AVAILABILITY ENHANCEMENTS FOR CAT IIIB LAAS*

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BIOGRAPHIES

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ABSTRACT

Standards are currently being developed for using LAAS for Category IIIb (CAT IIIb) operations. Since the performance requirements are more stringent than for CAT I, LAAS CAT IIIb faces the challenge of achieving adequate availability of good geometry from ranging sources used in the position solution. This paper analyzes the relative availability improvement predicted for various alternatives to enhance LAAS CAT IIIb availability. The analysis considers increasing the number of ranging sources by employing Wide Area Augmentation System (WAAS) geostationary satellites (GEOs) or airport pseudolites (APLs). Alternative error models are considered for airborne noise, airborne multipath and residual ionospheric delay. The potential of taking credit for reduction in airborne error contributions due to comparison among redundant navigation sensors such as via mid-value selection (MVS) is also explored. The analysis also includes use of a second frequency (e.g., L2) to remove ionospheric delay from the carrier smoothing process and thus permit better smoothing of multipath errors both on the ground and in the aircraft. A final alternative considered is use of highly accurate differential carrier phase measurements in a position solution to reduce vulnerability to loss of a critical satellite during the phase of the approach and landing below 500 feet above ground level. Availability under each scenario is computed for the vertical alert limit (VAL) ranging from 5.3 m (current requirement) to 10.0 m. Results show that if VAL is not increased beyond about 5.3 m, adequate (0.999) availability cannot be achieved by merely extending the CAT I architecture or by any one of the above methods alone, except using dual frequency in both the ground facility and the aircraft. However, since a second GPS civil frequency for use in the aircraft won’t be provided until circa 2014, several of the above techniques would need to be combined (e.g., reduced airborne error, reduced ionospheric error and dual frequency in the ground facility) to give adequate availability for VAL = 5.3 m. On the other hand, if VAL could be increased to approximately 7.5 m, some other alternatives alone and most pair-wise combinations of these alternatives would provide adequate availability. The final choice of a LAAS architecture for CAT IIIb will, of course, consider additional factors beyond availability, such as ease of extension from CAT I, cost and technical risk.

INTRODUCTION

Background

Standards have been previously developed for airborne and ground equipment to use the GPS Local Area Augmentation System (LAAS) for Category I (CAT I) precision approach operations [1, 2, 3]. Work is currently underway towards developing the corresponding standards for using LAAS for Category IIIb (CAT IIIb) operations. Since the performance requirements are more stringent for CAT IIIb, LAAS faces the challenge of

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achieving adequate availability of good geometry from ranging sources used in the position solution. Such availability is affected by the number of ranging sources and their accuracy in comparison to the vertical alert limit (VAL), which specifies the level the error may exceed with a very small probability. Preliminary studies have predicted that the availability of good ranging source geometry for CAT IIIb LAAS may not be adequate if based on simple extension of the CAT I architecture and error models, in relation to a required VAL of 5.3 m [1]. Previous work has proposed retaining the CAT I VAL of 10.0 m for CAT IIIb [4, 5]. On the other hand, values of VAL even smaller than 5.3 m have been proposed for CAT IIIb LAAS (e.g., 4.4 m [6]). As of this writing the required VAL for CAT IIIb LAAS is under review.

Purpose and Organization of Paper

This paper analyzes the relative availability improvement predicted for various alternatives (in addition to increasing VAL) to enhance LAAS CAT IIIb availability. The paper begins with a brief summary of how availability is computed, including the role of the satellite constellation state probabilities assumed. The primary availability criteria are then reviewed, including those reflecting requirements for accuracy, integrity and continuity. A brief description of the baseline scenario and each alternative is then presented, including basic concept and assumptions. Results are then given for the baseline scenario, which assumes standard constellation state probabilities. Next, results are presented for more optimistic historical constellation performance, to show the sensitivity of availability to the state probabilities assumed. Finally, availability results are presented for each alternative alone and in combination with other alternatives (assuming standard constellation probabilities). The reader who is familiar with the background of the model and the alternative concepts may wish to skip directly to the results and the general summary at the end of the paper.

AVAILABILITY CALCULATION

Instantaneous Availability

In this paper, availability of LAAS ranging source geometry at a particular location is computed as the 24-hour average of “instantaneous” availability values. These instantaneous values are computed at multiples of a fundamental time step Δt (5 minutes in this analysis). Each instantaneous value is computed by evaluating every possible specific set of the visible ranging sources that could be operating. These sets vary in size from all-in-view, down to a minimum of four ranging sources. If a particular set satisfies the criteria for providing CAT IIIb service (described below), the probability that specific set might be operating is included in the total instantaneous availability for the analysis time.

$$\text{Avail}_{24\text{-hr avg}} = \frac{1}{L}\sum_{j=0}^{L-1}\text{Avail}_{\text{instantaneous}}\left(j \times \Delta t\right) \; ; \qquad L = \frac{24\text{hrs}}{\Delta t} \tag{1}$$

$$\text{Avail}_{\text{instantaneous}} = \sum_{\text{Set Size}\,=\,4}^{\text{All-in-View}} B_{\text{set}} \times \text{Prob}\left\{\text{Set Operating}\right\} \tag{2}$$

$$B_{\text{set}} = \begin{cases} 1, & \text{if set meets CAT IIIb criteria} \\ 0, & \text{otherwise} \end{cases}$$

Constellation State Probabilities

The probability a particular set of ranging sources is operating will be illustrated assuming for simplicity that all ranging sources are satellites with the same failure characteristics. The extension to multiple types of ranging sources is not difficult, but beyond the scope of this paper. Let P(c,n,g) be the probability that a particular specific set of g satellites are operating out of n visible satellites in a total constellation of size c. In [7] it was shown that

$$\text{Prob}\left\{\text{Set Operating}\right\} = P\left(c,n,g\right) = \sum_{O=g}^{c-n+g} f\left(c,n,g,O\right) \times P\left(O,c\right) \tag{3}$$

where O is the total number of satellites operating with probability P(O,c) (known herein as the constellation state probability) and f(c,n,g,O) is a function involving factorials of its arguments. Therefore, only P(O,c) depends on the failure and restoration performance of the constellation.

Values for P(O,c) are sometimes derived indirectly from a Markov model involving the satellite mean-time-between-failures (MTBF), mean-time-to-restore (MTTR) and restoration strategy. However, in this analysis they are directly specified, based on information in the GPS Standard Positioning Service Standard (SPS Standard) [8]. Figure 2-3 of [8] gives values of “standard” probabilities for O from 21 to 24. However, these probabilities do not sum to 1.0, since the model considers the possibility of more than 3 satellite failures. Consequently, an extrapolation function was developed to compute all “standard” probabilities needed for this analysis as shown in Table 1 below. Figure 2-3 of [8] also gives values of “historical” probabilities for O from 21 to 24. There have never been fewer than 21 GPS satellites operating since full operating capability (FOC) was declared. Table 2 below gives the historical probabilities used in this analysis. As can be seen by comparing the two tables, the historical GPS constellation performance is somewhat better than that modeled by the “standard” constellation probabilities.

Table 1. Constellation State Probabilities (Fit to Standard in [8])

Number of Operating Satellites (O)	Probability of Exactly O Satellites Operating P(O,c)	Probability of At Least O Satellites Operating
24	0.95	0.95
23	0.03	0.98
22	0.012	0.992
21	0.0048	0.9968
< 21	$0.03 \times 0.4^{(23-O)}$	$0.9968 + 0.03 \times 0.4^{(23-O)}$

Table 2. Constellation State Probabilities (Historical [8])

Number of Operating Satellites (O)	Probability of Exactly O Satellites Operating P(O,c)	Probability of At Least O Satellites Operating
24	0.985056	0.985056
23	0.014839	0.999895
22	0.000104	0.999999
21	0.000001	1.0

AVAILABILITY CRITERIA

To provide service a specific set of ranging sources must satisfy the following two types of criteria:

- Protection Level Based on Set ≤ Alert Limit
- Number of Critical Ranging Sources in Set ≤ Limit

Protection Levels

Both lateral protection levels (LPLs) and vertical protection levels (VPLs) are specified for LAAS [1]. However, only VPLs are discussed here because requiring a protection level to be less than the alert limit is always more stringent in the vertical dimension. The following four vertical protection levels pertain to CAT IIIb availability

> VPL_{H0}: Limits integrity risk due to large fault-free ranging source errors
>
> VPLe: Limits integrity risk due to large undetected ephemeris error
>
> VPL_{H1}: Limits integrity risk due to large undetected reference receiver fault
>
> $PVPL_{H1}$: Limits continuity risk due to false detection of reference receiver fault

Although all four are considered, VPL_{H0} is the most stringent in the scenarios compared in the analysis. Therefore, for simplicity the detailed discussion that follows will deal only with VPL_{H0}. For further information the reader is referred to the LAAS CAT I MOPS [2] (VPLe) and to the LAAS MASPS [1] (VPL_{H1} and $PVPL_{H1}$).

VPL_{H0} represents a high probability bound on the vertical error, i.e., the level of vertical error that is exceeded with a very small probability expressed as

$$VPL_{H0} = K_{ffmd} \times \sigma_{vert} \tag{4}$$

where K_{ffmd} is derived assuming that the vertical error is Gaussian distributed (or overbounded by a Gaussian distribution) characterized by σ_{vert}. For LAAS CAT IIIb using 4 reference receivers, $K_{ffmd} = 6.673$ [1]. The vertical standard deviation σ_{vert} represents fault-free residual error contributions (after application of LAAS differential corrections) from all of the N ranging sources transformed into the position domain

$$\sigma_{vert} = \sqrt{\sum_{i=1}^{N} S_{i,vert}^2 \times \sigma_i^2} \tag{5}$$

where $S_{i,vert}$ is the vertical coefficient for the ith ranging source in the position solution [1]. The error standard deviation of the ith ranging source σ_i includes four components

$$\sigma_i = \sqrt{\sigma^2_{pr\_gnd,i} + \sigma^2_{air,i} + \sigma^2_{iono,i} + \sigma^2_{tropo,i}} \qquad (6)$$

The values of $\sigma_{pr\_gnd,i}$, $\sigma_{air,i}$, $\sigma_{iono,i}$ and $\sigma_{tropo,i}$ are often functions of the ranging source elevation angle θ_i and in certain cases functions of the distance from the LGF reference receiver centroid. These functions vary with the type of ranging source and error model scenario. For simplicity of notation, the subscript i will be omitted in the scenario descriptions below.

Although K_{ffmd} is derived from an integrity requirement [1], VPL_{H0} is essentially an accuracy measure, because it represents errors that occur when no faults are present. However, the value of K_{ffmd} is fixed by the assumption of Gaussian bounded errors and the bounding probability requirement. Consequently, the only way to make it easier for a given set of ranging sources to meet the criterion represented by VPL_{H0} is to either increase VAL or decrease the ranging source errors. (Of course, availability can also be increased by decreasing the probability of a ranging source failure or increasing the number of ranging sources.)

Critical Ranging Sources

A particular ranging source is said to be critical in a specific set, if the other sources in the set cannot provide service without it. That is, if a critical ranging source fails, the subset without it does not meet the requirement that $VPL_{H0} \leq VAL$. Loss of service during the approach is considered more problematic for CAT IIIb than for CAT I because LAAS provides vertical guidance much closer to the ground for CAT IIIb and also rollout guidance. Therefore, the continuity risk due to loss of adequate ranging source geometry must be appropriately limited for CAT IIIb. The main contributor to this risk is loss of a ranging source due to either actual failure or false detection of a ranging source fault by ground monitoring [1]. For a particular set of ranging sources in use, the total such continuity risk is proportional to the number of critical ranging sources in the set. A prospective continuity risk allocation in [1] set a limit of 2 critical ranging sources for CAT IIIb. Thus, when a particular set of ranging sources of size N is evaluated for availability, all N subsets of size N-1 must also be checked. If at most 2 critical satellites are allowed, at least N-2 of these subsets must provide $VPL_{H0} \leq VAL$.

SCENARIOS

Scenarios examined include a baseline and various alternatives. The alternatives include the following basic methods grouped by rationale for increasing availability:

- Decrease probability of satellite failures
  - Historical constellation state probabilities
- Increase number of ranging sources
  - Geostationary Satellites (GEOs)
  - Airport Pseudolites (APLs)
- Decrease ranging source errors
  - Reduced airborne error model
  - Reduced ionospheric error model
  - Dual frequency (ground only)
  - Dual frequency (ground and air)
  - Differential carrier
- Reduce position domain error
  - Credit for position redundancy in aircraft (mid-value selection)

The baseline scenario will be described in detail first, followed by each alternative. For simplicity, the description of each alternative will only note changes relative to the baseline.

Baseline

Locations Analyzed: 78 in conterminous U.S. (CONUS) identified as current ILS CAT III sites or prospective CAT III LAAS sites (results are not runway specific, except for APLs)

Constellation State Probabilities: Standard [8]

Distance X of analysis point from reference receiver centroid: 1/3 nmi (0.617 km) (100 ft AGL point if LGF reference receiver centroid is at glide path intercept point (GPIP))

Maximum Number of Critical Satellites Allowed: 2 [1]

Ground Error In this analysis the ground error will be assumed to consist of two components as presented in the LAAS Ground Facility (LGF) Specification [3]. The first component represents reference receiver measurement errors due to noise and multipath. The second component represents additional “residual” errors such as ground-to-airborne multipath and LGF reference receiver relative survey error (see Appendix D of [1]). The resulting general expression for the broadcast σ_{pr_gnd} is

$$\sigma_{pr\_gnd} = \sqrt{\frac{\sigma_{RR}^2}{M} + \sigma_{residual}^2} \qquad (7)$$

In this analysis the LGF is assumed to have M = 4 reference receivers of capability denoted by Ground Accuracy Designator C (GAD-C4) [1] giving

$$\begin{aligned}\sigma_{RR} &= 0.15 + 0.84 \times e^{-\frac{\theta}{15.5}} \text{ m} ; \ \theta \geq 35 \text{ deg} \qquad (8)\\ \sigma_{RR} &= 0.24 \text{ m} ; \ \theta < 35 \text{ deg}\end{aligned}$$

$$\sigma_{residual} = 0.04 \text{ m} \qquad (9)$$

Airborne Error The airborne equipment performance is assumed to be characterized by airborne accuracy designator B (AAD-B). The corresponding airborne error is the sum of contributions from receiver noise and airframe multipath [9] giving

$$\sigma_{air} = \sqrt{\sigma_{air\_noise}^2 + \sigma_{air\_multipath}^2} \qquad (10)$$

$$\sigma_{air\_noise} = 0.11 + 0.13 \times e^{-\frac{\theta}{4}} \text{ m} \qquad (11)$$

$$\sigma_{air\_multipath} = 0.13 + 0.53 \times e^{-\frac{\theta}{10}} \text{ m} \qquad (12)$$

Ionospheric Delay Error The residual error due to ionospheric delay is characterized by a standard deviation of the gradient $\sigma_{vert\_iono\_gradient}$ or decorrelation of the vertical delay over distance. A value of $\sigma_{vert\_iono\_gradient}$ = 4 mm / km is assumed for the baseline [9]. The impact of this gradient is captured in σ_{iono} in two ways. First, σ_{iono} includes a term related to the distance X of a user from the LGF reference receiver centroid. Second, since the approaching aircraft is moving with a velocity V_{air} through a spatial gradient of ionospheric delay, a relative error with standard deviation $\sigma_{vert\_iono\_gradient} 2\tau V_{air}$ accumulates due to first order smoothing of the ionospheric delay in both the LGF and aircraft with time constant τ = 100 s [9]. In the baseline scenario, the aircraft velocity is assumed to be 250 kts (0.129 km/s) (an extremely conservative value that is decreased in a later alternative scenario) [10]. Finally, the vertical ionospheric delay is amplified by an obliquity factor derived from the elevation angle to the satellite [2]. The model for residual ionospheric delay error is

$$\begin{aligned}\sigma_{iono} &= \text{obliquity}\\ &\times \sigma_{vert\_iono\_gradient}\left[X + 2\tau V_{air}\right]\\ &= \text{obliquity} \times 4 \text{mm} / \text{km} \qquad (13)\\ &\times \left[0.617\text{km} + 2 \times 100\text{s} \times 0.129\text{km} / \text{s}\right]\\ &= \text{obliquity} \times 0.106 \text{ m}\end{aligned}$$

Tropospheric Delay Error The residual error due to tropospheric delay depends on the difference in altitude between the aircraft and LGF [1]. However, due to the relatively small separation between the LGF and the availability analysis point (1/3 nmi) for simplicity it is assumed that

$$\sigma_{tropo} = 0 \qquad (14)$$

Comparison of Baseline Error Components

Figure 1 shows a plot of the baseline values for σ_{pr_gnd}, σ_{air} and σ_{iono} and the resulting total ranging error σ as a function of satellite elevation angle. It is interesting to note from the figure that the airborne error is the single

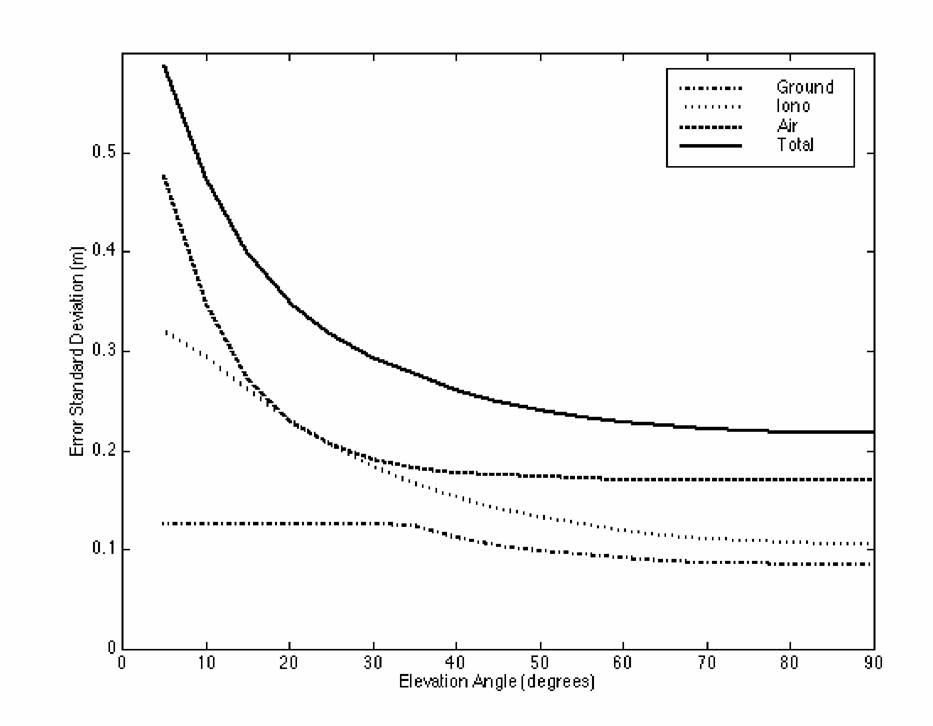

Figure 1. Pseudorange Error Components and Total Error for Baseline Scenario

largest component followed by the ionospheric error. On the other hand, the ground error component is small by comparison, especially at low satellite elevation angles.

Historical Constellation State Probabilities

In this scenario, the constellation state probabilities are reduced from the standard values in Table 1 to historical values in Table 2 (refer back to the discussion above in the section on Availability Calculation).

Geostationary Satellites (GEOs)

The baseline scenario considers only GPS satellites. However, the LGF must be capable of generating differential corrections for geostationary satellites (GEOs) that are part of the Wide Area Augmentation System (WAAS) [3]. Thus, appropriately capable airborne receivers can also make use of WAAS GEOs as ranging sources for LAAS. In the time frame in which LAAS CAT III might be introduced (circa 2008), GEOs under consideration for WAAS are planned to be located at 125 deg W (wide-band), 107 deg W (wide-band) and 98 deg W (narrow-band) Longitude [11]. Accuracy of the WAAS GEOs as ranging sources depends on the bandwidth of the signal they transmit. Wide-band WAAS GEOs transmit a signal with bandwidth comparable to GPS satellites and therefore are assumed to have ranging errors of the same size also. For narrow-band WAAS GEOs, the ranging performance (at minimum signal power) is specified in [2] as 1.8 m for the airborne error root-mean-square (RMS) and in [3] as 1.8 / 2 (M = 4) for the ground error RMS. However, significantly better performance has been observed from current narrow-band WAAS GEOs [12]. Therefore, in the availability analysis the GEO errors are all assumed to have the same standard deviation as for GPS, with the exception of the ground error contribution for narrow-band GEOs

$$\sigma_{pr\_gnd\_narrow\_GEO} = 0.3 \text{ m} \qquad (15)$$

Airport Pseudolites

An airport pseudolite (APL) would broadcast a GPS-like ranging signal from a transmitter located on the airport property. Consequently, it has always been recognized that a significant challenge associated with APLs is the significant variation in power of the received signal from the time of acquisition as the aircraft first enters the terminal area until the approach is completed near the APL. The power level of GPS satellites or GEOs is essentially constant during this time since they are so far away from the aircraft. Therefore, it is difficult to select the power of the APL so it is strong enough to be useful in the terminal area, but does not swamp out the relatively weak satellite ranging signals close to the airport. A detailed discussion of techniques to overcome this “near-far” signal power variation of APLs is beyond the scope of this paper. Suffice it to say that APLs have been successfully tested as ranging sources in conjunction with GPS satellites [13, 14].

The availability benefit of an APL depends on its location and ranging errors. The APL is basically a very low elevation ranging source (elevation angle is actually negative relative to an aircraft on approach). However, the direction of the APL range measurement relative to satellite ranging sources can vary significantly when the aircraft is at the 100 ft altitude point, depending on the specific location of the APL relative to the runway. Consequently, it is possible to optimally place an APL to compensate for loss of low elevation satellites (either due to failure or simply going out of view before another satellite rises) needed for good vertical geometry. However, it may not be possible to site the APL at such an optimal location at a particular airport. Therefore for the purpose of this analysis the APL is assumed to be at an arbitrary location, midway down a 9,000 ft runway and displaced 500 feet to the right side of runway centerline. The corresponding availability results are computed for a particular set of 36 CAT III runways spread throughout CONUS.

The ground component of the residual range measurement error for an APL was estimated in [13] to be characterized by σ_{pr_gnd} = 0.2 m (M = 2 reference receivers). Therefore, since corrections are assumed to be averaged over 4 reference receivers, the availability analysis is based on

$$\sigma_{pr\_gnd\_APL} = 0.14 \text{ m} \tag{16}$$

The airborne receiver error for APLs has not been characterized in detail. Therefore, for simplicity, the airborne error was assumed to be the same as a GPS satellite at 3 degrees elevation angle giving

$$\sigma_{air\_APL} = \sigma_{air}(3 \text{ deg}) = 0.55 \text{ m} \tag{17}$$

The resulting total error standard deviation of approximately 0.6 m for ground and airborne error contributions is consistent with the total ranging errors observed in [14].

Reduced Airborne Error Model

The baseline model for airborne noise and multipath [9] was developed from flight test data using GPS receivers with standard correlator spacing [15]. However, some limited additional data using narrow correlator receiver technology with better multipath rejection was also gathered in subsequent flight tests [15]. The errors observed for the narrow correlator receiver [15] were actually closely bounded by a prospective multipath error model (less conservative than the baseline model from [9]) in the original LAAS MASPS [1]. As of this writing a new airborne multipath data collection effort is being planned. However, in lieu of extensive new data, the original model from [1] will be assumed in evaluating the potential of reducing the airborne multipath (and noise). The corresponding value of σ_{air} is given by the model for AAD-B from [1]

$$\sigma_{air} = 0.074 + 0.18 \times e^{-\frac{\theta}{27.7}} \text{ m} \tag{18}$$

Reduced Ionospheric Error Model

The baseline model for the ionospheric residual error assumes a $\sigma_{vert\_iono\_gradient}$ of 4 mm / km. This value was initially proposed as a reasonable bound for most foreseeable ionospheric conditions. However, it may be too conservative when the ionosphere is quiet, but not nearly large enough when the ionosphere is disturbed [10]. Moreover, recently identified ionospheric anomalies such as “wave fronts” may produce large transients of ionospheric delay, that arrive at the aircraft and LGF at different times and are, therefore, not fully compensated by the differential corrections [16]. A possible solution to this dilemma is to use a smaller value of divergence rate most of the time (say 1 mm / km [10]). Wave front detectors in the aircraft and on the ground could provide mitigation for local wave fronts. Possible means for using real time ionospheric delay information to increase the value of $\sigma_{vert\_iono\_gradient}$ above 1 mm / km when necessary include longer baseline measurements at a single LAAS site, networking of multiple LGFs or via a connection to WAAS. The baseline model also conservatively used an aircraft velocity of 250 kts (0.129 km/s). However, a more representative aircraft velocity on approach is thought to be no more than 180 kts (0.093 km/s). With the above changes from the baseline, the reduced ionospheric error model becomes

$$\begin{aligned} \sigma_{iono} &= \text{obliquity} \times 1 \text{ mm/km} \\ &\times [0.617\text{km} + 2 \times 100\text{s} \times 0.093\text{km/s}] \\ &= \text{obliquity} \times 0.019 \text{ m} \\ &(\text{as compared to baseline of } 0.106 \text{ m}) \end{aligned} \tag{19}$$

Dual Frequency (Ground Only)

Techniques for using GPS signals transmitted on a second frequency to overcome limitations of LAAS due to ionospheric delay have been previously proposed in detail in [17]. The ionospheric delay present on a satellite’s range measurement varies over time. The delay varies as the path length through the ionosphere changes with satellite elevation angle. The delay may also vary due to changes in the ionosphere itself over time and location of the pierce point of the satellite’s signal through the ionosphere to the aircraft or LGF. The LAAS carrier smoothing time constant of 100 s was chosen as a compromise between providing some reduction in noise and multipath, without introducing too much lag in following the ionospheric delay variation. One of the most promising dual frequency techniques in effect removes the ionospheric delay from the measurements that are carrier smoothed. Such “ionospheric divergence-free” smoothing might allow the smoothing time constant to be increased significantly and the noise and multipath correspondingly reduced. If the aircraft is not using dual frequency techniques, there might be an additional error due to smoothing of the ionospheric delay in the aircraft, but not in the LGF. A method proposed in [18] would smooth in parallel the ionospheric delay in the LGF with the usual 100 s time constant and add it back into the differential corrections before broadcast. The analysis assumes that the smoothing time can be increased enough to reduce σ_{RR} by a factor of two giving

$$\sigma_{pr\_gnd\_dual\_frequency} = \sqrt{\frac{\left(\sigma_{RR}/2\right)^2}{4} + \sigma^2_{residual}} \qquad (20)$$

This potential reduction in ground errors does not come without a price. The longer smoothing time constant requires a longer time for the filter to reach steady state after the satellite is first acquired. Moreover, a smoothing time increase by a factor of 8 (rather than the theoretical value of 4) was found to be necessary to reduce σ_{RR} by a factor of two [19]. Delaying use of a satellite for two smoothing time constants (1,600 s) after first being acquired would seriously impact availability and defeat the whole purpose of using dual frequency techniques [19]. However, it was found that merely inflating the broadcast value of σ_{pr_gnd} by a decreasing amount during the long initial filter settling time gives availability results nearly identical to those assuming the satellite can be fully utilized at the usual elevation angle of 5 degrees [19]. Therefore, for simplicity in this analysis, availability using dual frequency in the LGF is computed assuming that the increased filter settling time can be ignored.

Dual Frequency (Ground and Air)

Prospective techniques for using dual frequency in both the LGF and aircraft are described in detail in [17, 18]. The additional effects of also using ionospheric divergence-free smoothing techniques in the aircraft are two-fold. First, it is assumed that the carrier-smoothing time constant can be increased to the same value used in the LGF with the similar effect of decreasing the airborne noise and multipath standard deviation by a factor of two

$$\sigma_{air\_dual\_frequency} = \frac{\sigma_{air\_single\_frequency}}{2} \qquad (21)$$

Second, it is assumed that since the ionospheric delay is no longer being smoothed in the aircraft, there is no error component due to the aircraft velocity through the ionospheric spatial gradient. Therefore, σ_{iono} no longer includes the $\sigma_{vert\_iono\_gradient}\, 2\tau V_{air}$ term and becomes

$$\sigma_{iono} = \text{obliquity} \times \sigma_{vert\_iono\_gradient} \times X \qquad (22)$$

Differential Carrier

In this paper the term differential carrier refers to a method in which the aircraft uses the standard carrier-smoothed code (CSC) solution down to a relatively low altitude (say 500 ft AGL), but uses carrier measurements only thereafter (adjusted by differential corrections broadcast from the LGF) [20]. The aircraft position during low altitude flight, touchdown and rollout is based on the CSC solution at 500 ft AGL propagated forward by subsequent carrier measurements alone. Thus, the position error in the later phases of the procedure consists of a fixed component present in the CSC solution at 500 ft AGL plus random errors in the carrier measurements thereafter. The main original motivation for using such a differential carrier method arose from concerns about multipath at the aircraft antenna due to reflections off of the ground or water and objects (such as other aircraft) on the airport surface [20]. The impact of multipath on carrier phase measurements is much smaller than on code measurements. However, the author of [20] also recognized the potential for highly accurate differential carrier measurements to make it easier for ranging source geometry to meet the critical ranging source constraint for CAT IIIb.

This potential benefit is best illustrated by considering a corresponding vertical protection level proposed for differential carrier [21]. Recall that the general form of VPL_{H0} is

$$VPL_{H0} = K_{ffmd} \times \sigma_{vert} \qquad (23)$$

Let t_{500} be the time at which the aircraft descends to 500 ft AGL and switches from a CSC solution to a differential carrier position solution. Then the vertical error $E_{vert}(t)$ for $t > t_{500}$ is the sum of a CSC component and a carrier component

$$E_{vert}(t) = E_{vert\_CSC}(t_{500}) + E_{vert\_carrier}(t) \; ; \quad t > t_{500} \tag{24}$$

Therefore, σ_{vert} in the VPL_{H0} is the root-sum-square (RSS) of two corresponding terms

$$\sigma_{vert} = \sqrt{\sigma^2_{vert\_CSC}(t_{500}) + \sigma^2_{vert\_carrier}(t_{100})} \tag{25}$$

Note that the CSC solution errors are characterized at the 500 ft altitude point, but the carrier errors are characterized at the 100 ft altitude point (as is the VAL to which VPL_{H0} is compared in the availability criterion). Recall that the 100 ft altitude point corresponds to a distance X = 1/3 nmi from an LGF reference receiver centroid located at the GPIP. The corresponding 500 ft altitude point would be at X = 1.67 nmi. Note that in general both the CSC and carrier measurement errors are functions of X. However, as of this writing no detailed dependence of the carrier errors on X has been formulated. Moreover, the ranging source geometry is very nearly the same at t_{500} and t_{100}. Therefore, for simplicity in the analysis the vertical standard deviations are formulated as follows

$$\sigma^2_{vert\_CSC}(t_{500}) = \sum_{i=1}^{N} S^2_{vert,i}(t_{100})\sigma^2_{CSC,i}(t_{500}) \tag{26}$$

$$\sigma^2_{vert\_carrier}(t_{100}) = \sum_{i=1}^{N} S^2_{vert,i}(t_{100})\sigma^2_{carrier,i} \tag{27}$$

From the above discussion it is clear that VPL_{H0} used in the availability criterion for differential carrier is actually slightly larger than for the usual CSC solution, because a carrier error is included and the CSC errors are evaluated at a greater distance from the LGF. Even so, it will now be made clear why differential carrier provides a potential benefit with regard to meeting the critical ranging source limit for CAT IIIb.

Recall that loss of a critical ranging source is a continuity risk because the remaining ranging sources cannot provide adequate vertical guidance. However, this concern applies only to a relatively short period of time when the aircraft is close to the ground (certainly below 500 ft AGL). Therefore, the critical satellite consideration (limit of 2 herein) applies only to the satellites used in the differential carrier solution and thus only to $\sigma_{vert\_carrier}$. Due to the much smaller error of the carrier, this critical ranging source limit is expected to be much easier to meet than when it applies to the CSC solution. Moreover, the performance is anticipated to approach that of CSC at 500 ft AGL without any limit on the number of critical satellites. This was found to be the case for values of $\sigma_{carrier}$ up to about 25 mm [22]. Since this value is considered to be very conservative (it is a significant fraction of the carrier wavelength of 192 mm) the availability analysis assumes that

$$\sigma_{carrier,i} = 0.025 \text{ m} \tag{28}$$

Mid-Value Selection (MVS)

It has recently been pointed out that the LAAS protection levels do not take advantage of redundant position comparisons commonly done in CAT III capable aircraft [23]. Since separate GPS antennas and receivers are used, it is appropriate to consider that both the airborne multipath and receiver noise are independent among the multiple positions computed. Therefore, such a comparison can in effect reduce the airborne contribution to the total position error. One such method (know as mid-value selection) selects the median of three independently generated positions. Under the assumption that the airborne errors in each position are Gaussian distributed, mid-value selection corresponds to halving the error variance of airborne error from a single receiver [24]. Thus, the benefit of mid-value selection is represented by

$$\sigma_{air\_mvs} = \frac{\sigma_{air}}{\sqrt{2}} \tag{29}$$

Comparison of Total Error for Alternatives

Figure 2 shows a plot of the total differential ranging error σ as a function of satellite elevation angle for the baseline scenario and 5 of the above alternatives. Note that most alternatives alone give relatively small reductions in the total error. However, use of dual frequency in both the LGF and aircraft would potentially reduce the total ranging error σ by more than a factor of two.

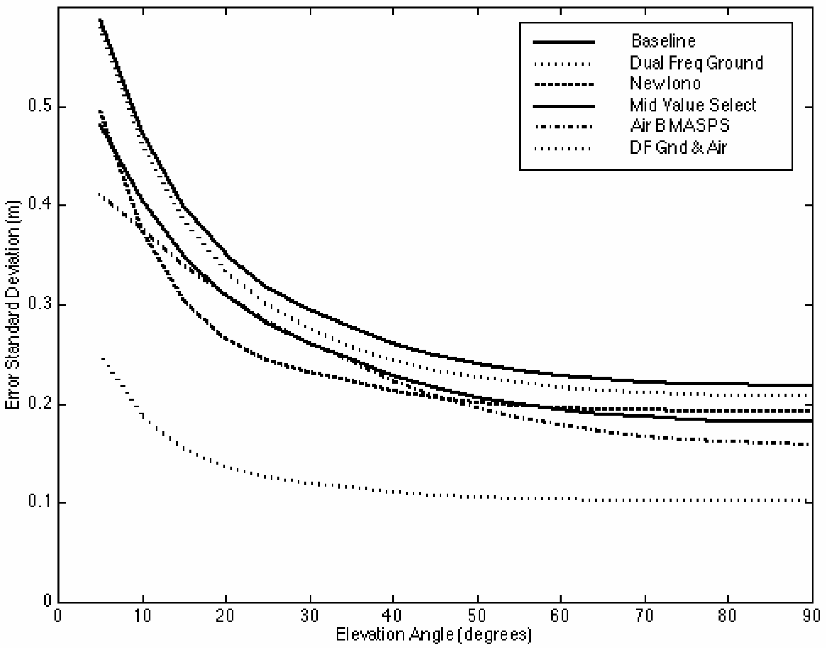

Figure 2. Total Pseudorange Error for Alternative Scenarios

RESULTS

In this analysis for availability performance to be considered acceptable, the following two conditions must be satisfied at every location analyzed:

- Daily average availability at least 0.999
- No outages with all ranging sources operating

Baseline Availability

Results for the baseline scenario are given in Figure 3. The figure actually shows unavailability = 1 – availability, which is more informative than availability in a graphical presentation of this type. Therefore, lower (rather than higher) values in the figure represent better performance and a value of 0.001 corresponds to 0.999 availability. Each group of unavailability bars represents the 24-hour average results for all 78 locations analyzed. The groups correspond to VAL ranging from 5.3 m to 10.0 m. Note that for VAL = 5.3 m, unavailability at most locations is about a factor of 50 too large, with the worst performance being nearly a factor of 100 too large (availability ~ 0.9). However, as VAL is increased to 7 – 8 m, results at some locations drop by an order of magnitude in comparison to the worst locations. This is due to the worst locations having outages even when all satellites are operating, whereas the drastically better locations do not. When, the VAL is increased to 10.0 m, the performance is roughly the same at all locations and is also somewhat better than 0.001. Thus, for the baseline scenario, VAL would have to be increased to 10.0 m for acceptable availability.

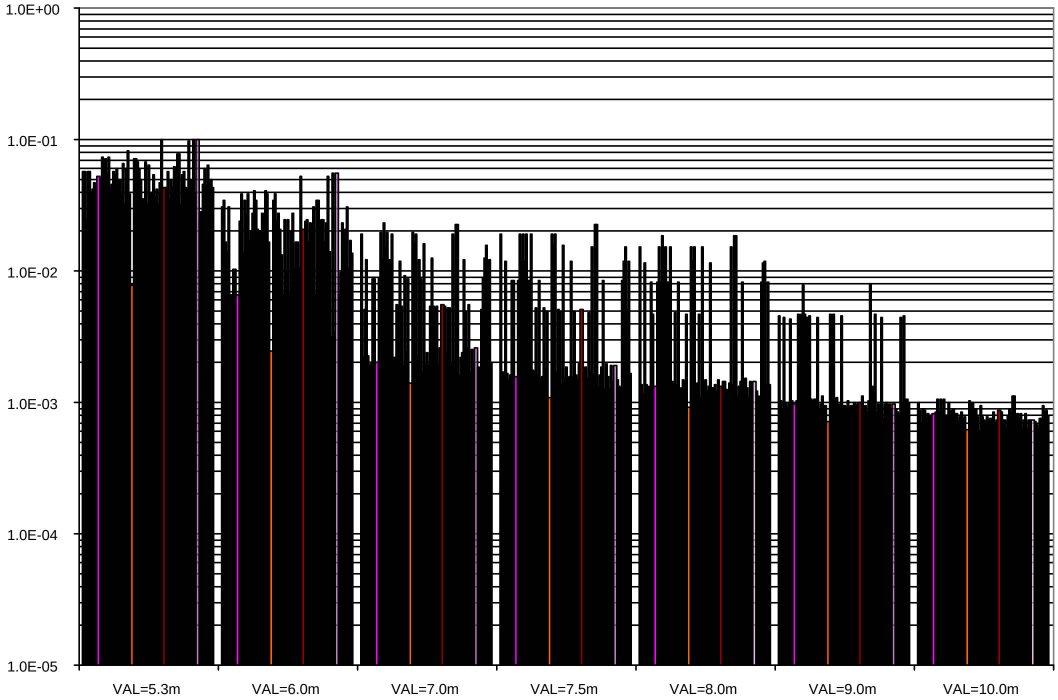

Figure 3. Unavailability Results for Baseline

Historical Constellation State Probabilities

Results for more optimistic historical (rather than standard) constellation state probabilities are shown in Figure 4. Note that the results for VAL = 5.3 to 6.0 m are essentially unchanged relative to the baseline in Figure 3. This is due to the fact (as mentioned above) that results for these VALs are driven by outages when all satellites are operating. By referring to Tables 1 and 2, it can be seen that the probability of all 24 satellites operating is very nearly the same for both standard and historical values. However, the state probabilities corresponding to several satellite failures differ by a relatively large amount. Thus, as VAL is increased and results for some locations are driven by the probability of more satellite failures, the difference in performance between the best and worst locations becomes considerably larger than it was for the standard probabilities (Figure 3). However, the situation of outages when all satellites are operating still persists at the worst locations until VAL is increased to 10.0 m as it did for the baseline scenario. Therefore, the availability challenge faced by LAAS CAT IIIb would not be mitigated simply by assuming more optimistic satellite failure and restoration performance. In lieu of increasing VAL to 10.0 m, additional alternatives such as those described above to increase the number of ranging sources or decrease ranging source errors must be considered.

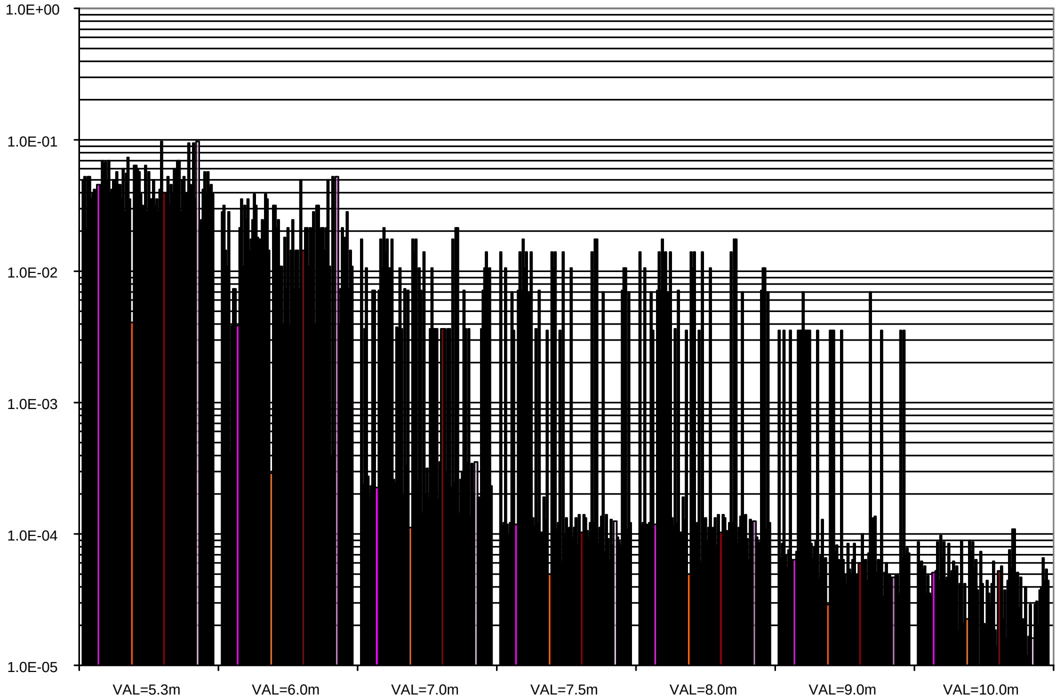

Figure 4. Unavailability Results for Historical Constellation State Probabilities

Individual Alternatives

Availability (unavailability) results similar to those shown in Figure 3 for the baseline were also computed for all of the alternative scenarios analyzed (also assuming standard constellation state probabilities). However, rather than presenting such detailed results, the remainder of the paper will focus merely on the minimum VAL that provides acceptable availability for a given scenario. These results for using the alternatives individually are compared in Figure 5. From the figure it can be seen that

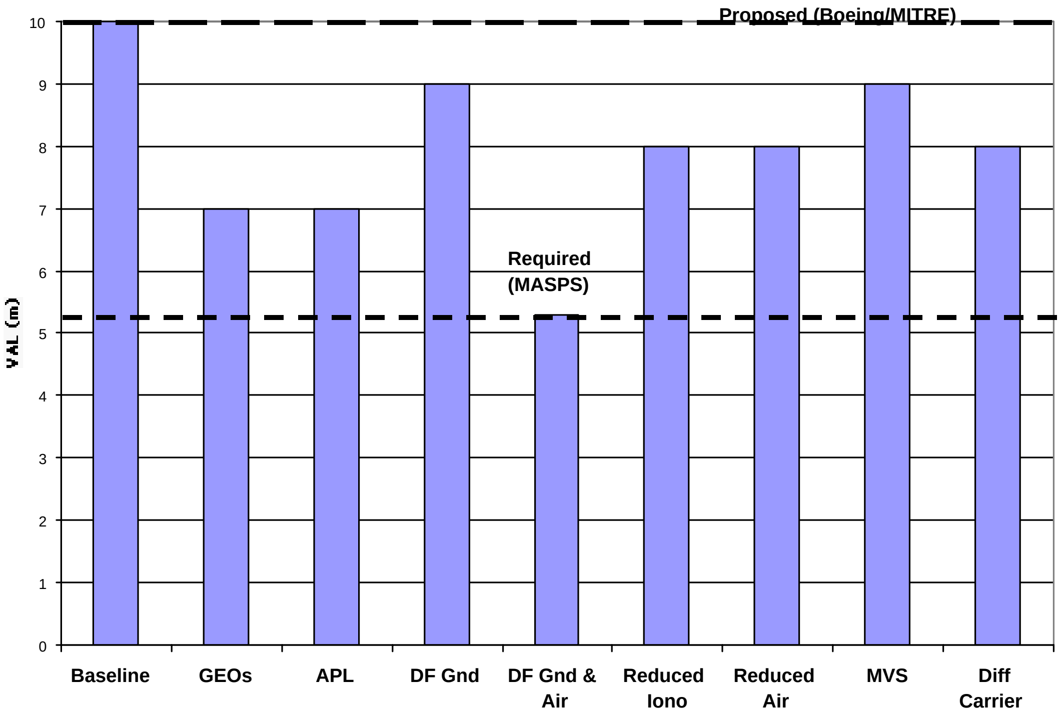

Figure 5. Smallest VAL for Acceptable Availability – Individual Alternatives

the only alternative that has the potential to provide acceptable availability for VAL = 5.3 m is use of dual frequency in both the LGF and the aircraft. It should be pointed out that the LGF might use the currently provided military L2 signal as the second frequency even though it is not protected from interference. However, a second civil frequency (L5) that is protected and therefore acceptable for use in the aircraft is not expected to reach full operating capability (FOC) until circa 2014. Therefore, since the goal is introduction of CAT IIIb capable LAAS circa 2008, using dual frequency in both the LGF and aircraft is not really a viable alternative. If VAL could be increased to some intermediate value (say 7.0 – 7.5 m), GEOs alone or APLs alone have the potential to give acceptable availability for LAAS CAT IIIb. However, even with VAL increased to this level, there may be some risk in linking the success of LAAS CAT IIIb to the presence of WAAS GEOs at the assumed locations. Moreover, APLs may face difficulties in siting and compatibility with airborne equipment. Therefore, it is desirable to consider combinations of alternatives that could provide adequate availability at an intermediate VAL or (of course) the original VAL = 5.3 m.

Pair-wise Combinations of Alternatives

If VAL can be increased to approximately 7.5 m, some pair-wise combinations of alternatives (in addition to the individual alternatives GEOs, APLs, and dual frequency in both LGF and aircraft) might provide adequate availability. Results for the remaining alternatives (differential carrier, dual frequency in LGF only, mid-value selection, reduced airborne error model and reduced ionospheric error model) used in pairs are shown in Figure 6. For each combination, Figure 6 shows the unavailability at the best location, median location and worst location assuming VAL = 7.5 m. Note that for 5 of the 10 possible combinations of alternatives, the unavailability at even the worst location meets the strict criterion of being less than 0.001 (0.999 availability). Note also that for 4 other combinations unavailability at the worst location only slightly exceeds 0.001 while the median performance is well under this requirement. Even for the combination of differential carrier and mid-value selection for which unavailability at the worst location is approximately 0.005, the median value is less than 0.001. Thus, most pair-wise combinations of alternatives would essentially provide adequate availability (some with significant margin) for an intermediate VAL of 7.5 m.

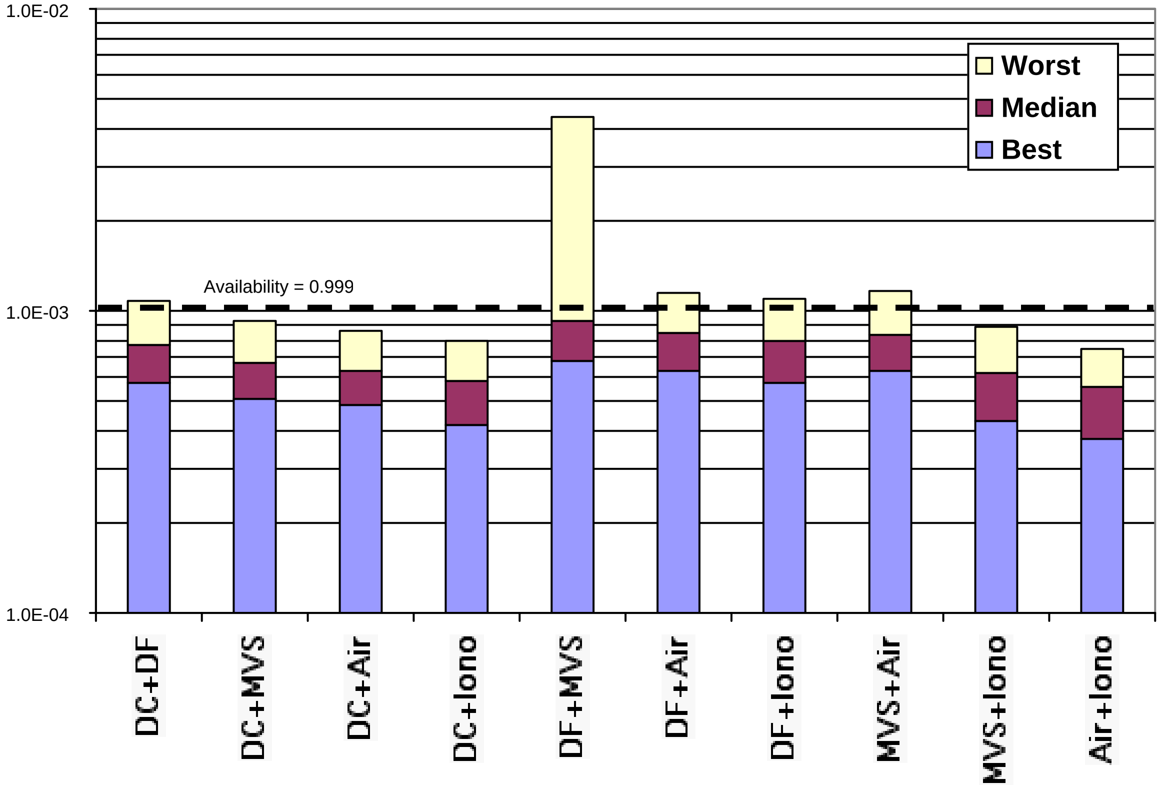

Figure 6. Unavailability Results for Alternative Pairs (VAL = 7.5 m)
(DC = Differential Carrier, DF = Dual Frequency Ground Only, MVS = Mid-Value Selection,
Iono = Reduced Ionospheric Error Model, Air = Reduced Airborne Error Model)

Combinations of Alternatives in Triples

If VAL must remain at 5.3 m, combinations of alternatives taken three at a time need to be considered. Figure 7 shows results for all 10 possible combinations of three alternatives (excluding GEOs, APLs, and dual frequency in both LGF and aircraft). Note that 5 such combinations would not be acceptable, because performance at the worst location is seriously degraded, even though the median value approximately meets the goal (0.001 = 0.999 availability). However, the remaining 5 combinations would likely be acceptable because performance at the worst location is just slightly worse than this goal while the median value provides good margin relative to acceptable availability performance.

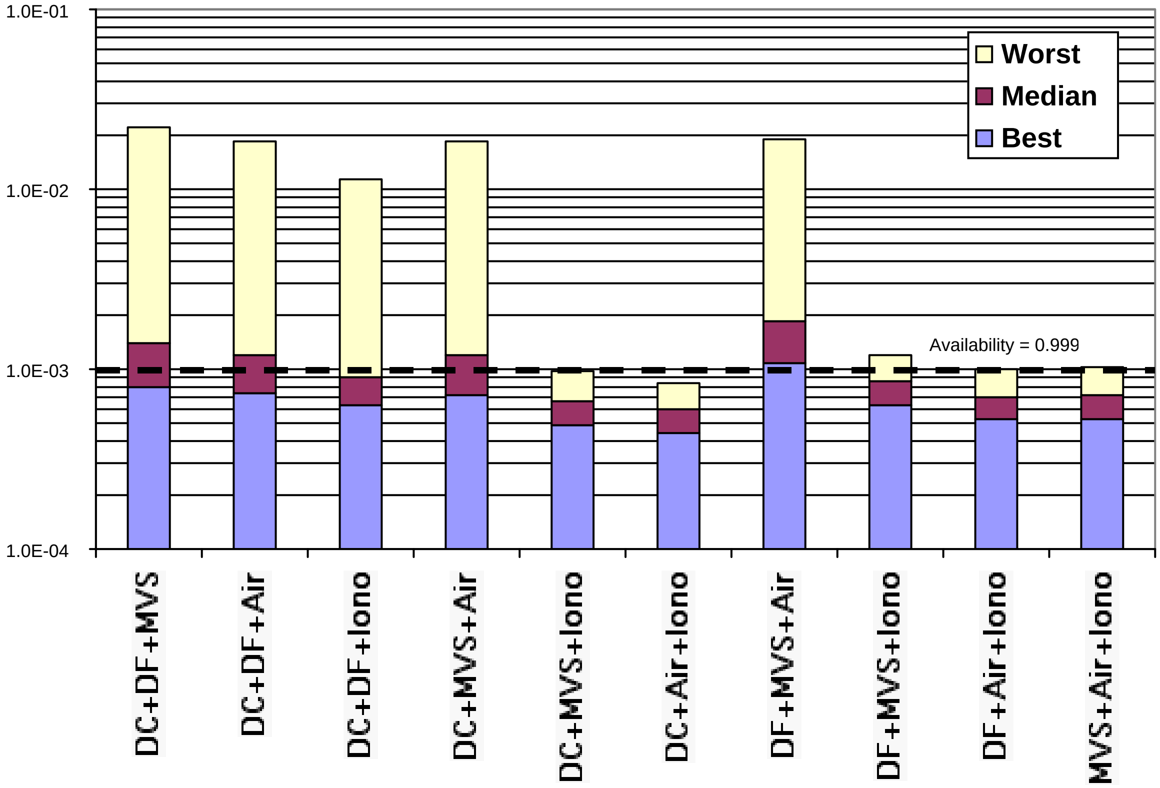

Figure 7. Unavailability Results for Alternative Triples (VAL = 5.3 m)
(DC = Differential Carrier, DF = Dual Frequency Ground Only, MVS = Mid-Value Selection,
Iono = Reduced Ionospheric Error Model, Air = Reduced Airborne Error Model)

SUMMARY

- Availability of good ranging source geometry of LAAS for CAT IIIb has been analyzed at 78 locations in CONUS for a baseline scenario and a variety of alternative scenarios with the potential to enhance availability performance.

- Results for the baseline (standard GPS constellation probabilities and simple extension of LAAS CAT I architecture) indicate that VAL must be increased from the previously specified value of 5.3 m to a recently proposed value of 10.0 m for adequate availability at all locations analyzed.

- This inadequate availability for VAL < 10.0 m would not be mitigated by merely assuming more optimistic GPS historical constellation state probabilities in the analysis.

- Consequently, other alternatives to increase the number of ranging sources or reduce ranging source errors were considered, including WAAS GEOs, APLs, dual frequency in LGF, dual frequency in LGF and aircraft, reduced ionospheric error model, reduced airborne error model, mid-value selection of position solutions in aircraft and differential carrier position solution below 500 ft AGL.

- The only single method that would provide adequate availability for VAL = 5.3 m is use of dual frequency in both the LGF and aircraft. However, a protected second GPS civil frequency for use in the aircraft will not be fully operational until several years after the planned initial introduction of CAT IIIb LAAS.

- If VAL can be increased to about 7.5 m, WAAS GEOs alone or APLs alone would provide adequate availability. However, WAAS GEOs may not be in the optimal locations assumed and APLs face difficulties in siting and compatibility with airborne equipment.

- Most pair-wise alternatives (excluding GEOs and APLs) would also provide adequate availability if

VAL were increased to about 7.5 m, some achieving significant margin.

- If VAL must be kept low (about 5.3 m), adequate availability would be provided by some combinations of three alternatives (excluding GEOs and APLs).

- The final choice of architecture for LAAS CAT IIIb will depend on the VAL required and other factors such as cost, complexity, ease of extension from CAT I, ground equipment siting constraints and maturity or technical risk in general.

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