DOAs held constant. The maximum from this step is the input to the third and final step, which is refinement of all L source DOAs in an alternating maximization fashion using signal covariance matrix MLE $\hat{\mathbf{S}}_{\mathbf{s}}^{(L)}$ from (17) and noise power MLE $\hat{\sigma}_w^{2(L)}$ from (15). The detection decisions follow from Section III.

The MLEI implementation is similar to MLEG except that signal power MLE $\hat{\sigma}_l^2$ in (31) and noise power MLE $\hat{\sigma}_w^{2(L)}$ in (15) are used in all three steps. Noise power MLE $\hat{\sigma}_w^{2(L)}$ is initialized from (16). The off-diagonal terms of signal covariance matrix $\hat{\mathbf{S}}_{\mathbf{s}}^{(L)}$ are zero following the independent signals model.

The reason for the above involved procedures for MLEG and MLEI is that a closed form solution for LLF γ_L cannot be found due to their imposed restrictions on signal covariance matrix MLE $\hat{\mathbf{S}}_{\mathbf{s}}^{(L)}$ (positive definite, independent). This is contrasted to LLF γ_L in (19) where no restrictions are imposed on signal covariance matrix MLE $\hat{\mathbf{S}}_{\mathbf{s}}^{(L)}$ and a closed form solution for LLF γ_L can be found.

An alternate simpler implementation for MLEG may be preferred [6]. The positive definite restriction for signal covariance matrix MLE $\hat{\mathbf{S}}_{\mathbf{s}}^{(L)}$ is relaxed for purposes of DOA estimation-only per model order. That is, the DOA search procedure is identical to the SAIC, SMDL, and BPD DOA search described below to minimize (20). A signal covariance matrix MLE $\hat{\mathbf{S}}_{\mathbf{s}}^{(L)}$ with a positive definite restriction is computed using the procedure above for these fixed DOA MLEs. Little difference in performance has been seen.

V. PERFORMANCE RESULTS

Performance was compared among UAIC and UMDL [26], SAIC and SMDL [25], BPD [13], MVDR [11], the presented MLE approach with a general signal model (MLEG), and the presented MLE approach with an independent signal model (MLEI). A standard uniform linear array was used.

Among the available choices, the number of performance comparisons must necessarily be limited. The above approaches were selected for two reasons. First, with the exception of MVDR, all follow the same mathematical model culminating in the LLF in (6). Second, the other approaches besides MLEG and MLEI are the most commonly used approaches.

UAIC and UMDL were implemented straightforwardly according to [26]. MVDR was implemented without diagonal loading because of the large number of selected snapshots ($K = 200$). The MVDR detection threshold was a selected factor times the median MVDR power over all DOAs. SAIC, SMDL, BPD, MLEG, and MLEI all require a multidimensional optimization (i.e., minimization or maximization) over source DOAs (one-dimension per source). For SAIC, SMDL, and BPD, the multidimensional search to minimize data term $\mathfrak{L}$ in (20) was implemented sequentially over model order L, analogous to Table III. At each model order L, two steps were performed. First, an initial one-dimensional search for the minimum data term $\mathfrak{L}$ in (20) was performed over the "new" source DOA with previous source DOA estimates held constant. Second, all source DOAs in the resulting peak were each refined in an alternating minimization fashion. The SAIC, SMDL, and BPD criteria can be computed with their respective penalty function ([13, (37), (39), (40)]) per model order. The estimated model order was the respective minimum over model order. MLEG and MLEI were implemented as described in Section IV.

While SAIC, SMDL, and BPD are potentially affected by signal covariance matrix MLE $\hat{\mathbf{S}}_{\mathbf{s}}^{(L)}$ not necessarily being positive definite as discussed in Section IV, no correction was made to (20) in the minimization step to require positive definiteness, as it did not appear necessary. Because of the similar sequential implementations, SAIC, SMDL, BPD, MLEI, and MLEG all have similar computational complexity. UMDL, UAIC, and MVDR are significantly less complex to implement.

A. No Signals Present (False Alarm) Performance

The point of view is taken that false alarm, or no signals present, performance is of high importance. In particular, signal(s) present detection performance must be evaluated with regard to signal absent detection performance. False alarm performance is not usually shown in the array detection literature.

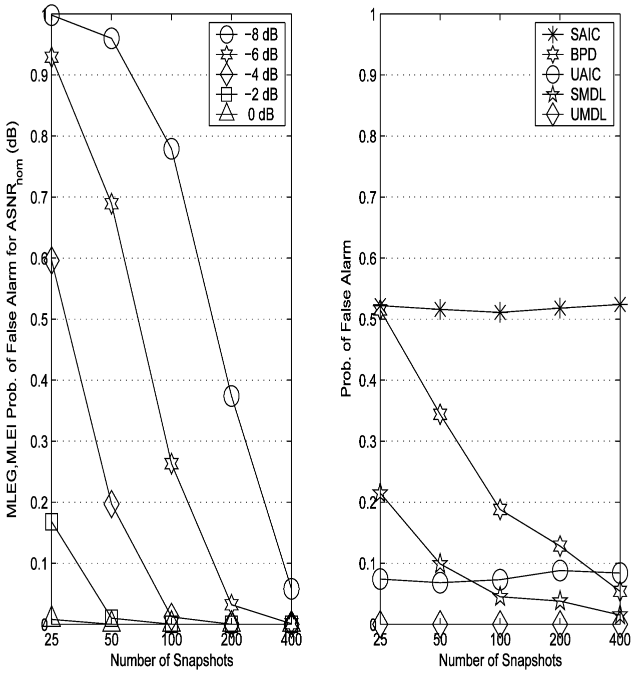

Fig. 1. False alarm performance versus number of snapshots and various tuning parameters $ASNR_{nom}$ in MLEG and MLEI, with N = 6, trials = 1000.

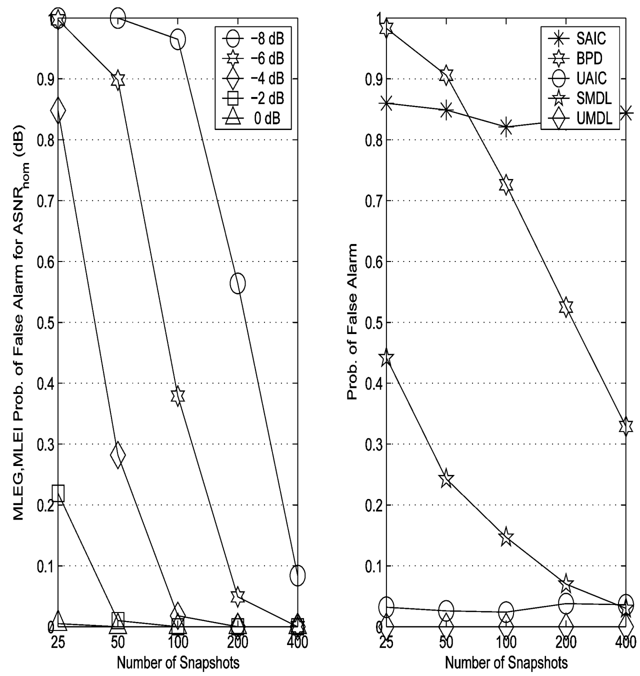

Fig. 2. False alarm performance versus number of snapshots and various tuning parameters $ASNR_{nom}$ in MLEG and MLEI, with N = 15, trials = 1000.

Consistent with this point of view, there must be a minimum acceptable P_f, which is conservatively chosen as nominally 0.1 for our purposes here.

In Figs. 1 and 2, P_f is plotted against number of snapshots K for 1000 trials with number of sensors N equal 6 and 15, respectively. P_f is the probability that any number of sources other than zero is detected. The left-hand panels of Figs. 1 and 2 show the presented MLEG and MLEI approaches. Each curve corresponds to a selected tuning parameter $ASNR_{nom}$. As expected, MLEG and MLEI false alarm performance is identical. The right-hand panels of Figs. 1 and 2 show the other approaches with each curve corresponding to a particular approach.

In general, P_f performance improves with higher number of snapshots K (UAIC and SAIC excepted) and lower number of sensors N (UAIC again excepted). The reason for the latter is that higher N means more independent "looks" over DOA and, therefore, more opportunities to make a (false) detection. For fixed number of snapshots K and number of sensors N, P_f is fixed for UAIC, UMDL, SAIC, SMDL, and BPD. In contrast, Figs. 1 and 2 demonstrate that for fixed number of snapshots K and number of sensors N, MLEG and MLEI can be set to any desired P_f by adjusting tuning parameter $ASNR_{nom}$.

P_f is determined experimentally as shown in Figs. 1 and 2. Future work would be the analytical determination of P_f. UMDL has the lowest P_f performance of any approach. In cases where P_f is saturated at zero (no detected false alarms), more trials are required to adequately estimate false alarm performance, particularly for UMDL.

To meet the criterion of P_f approximately equal to or less than 0.1, all subsequent signal(s) present cases are run with number of sensors N equal 6 and number of snapshots K equal 200. MLEG and MLEI are run at tuning parameter $ASNR_{nom}$ of −6 and −2 dB. All the approaches except SAIC meet that P_f criterion. SAIC performance is omitted in some figures due to its poor P_f performance.

B. One Signal Present Performance

In Fig. 3, probability of correct detection P_{cd} is plotted against actual ASNR of a single signal. In all cases, P_{cd} is the probability that the correct number of sources are detected where the possible choices are 0 to a maximum L_{max}, which is bounded by $N - 1$. The number of trials is 200. The top panel shows MLEG and MLEI performance for tuning parameter $ASNR_{nom}$ equal −6 and −2 dB. The bottom panel shows performance of the other approaches. The same format is used for the two signals present cases.

The primary conclusion is that P_{cd} performance is highly dependent on P_f performance in all approaches. The best performer is BPD, but that must be considered against poorer false alarm performance than the other approaches. Similarly, the worst performer is UMDL, but that must also be considered against better false alarm performance than the other approaches. The same is true for MLEG and MLEI where both P_{cd} in Fig. 3 and P_f

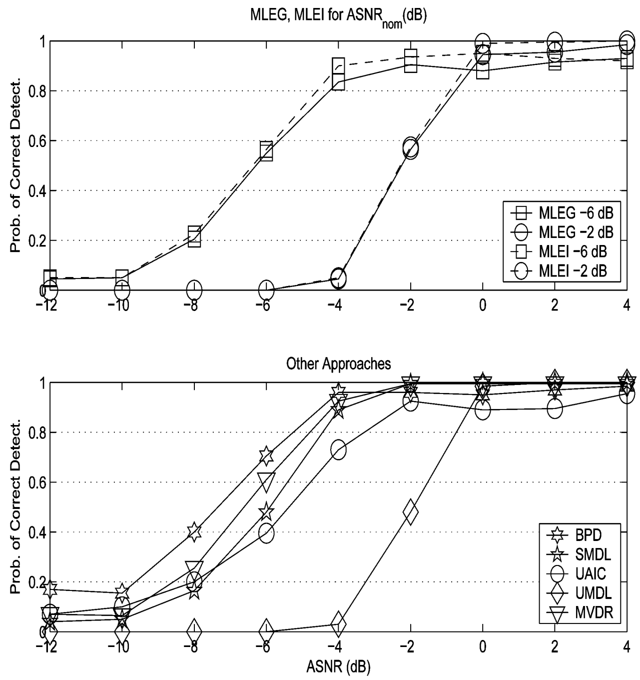

Fig. 3. Probability of correct detection versus ASNR for one signal present with $N = 6$, $K = 200$, trials = 200.

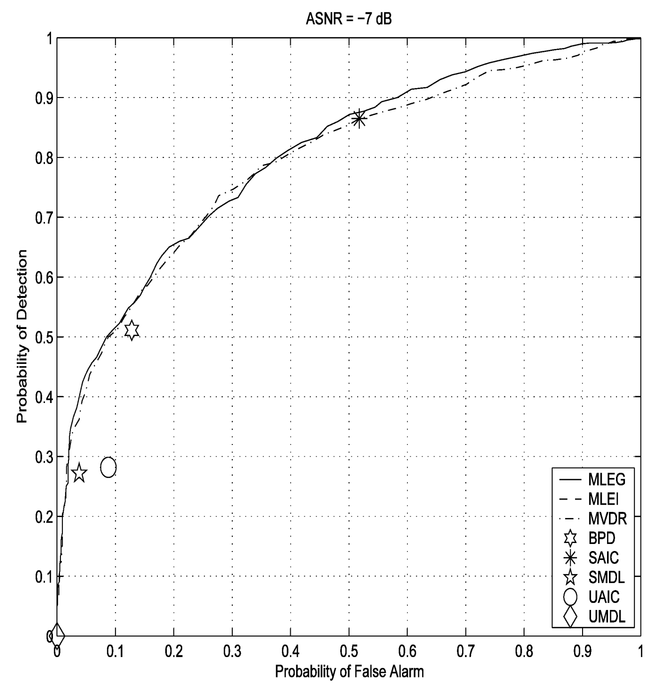

Fig. 4. One signal ROC performance, with $N = 6$, $K = 200$, ASNR = −7 dB, trials = 1000.

in Figs. 1 and 2 are monotonically inversely related to tuning parameter ASNR_{nom}. As for false alarm performance, MLEG and MLEI one signal present performance is nearly identical. As a last example, both P_{cd} and P_f performance is roughly comparable among SMDL, UAIC, MVDR, MLEG, and MLEI with tuning parameter ASNR_{nom} equal −6 dB. MVDR was tuned for a P_f of 0.05.

In the high ASNR region, MLEG and MLEI P_{cd} performance is affected by overestimation errors, which are false alarms by the hypothesized signal corresponding to a false signal. As a consequence, P_{cd} performance is better for higher tuning parameter ASNR_{nom} in the high ASNR region even though the converse is true at low ASNR. The same is true in the two signal cases. BPD and SMDL do not exhibit the same effect in the high ASNR region.

Another view of signal present detection performance is a ROC curve. The purpose of a ROC curve is to explicitly demonstrate the dependence of signal present performance on signal absent performance. To this end, the test conditions for our computation of ROC curves match the classical ROC conditions as closely as possible. In particular, a binary detection decision case is assumed, where only two detection decisions are allowed: either no signals present or one signal present.

The ROC curve eliminates detection threshold (if it exists) as an intermediate variable and plots P_d directly against P_f for a fixed actual ASNR. Note that model order overestimation errors are eliminated, and only model order underestimation errors contribute to P_d. This is consistent with the classical ROC curve conditions where model order overestimation errors are nonexistent.

Fig. 4 shows single signal ROC performance for an actual ASNR of −7 dB for all approaches. The test conditions are identical to Figs. 1 and 3 but with 1000 trials. There are two key points illustrated in Fig. 4. First, under these test conditions, there is no inherent performance difference among MLEG, MLEI, MVDR, BPD, and SMDL. That is, all of the (P_f, P_d) points lie nearly on the same curve. All that is accomplished is a trade-off of P_f and P_d over the approaches for the test conditions. Second, MLEG, MLEI, and MVDR (P_f, P_d) performance is a true continuum rather than discrete points as with the other approaches. This allows a selection of the operating point on the (P_f, P_d) curve.

There is a loss in performance for UAIC due to its unstructured model. The extreme P_f performance of UMDL (very low with an attendant loss in detection sensitivity) and SAIC (unsatisfactorily high) limits their utility and makes meaningful comparisons to the other approaches difficult.

A single figure can only show a narrow view of detection performance. We have selected the figures to give as broad and diverse a view of detection performance as possible in a necessarily limited number of figures. P_f in Figs. 1, 2, and 4 includes only model order overestimation errors for the no signal present case. P_{cd} in Fig. 3 includes both model order underestimation and overestimation errors for the one signal present case. P_d in Fig. 4 includes only model order underestimation errors for the one signal present case. Each of the figures serves a purpose.

The same philosophy is used for the two signals present cases.

This performance is as expected. For one signal, issues of structured versus unstructured, independent versus correlated signal models, and choice of detection statistic have small effect. The advantage of MLEG, MLEI, and MVDR is the ability to control P_f. These results for the one signal present case are applicable to multiple signals present cases with widely separated DOAs, where source resolution performance is not an issue.

C. Two Signals Present Performance

Three cases are considered for two signals present performance. The difference among the cases is the degree of correlation between the two signals. The closely spaced angular DOAs θ_1 and θ_2 are 15° and 20°. Thus, source resolution performance is an issue. Actual ASNR is the same for both signals.

ROC performance is also given as another view of two signals present detection performance similar to the one signal present case with the same motivations. In particular, we try to match the classical ROC conditions as closely as possible. Only three detection decisions are allowed: no signals present, one signal present, or two signals present. P_f is the probability that any number of signals other than zero is detected for the no signals present case. P_d is the probability that two signals are detected for the two signals present case. As in one signal present performance, model order overestimation errors are eliminated, and only model order underestimation errors contribute to P_d.

Figs. 5 and 6 show the independent signals case. Correlation coefficient $\rho_{12} = \mathrm{E}[s_1 s_2^*]/(\sigma_1 \sigma_2) = 0$. There is a clear advantage for MLEG and MLEI with MLEI higher and the expected trade-off of P_{cd} and P_f over tuning parameter $\mathrm{ASNR}_{\mathrm{nom}}$. In two (and higher) signals cases, issues of structured versus unstructured and independent versus correlated signal models come into play as well as the choice of detection statistics. The independent signal model in MLEI is optimum for this case and contributes to its performance. The correlated signal model in BPD and SMDL is not optimum for actual independent signals. In spite of its correlated signal model, MLEG performs well due to its separate detection statistics. UAIC and UMDL suffer due to the unstructured model. MVDR has poor performance on all the two signals cases due to its lack of resolvability for these closely spaced sources.

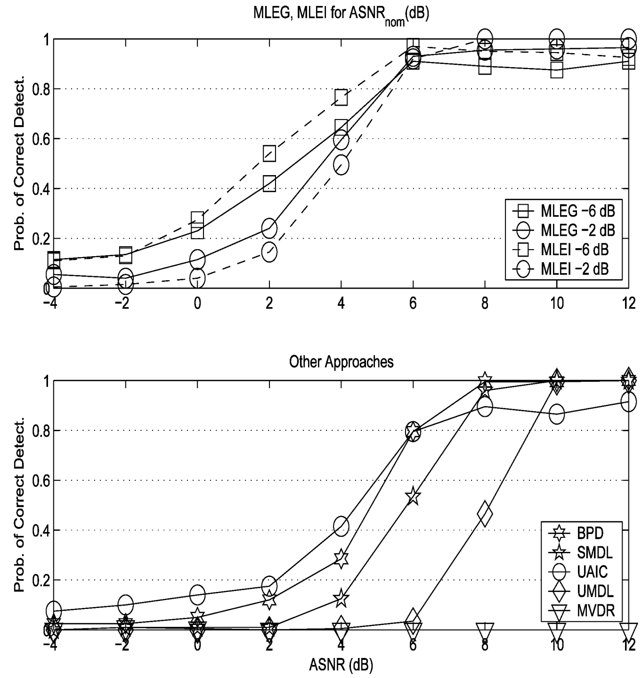

Fig. 5. Probability of correct detection versus ASNR for two equal power independent signals ($\rho_{12} = 0$) with $\theta_1 = 15°$, $\theta_2 = 20°$, $N = 6$, $K = 200$, trials = 200.

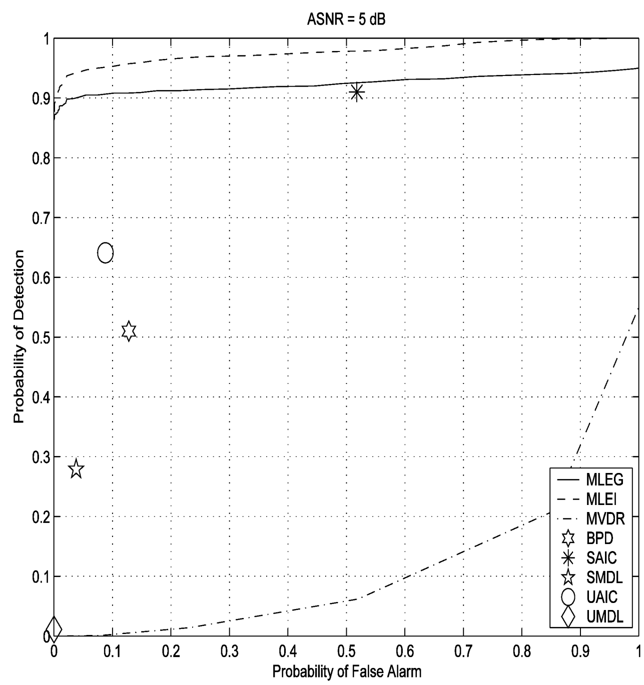

Fig. 6. ROC performance for two equal power independent signals ($\rho_{12} = 0$), with $\theta_1 = 15°$, $\theta_2 = 20°$, $N = 6$, $K = 200$, ASNR = 5 dB, trials = 1000.

Fig. 6 reveals that there is a resolvability saturation effect for MLEG and MLEI at higher P_f. The only model order relevant for determining P_d is $L = 2$. If the two actual sources are correctly resolved, they are represented by two hypothesized sources with high signal powers. This case controls P_d in the low P_f region where it increases rapidly. On the other hand, if the two actual sources are not correctly resolved, they are represented by only one hypothesized source with high signal power. The other hypothesized source is a random noise spike with low signal power. This case controls P_d in the higher P_f region

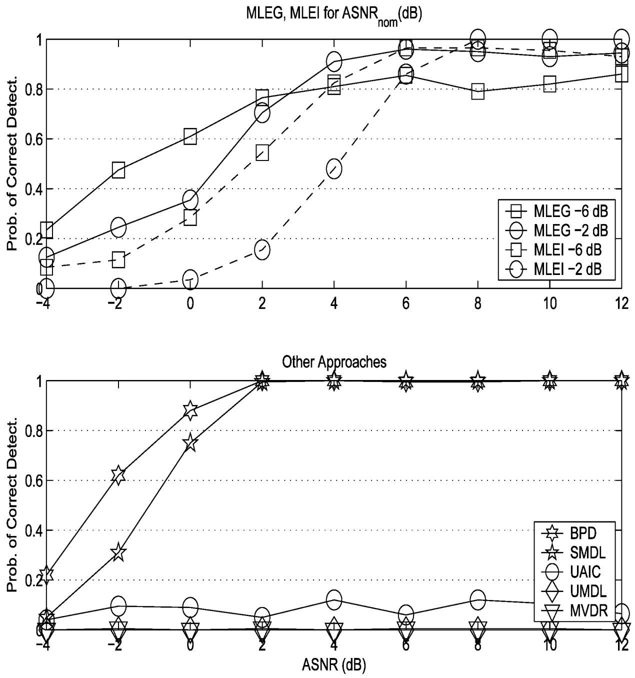

Fig. 7. Probability of correct detection versus ASNR for two equal power fully correlated signals ($\rho_{12} = j$) with $\theta_1 = 15°$, $\theta_2 = 20°$, $N = 6$, $K = 200$, trials = 200.

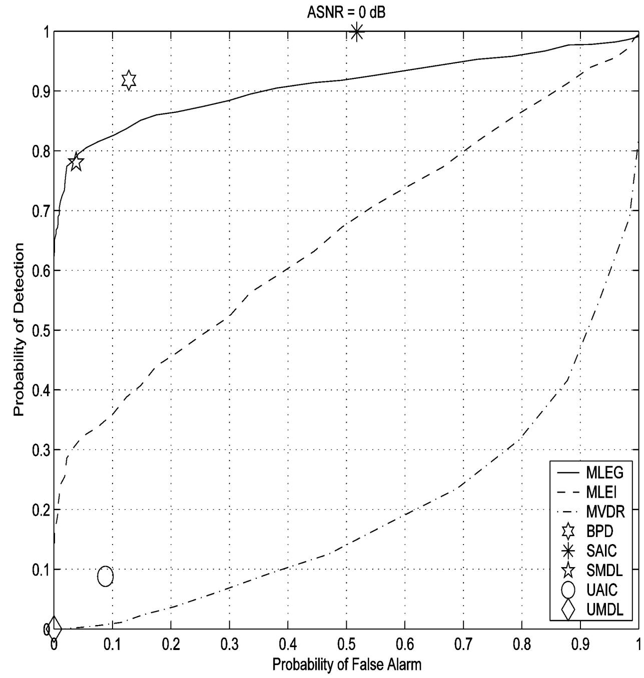

Fig. 8. ROC performance for two equal power fully correlated signals ($\rho_{12} = j$), with $\theta_1 = 15°$, $\theta_2 = 20°$, $N = 6$, $K = 200$, ASNR = 0 dB, trials = 1000.

where it increases slowly. This effect comes into play since estimated DOAs are ignored. Other metrics of detection performance could potentially take into account the plausibility of estimated DOAs versus actual DOAs.

Figs. 7 and 8 show the fully correlated signals case with a 90° phase difference. Correlation coefficient $\rho_{12} = \mathrm{E}[s_1 s_2^*]/(\sigma_1\sigma_2) = j$. There is a clear advantage for BPD, MLEG, and SMDL with the difference between them predicted by P_f. The correlated signal model in BPD, MLEG, and SMDL is optimum for this case and is the reason for their performance. The resolvability saturation effect for MLEG and MLEI can be seen in Fig. 8 but is not as pronounced as in Fig. 6. MLEI does not model signal correlation and, thus, is unable to take advantage of signal correlation if it exists. However, performance is not degraded with respect to independent signals. The same is not true for UAIC and UMDL. The unstructured model renders them ineffective in this case. All an unstructured approach can do is determine the rank of the signal subspace, which is one for two fully correlated signals.

A partially correlated signals case is shown in [8]. This is the "in between" case of independent and fully correlated signals. Performance is consistent with the independent and fully correlated signals cases.

In all cases, when the correct number of signals is detected, DOA estimation performance is the same as standard ML performance. At high SNR, the mean-square estimation error (MSE) is predicted by the Cramér-Rao bound. At low SNR, there is a reduced threshold effect, since many false estimates which normally contribute to the increase in MSE are eliminated. Plots of estimation performance are not included here but can be found in [5].

VI. SUMMARY AND CONCLUSIONS

A new technique for combined detection and DOA estimation of narrowband sources of energy has been presented. Existing approaches with satisfactory performance on closely spaced sources have no ability to control false alarms. The primary contribution of the presented approach is the ability to control false alarm and signal(s) present performance rather than having to accept predetermined levels which may not be satisfactory. This is accomplished by the use of ASNR MLEs as detection statistics. This contribution puts the array detection problem on the same footing as other detection problems, where trade-off of P_d and P_f is standard design procedure. The value of this trade-off is demonstrated in the performance results where signal(s) present performance is shown to be highly dependent on false alarm performance as expected. This is true even for existing approaches, where no such trade-off is possible. Multiple signal performance exceeds performance of existing approaches for independent signals and is comparable for correlated signals, with similar computational complexity.

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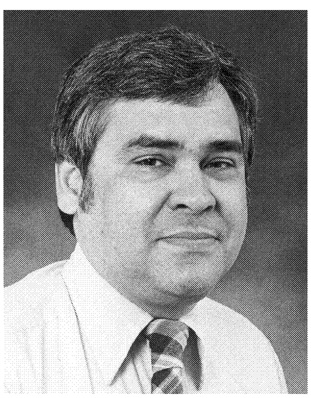

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