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Analysis of Fix Displacement Tolerances for the Development of Instrument Procedures Criteria for GPS

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Abstract

The current airway structure is based on conventional ground-based navigational aids (navaids) such as very high frequency (VHF) Omnidirectional Range (VOR) and Distance Measuring Equipment (DME). In this structure, various fixes are defined along with their associated fix displacement tolerances (FDTs), from which obstacle clearance areas are designed. In the existing system, FDTs are formulated using 95 percent values and depend upon which navaids are used to establish the fix and upon the relative geometry of the aircraft, path, and facilities. The fix tolerance can be as small as ±0.5 nautical miles (nmi) along track for a DME fix.

While an FDT is a 95 percent value, the error with conventional navaids is rather tightly bounded by a local ground monitor. On the other hand, while Global Positioning System (GPS) is generally much more accurate than ground-based navaids, GPS does not have a local monitor. Therefore, direct application of a 95 percent GPS accuracy value is not appropriate for an FDT. For the case of GPS, it is necessary to use analyses of geometry and probabilistic computations in order to define the limits of the GPS position error. This paper analyzes the problem to answer three related questions. The first question is, “Does the GPS user equipped with a Technical Standard Order (TSO)-C129 receiver have compatible accuracy and integrity to fly over existing airways?” This question leads to a more fundamental, second question, “How to derive the FDT for the GPS users with a TSO-C129 receiver?” With the deployment of GPS/Wide Area Augmentation System (WAAS) in the National Airspace System (NAS) imminent, it is also necessary to answer the third question, “How to derive the FDT for the GPS/WAAS users?” This analysis has been performed in support of the Federal Aviation Administration (FAA) development of fix tolerance areas and is part of the effort to develop area widths and other parameters for instrument procedures criteria.

KEYWORDS: Global Positioning System, GPS, TSO-C129, fix displacement tolerance, FDT, GPS integrity, receiver autonomous integrity monitoring, RAIM, RNAV

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Biographies

Dr. Young C. Lee is a Lead Engineer in the Center for Advanced Aviation System Development at The MITRE Corporation in McLean, Virginia. He has been working in the areas of GPS integrity and aviation applications of GPS for the last 12 years. He has provided technical support to the FAA Satellite Operational Implementation Team (SOIT) for the development of Technical Standard Order (TSO)-C129 and the FAA guidance material for the use of GPS as a Primary Means of Navigation in Oceanic/Remote Airspace, and for approval of special navigation use of GPS in Alaska. He has also actively participated in RTCA Special Committee (SC)-159, especially in its Fault Detection and Exclusion and Wide Area Augmentation System Working Groups, and is currently serving as the secretary of the committee. He received his BS from Seoul National University and Ph.D. from the University of Virginia, both in electrical engineering.

Ralph Sexton has worked in areas associated with air navigation and missile guidance systems for more than 35 years. While employed with the Federal Aviation Administration, he was involved in flight test projects and analyses for development of criteria for the US Standard for Terminal Instrument Procedures (TERPS); was supervisor of the section responsible for writing TERPS; and was an advisor to the International Civil Aviation Organization (ICAO) Obstacle Clearance Panel (OCP), developing criteria for the ICAO Procedures for Air Navigation Services-Operations (PANS-OPS). He is currently involved in flight test projects and development of equipment performance requirements supporting implementation of GPS for air navigation. He currently instructs courses in PANS-OPS for international instrument procedures personnel. He has a BS in physics, has studied electrical engineering, aeronautical engineering and mathematics, and is an instrument-rated pilot.

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Section 1

Introduction

One of the critical issues for implementation of Global Positioning System (GPS) in the National Airspace System (NAS) is the allocation of tolerances for fixes. Fixes may designate a point where an aircraft is to turn, descend, report, initiate a missed approach or initiate other action based upon geographic location for use in en route, terminal and approach phases of flight. Establishing fix tolerances will ultimately be a key factor in determining airspace requirements, criteria for instrument approach procedures, air traffic procedures and operational procedures. With the publication of the Technical Standard Order (TSO)-C129, the first question the Federal Aviation Administration (FAA) had to ask was, “Is it acceptable to use existing fixes in the current airways with a GPS TSO-C129 receiver?” This question leads to a more fundamental question of how to derive the Fix Displacement Tolerance (FDT) for stand-alone GPS and GPS/Wide Area Augmentation System (WAAS).

The current airway structure is based upon ground-based navigational aids (navaids) such as very high frequency (VHF) Omnidirectional Range (VOR) and Distance Measuring Equipment (DME). In the existing system, FDTs are formed by using 95 percent accuracy values for the navigation systems upon which the fix is based, and the size depends on how close the navaids are to the fix. The FDT can be as small as ±0.5 nautical mile (nmi) along-track for a DME fix to be used by an aircraft in en route or terminal phase of flight. While an FDT is a 95 percent value, the error with ground-based navaids is rather tightly bounded by a ground monitor.

For stand-alone GPS, such bounding of errors must rely on Receiver Autonomous Integrity Monitoring (RAIM), or equivalent functions, implemented in the GPS airborne system. For this reason, the US Standards for Terminal Instrument Procedures (TERPS) criteria [1] and the criteria proposed for the International Civil Aviation Organization (ICAO) for the Procedures for Air Navigation Services–Operations (PANS-OPS) [2] are based on RAIM integrity limits defined for different phases of flight rather than the 95 percent accuracy values. Obviously, this was a conservative approach that would ensure no compromise in safety. However, because RAIM is based on a series of quite conservative assumptions, the error bounds derived on the basis of RAIM integrity limits are too conservative to achieve the potential capabilities of GPS. For example, RAIM bounds for a user flying in en route

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or terminal phase of flight cannot support 0.5 nmi along-track tolerance (ATT) for a DME fix. It would really not make sense if obstacle clearance areas (OCAs) would have to be enlarged with the use of much more accurate GPS. For these reasons, it has become clear that a close examination of the issue is needed.

The purpose of this paper is to closely examine the relationship of GPS system performance and the FDT and to develop FDT criteria on more realistic and consistent foundations. Ideally, one would start with a target level of safety and calculate the smallest allowable FDT that would satisfy the target level of safety for different phases of flight. However, such a baseline analysis would require data that has not yet been collected. For this reason, we find it most logical to select threshold values of FDT that would be operationally meaningful and then to perform the analysis relative to these values. A more extensive analysis could be done in the future as we collect more data and gain fresh insights.

The analysis in this paper assumes that the integrity monitoring is provided by RAIM, such as the case of stand-alone GPS with a TSO-C129 receiver. However, the criteria developed in this paper would also be valid for GPS/WAAS for the following reason. According to the current GPS/WAAS avionics specifications, a GPS/WAAS receiver may dynamically switch for integrity between WAAS signal and RAIM in real time depending on whether WAAS signal can provide integrity at the time. The two integrity methods for a GPS/WAAS receiver operate with integrity limits (i.e., horizontal alert limits) that are identical with RAIM integrity limits for a TSO-C129 receiver for the same phases of flight. Therefore, the criteria developed in this paper would also apply as an upper bound for GPS/WAAS. A legitimate argument can be made that the loss of WAAS is a sufficiently rare event that WAAS accuracy and integrity may be used as the basis for establishing FDT. However, since the objective is to have a sufficiently robust system to allow en route, terminal and nonprecision operations to continue without being degraded, this paper considers only this case with RAIM for these operations.

The next section reviews how the existing FDTs have been derived and how RAIM works to provide position data integrity. The subsequent section analyzes the issue. The last section provides a summary and recommendations.

Section 2

Review of Background

To facilitate the discussions in our analysis, two topics will be reviewed. First, existing FDT criteria for ground-based navaids and the currently proposed criteria for Global Navigation Satellite System (GNSS)/GPS FDTs will be reviewed. This will then be followed by a review of how RAIM works.

2.1 Existing Fix Displacement Tolerance Criteria

For an examination of the background method of establishing fix tolerance areas, the VOR will be considered, followed by the evolution and extrapolation of the basic principles as they were applied to area navigation (RNAV) systems.

2.2 Fix Displacement Tolerance for Very High Frequency (VHF) Omnidirectional Range (VOR)

The following is the method of establishing fix tolerance areas for the VOR, as described in the PANS-OPS [3]. The VOR FDT is established based upon the performance of the system at a 95 percent accuracy. The tolerance is considered in two dimensions: along track and cross track. The magnitudes of the along-track and cross-track dimensions are formed by considering the components of each navigation system's tolerance at 95 percent values and combining them by the root sum square (RSS) technique. For determination of cross-track tolerance (XTT), the component of flight technical tolerance (FTT) is included, while it is not included for ATT. The FTT is the error budget item that represents the pilot's accuracy in maintaining the aircraft on the designated track.

The dimensions for VOR ATT and XTT are shown in Table 1. Figure 1, which is simplified from an illustration in the PANS-OPS [3], shows fix tolerance areas formed by VOR/DME and VOR/VOR.

Table 1. Along-Track Cross-Track Tolerances for VOR (95 percent values)

COMPONENT	Along-track (degree)	Cross-track (degree)
Ground System	±3.5	±3.5
Monitor	±1.0	±1.0
Receiver	±2.7	±2.7
FTT		±2.5
Total (RSS of the component errors)	±4.5	±5.2

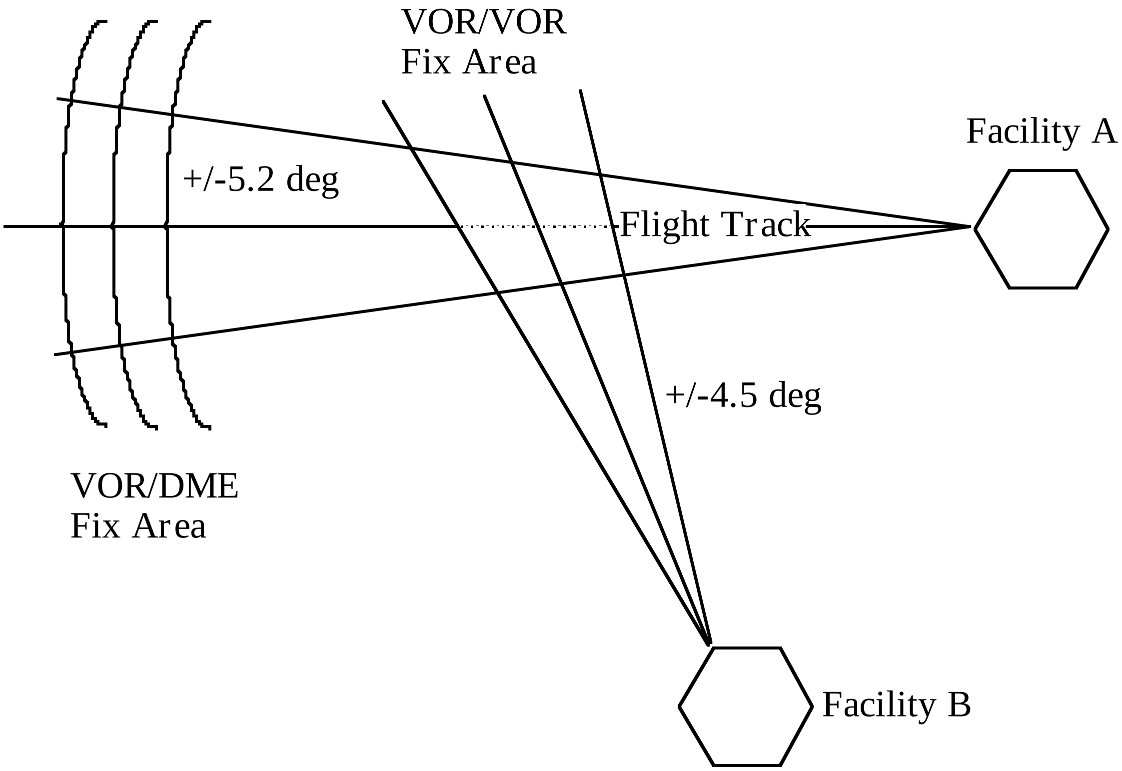

Figure 1. Fixed Tolerance Area Formed by VOR/DME and VOR/VOR

The width of fix tolerance area and the minimum width of the OCA consideration are determined by similar processes. The FDT remains at approximately two times the standard deviation (2-sigma) while the minimum width of the OCA is determined by multiplying the 2-sigma value by 1.5 and thereby approximating a 3-sigma value or 97.6 percent. The minimum width of the OCA for a VOR track would be 1.5 x ±5.2 degrees (deg) = ±7.8 deg. Therefore, the minimum width of the OCA would be determined by the minimum specified for the phase of flight or the width of the area determined by the splay of ±7.8 deg from the facility, whichever width is greater. This is illustrated in Figure 2, extracted from the PANS-OPS [3]. The important point here is the relationship of the fix tolerance area to the 95 percent value of the navigation system performance accuracy and the relationship of the minimum OCA to this value and the assumptions considered in establishing each value.

In the case of TERPS [1], the minimum width of the OCAs is determined by using the 4.5 deg (same value as PANS-OPS [3] without FTT) for the primary area and 6.7 deg for the minimum total semi-width of the OCAs. This is slightly smaller than the PANS-OPS values,

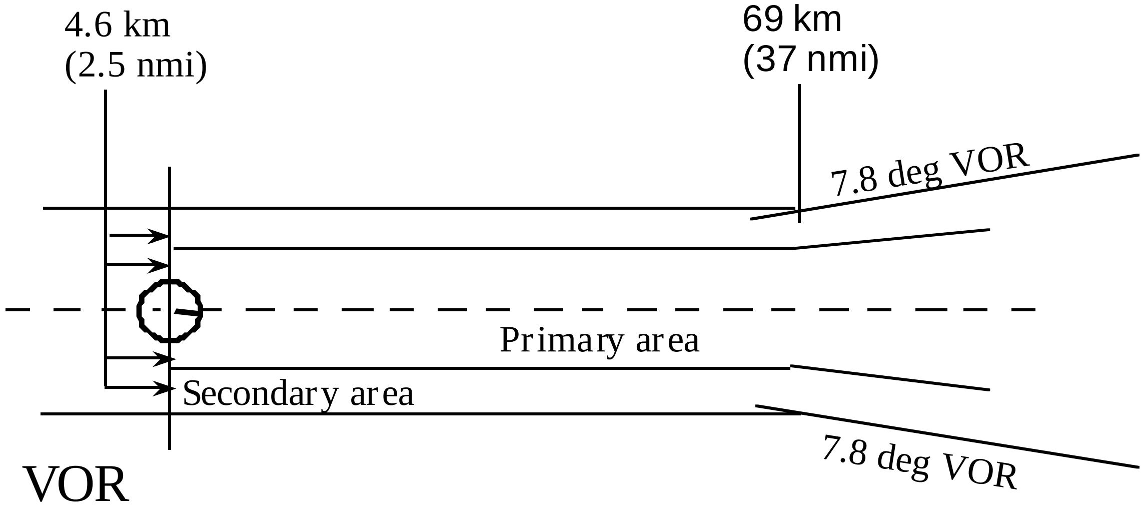

Figure 2. Initial Approach Area Utilizing Straight Tracks

but still utilizing the ratio of approximately 1.5 for the semi-width to the 95 percent navigation system tolerance.

2.3 Fix Displacement Tolerance for VOR/Distance Measuring Equipment (DME) Area Navigation RNAV

The method of defining the fix tolerance area for VOR/DME RNAV resolves the VOR angular tolerance and the DME tolerance into along-track and cross-track dimensions [3]. To achieve the semi-width of the OCA, a slightly different technique is used: multiplying the resultant 95 percent value by 1.5, as before, to achieve an approximated 3-sigma, but then adding a *buffer*. A 95 percent value for FTT is combined in forming cross-track tolerance. The width of the buffer is determined by the phase of flight.

The reason for the practice of adding a buffer in developing the areas will be relevant to the later consideration of the process as applicable to establishing fix tolerance areas for GPS. The area navigation system now includes additional components and processes, which not only relate to accuracy but to integrity as well. The final position solution now rests not with the VOR/DME alone, but also with the airborne RNAV equipment, which includes a computer and algorithms for computing the final position solution. Including such computing elements not only introduces additional error components, which, although quite small, must be taken into account, but also means that the preservation of integrity now calls for some scheme to include the airborne equipment. It is important to note that the basic integrity of the VOR/DME signals-in-space still remains in place.

Accommodating the integrity of the airborne system additional components can possibly be accomplished by an algorithm that computes a probabilistic model and confidence level of the solution. In a sense, this provides an additional check and confidence for the final position solution that includes all system elements. This method necessarily increases the bounds of uncertainty of the solution and increases the accuracy limit that assures the integrity requirements. Although no clear rationale is stated, it is likely that the addition of a buffer was to accommodate the integrity limit. The size of an appropriate increase is an important factor to be determined. In the case of GNSS, the basic integrity scheme for the satellite service is always functional, but increased measures must be taken. It is also necessary for the integrity function to meet the time-to-alarm requirements, which are 30 sec for en route and 10 sec for both terminal and nonprecision approach. These times cannot be assured to the required certainty without RAIM.

2.4 Fix Displacement Tolerances for GPS/Global Navigation Satellite System (GNSS)

While availability and outage duration statistics analyzed above are important parameters, continuity of service (or equivalently, loss of continuity) is also a key factor. Loss of continuity of GPS service can occur because of one of three different events: false integrity alert, genuine integrity alert, and unexpected loss of RAIM because of unpredicted satellite outage. These events are discussed below.

The accuracy specified for GPS is 100 m, 95 percent, and 300 m, 99.99 percent. By using these accuracy values and applying the method previously used to establish fix tolerances, we can calculate FDT values of 0.054 nmi and 0.16 nmi respectively with the latter being conservative. Applying the original concept for area width, 100 m x 1.5 = 150 m (0.08 nmi) or, applying the technique more adversely by using 99.99 percent accuracy, 300 m x 1.5 = 450 m (0.24 nmi). (FTT would be included for XTT). However, the issue is now one of integrity rather than accuracy. This issue should be addressed by specifying failure modes and frequencies of failures, and then determining how likely it would be that the failure modes would produce a position error in excess of a specified limit, and further determining what the probability might be that the resulting position error would escape detection by integrity mechanisms other than RAIM.

Another issue is whether the use of the RAIM integrity limit is appropriate for this purpose, or whether there is significant reduction in probability of missed detection below the RAIM itself or other checks. Consider, for instance, the probability of a failure that could result in a 2.0 nmi (en route) position error and the combined probability that such a failure could go undetected up to the RAIM integrity limit. Although no data exists that could empirically determine this, it is undoubtedly quite a small probability (e.g., less than 10^{-7}/hr). For this example, the considerably large limit of 2 nmi provides a comfortable margin of assurance both that the system will be available in the presence of a very significant failure and that the failure would be detected within the required time.

There are two possible types of lost services: the loss of the RAIM function (in itself not a loss of accuracy); the failure of the RAIM fault detection algorithm to calculate a sufficiently high probability that the position solution is within the limit (even though there would be a high probability that it was). A failure in the satellite or ground system could result in one of these two types of lost services. There are also other layers of integrity (e.g., GPS Ground Monitoring Network, airborne step detection function) besides RAIM, and it is extreme to establish the FDTs based upon the proposition that RAIM is the only functioning integrity mechanism. Again, the issue is that the RAIM integrity limit is set to meet the time-to-alarm requirements and is large to assure acceptable operation even during rare periods of degraded performance.

Consider the initial GPS allocation for FDT (or waypoint tolerance) as used in the US material, FAA Order 8260.38, Civil Utilization of Global Positioning System (GPS) [4]. The fix displacement ATT/XTT are ±2.0/2.8 nmi for en route, ±1.0/1.5 nmi for terminal; and ±0.3/0.5 nmi for approach phases of flight, respectively. The along-track values are the values for the RAIM thresholds specified in TSO-C129 [5], while the cross-track values are RAIM integrity limits expanded by including FTT.

The GNSS FDTs proposed in the pending criteria for the PANS-OPS [2] follow a similar practice. The RAIM integrity limit is used directly as the ATT, with the RAIM integrity limit plus FTT (direct addition) producing the XTT, and the area semi-width calculated as twice the XTT. The FTT in this case was estimated to be from one-half to two-thirds of full scale deflection (FSD), depending on the phase of flight. It is clear in this case as in the previous criteria that the conservative nature of the probabilistic model results in both a departure from the method used with ground-based navigation and a conservative approach. Presumably this is because of the lack of operational data and a lengthy period of operational experience with the system.

In summary, the method used for determining FDTs and OCA widths has moved from the direct application of the 95 percent accuracy value for FDT and multiplying the value by 1.5 for OCA width, to the use of the RAIM integrity limit for FDT and multiplying the limit by 1.5 and then adding a buffer for OCA width for GPS. Furthermore, another method was introduced in the proposed criteria—adding the RAIM integrity limit to the FTT and doubling the result for the OCA width. This sequence seems to illustrate a loss of consistency and the absence of defined rationale in the method of calculating the values for fix tolerance and OCA widths. The key to achieving the most realistic FDT lies in achieving integrity and availability requirements while utilizing the most realistic probabilistic model. Some means for reducing FDT may also lie in achieving integrity through other means such as an inertial navigation system or other cross-referenced airborne navigation systems. This paper examines the relationship of system performance for GPS and the FDT and proposes a method for developing the appropriate relationship on solid foundations.

One final note is that, although this paper addresses the application of system accuracy and integrity limits for GPS to the development of fix tolerance areas and widths of OCAs, the methods are also applicable to examining the relationship between GPS performance and a specified Required Navigation Performance (RNP). The issues and techniques are the same, although different values might result based on the definition of accuracy, availability, integrity and continuity necessary to meet an RNP.

2.5 Receiver Autonomous Integrity Monitoring (RAIM)

RAIM performance can be examined with a plot shown in Figure 3. The figure plots the position error on the vertical axis versus the test statistic on the horizontal axis. RAIM protects the position error against a selected position error limit by observing the test statistic, which is determined as a function of range residuals. A range residual is the difference between the measured pseudorange to each satellite and the predicted pseudorange to the same satellite, using the current position estimate and the satellite position contained in the navigation message. An integrity decision threshold for the test statistic is selected on the basis of a nuisance alarm rate in a no-failure condition. If the test statistic exceeds the decision threshold, a detection alarm is raised. A pair of position error and test statistic at any given time falls in one of four regions:

- Normal (A1 and A2 in Fig. 3)
- Early detection (B in Fig. 3)
- Missed detection (C in Fig. 3)
- Valid detection (D in Fig. 3)

If one of the satellites used for navigation fails and its error dominates the errors of the other satellites, then the pair of the position error and test statistic would rather closely follow a straight line, whose slope depends on the user-to-satellite geometry and also on which satellite has the dominant error (e.g., satellite j vs. satellite k in Figure 3). Because of the presence of relatively small errors on the other non-failed satellites, actual variations would be wiggly lines as shown in Figure 3. Depending on the proximity of the line to the missed detection region, a missed detection is more or less likely to occur. In other words, depending on which satellite fails, the failure can be more or less easily detected for a given geometry.

One critical parameter that characterizes RAIM performance is horizontal protection level (HPL), which is determined as a function of the user-to-satellite geometry. RAIM guarantees that any position error larger than HPL will be detected with a probability of 0.999 upon occurrence of a failure regardless of which satellite fails. The probability of detection changes

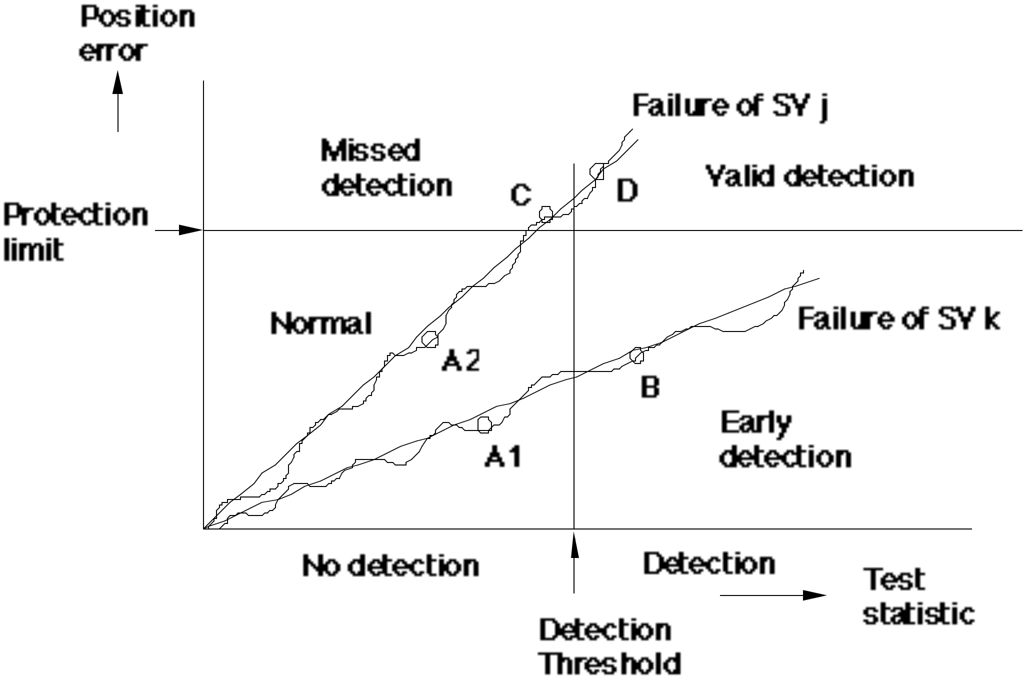

Figure 3. Illustration of Receiver Autonomous Integrity Monitoring (RAIM) Performance

depending on which satellite has failed, and the detection probability of 0.999 has to be guaranteed even for a failure of the satellite that is the most difficult to detect. Therefore, HPL is derived for each geometry assuming that a failure is always on the satellite that is the most difficult to detect among the satellites used for navigation.

Another parameter of interest is horizontal alert limit (HAL). HAL is the position error that should be protected against for satisfactory navigation in a given phase of flight. HAL is 2 nmi, 1 nmi, and 0.3 nmi for en route, terminal, and nonprecision approach phases of flight, respectively. If HPL for a given geometry is equal to or smaller than HAL, RAIM is declared available for that phase of flight, and vice versa.

While HPL is derived under the assumption that a failure always occurs on the satellite that is the most difficult to detect for each geometry, that is not a realistic assumption. Failures are actually equally likely to occur among the satellites used for navigation by the receiver. In this case, the average detection probability is of interest and is shown below to be significantly larger than the detection probability that one would obtain under the worst case assumption.

Figure 4 plots detection probability as a function of position error for a geometry with an HPL of 2 nmi, as an example. The probability is plotted for two different cases:

- With an assumption that a failure is always on the most-difficult-to-detect satellite
- With the assumption that the failures are equally likely among the satellites used for navigation.

The figure shows that the detection probability is dramatically larger in the second case than in the first case. For example, for a position error of 0.5 nmi, the detection probability is less than 1 percent and over 80 percent, respectively. Because a failure occurs at random with equal likelihood, our analysis will be based on this second assumption. Also note that the detection probability plotted in Figure 4 is the fault detection capability of RAIM upon occurrence of a failure and that there exists in the GPS system itself many layers of integrity checks (e.g., GPS Ground Monitoring Network, airborne step detection function) that would reduce the probability of integrity failure even without RAIM.

It will be seen in the next section that the assumption of equal likelihood of a failure among the satellites in view is critical in our analysis for establishing fix displacement tolerances for GPS.

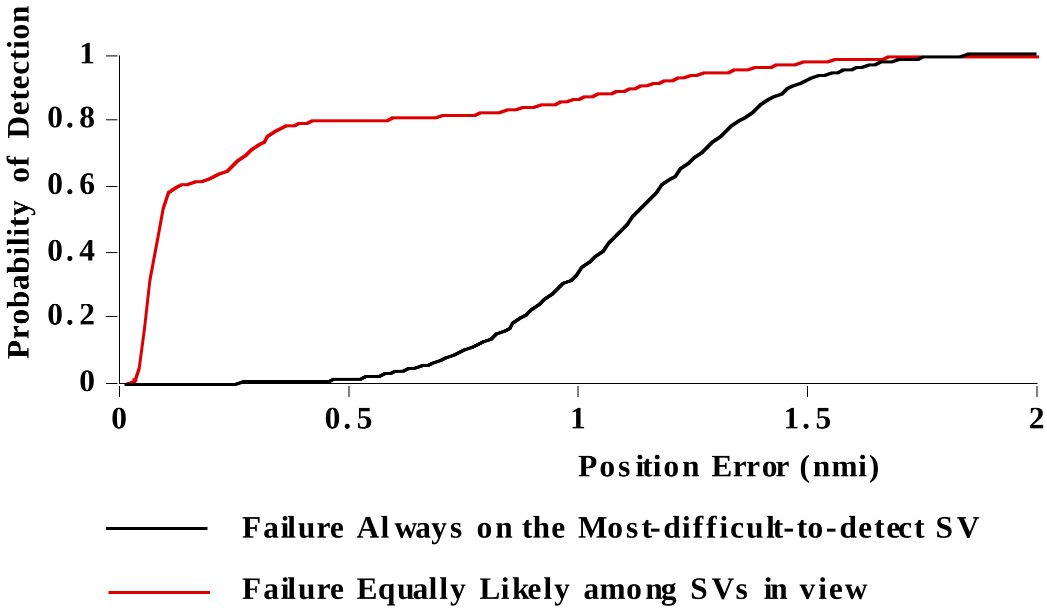

Figure 4. Probability of Detection Vs. Position Error (For a Geometry with HPL = 2 nmi)

Section 3

Analysis

Because of disparity between the error characteristics of ground-based navaids and GPS, one cannot directly relate quantities of the two. For this reason, while use of a 95 percent value may not be appropriate for GPS, neither may be a RAIM integrity alert limit. This has led the authors to restate the problem in a slightly different manner. It is desired for a GPS FDT to find a value that a GPS-based position error would exceed with an extremely small probability, say, on the order of 10^{-7}/hr. (The European Organization for Civil Aviation Equipment (EUROCAE) document on VOR ground systems requires Pr {hazardously misleading information} < 2 x 10^{-7}/hr). Because of the appropriate safety margin provided by the OCA, it is believed that this level of probability would acceptably make a GPS FDT comparable with VOR/DME tolerances.

The analysis in this section assumes that the integrity monitoring is provided by RAIM, such as the case of stand-alone GPS with a TSC-C129 receiver. However, as was described in Section 1, the criteria derived here would also apply as an upper bound for GPS/WAAS.

Our analysis will be presented in the order of en route, terminal, and nonprecision approach phases of flight. For each phase of flight, a target protection level of horizontal position error will be selected. The analysis will determine at what rate an undesirable event may occur where a satellite fails and causes an undetected position error exceeding that target level of protection. If the rate is acceptably small, it is to be recommended to use the target level of protection as the along track tolerance for the phase of flight. For the cross track tolerance, FTT needs to be included, and this will be discussed separately.

3.1 Proposed Fix Displacement Tolerance for En-Route Phase of Flight

For the en route phase of flight, a target protection level of horizontal position error of 0.5 nmi is selected. This value is selected because it is the smallest FDT used in en route phase of flight in the existing airway structure, and also it is unlikely that use of a smaller value would be of any advantage for en-route phase of flight.

As a user aircraft is flying, HPL is determined as a function of the user-to-satellite geometry. Depending on the HPL value, the user in the en route phase of flight will be in one of the following three situations:

- HPL is larger than 2 nmi
- HPL is between 0.5 nmi and 2 nmi.
- HPL is smaller than 0.5 nmi

It is assumed in our analysis that designated actions are taken at fixes only when integrity is provided (either by WAAS signal, or by RAIM in a fallback mode when the WAAS signal cannot provide integrity). With this assumption, the first case (i.e., the case with HPL > 2 nmi) poses no problem because RAIM will be declared unavailable in the en route phase of flight. The second case needs to be carefully examined. On the other hand, the third case also poses no problem because RAIM guarantees detection of a position error exceeding the HPL of 0.5 nmi with a probability of 0.999 even if HPL is exactly 0.5 nmi and it is always the most-difficult-to-detect satellite that fails. Therefore, the probability of an undetected position error associated with the third case is negligible.

For the above reason, the rate of occurrence of an undetected position error exceeding the target level of protection of 0.5 nmi is governed by the product of the following probabilities:

- Pr {GPS satellite failure per hour} (P1)
- Pr {0.5 nmi < HPL < 2 nmi} (P2)
- Pr {missed detection of position error exceeding 0.5 nmi target level of protection | 0.5 nmi < HPL < 2, GPS satellite failure} (P3)

Each of these probabilities is evaluated as follows.

Pr {GPS satellite failure} (P1)

Failures observed thus far have been mostly with Block I satellites [6, 7]. Furthermore, while some Block II satellite failures have occurred, none of them had any significant impact on civil users (that is, none of them caused pseudorange errors larger than 200 m) and some of the problems have been removed [8]. According to the GPS Signal Specification [9], major GPS service failures should occur no more than three times a year, and realistically once a year. In this case, a major service failure is defined to be any one that results in a pseudorange error exceeding 150 m when the health word indicates that the satellite is healthy [9].

Assuming that 6 satellites are in view of the user (and also used for navigation) and using the realistic annual failure rate, the hourly rate of a failure occurring on any one of the six satellites is given by*

$$\begin{aligned} P1 &= \Pr\{\text{major service failure per hour (causing a pseudorange error > 150 m)}\} \\ &\leq (6 \text{ satellites} \times 1 / \text{year - constellation}) / (24 \text{ satellites /constellation}) \\ &= 2.85 \times 10^{-5} \end{aligned} \tag{1}$$

In case fewer than six satellites are in view, P1 would be smaller. In case more than six satellites are in view, P1 would be larger; however, one can almost guarantee that HPL would be less than 0.5 nmi. According to an analysis shown in the Appendix, the likelihood of HPL being less than 0.5 nmi is greater than 99.9 percent in case seven satellites are in view of the user. For geometries with more than seven satellites visible, HPL tends to be even smaller.

Pr {0.5 nmi < HPL < 2 nmi} (P2)

The distribution of HPL depends on the number of satellites in the constellation as well as on the elevation mask angle and user location. A representative distribution of HPL is shown in Table 2 as a function of the number of satellites in the constellation. The table also contains the probability of different operational states of the constellation. If these two sets of probabilities are considered together, P2 is given by

$$P2 \leq (0.7 \text{ x } 0) + (0.24 \text{ x } 0.03) + (0.05 \text{ x } 0.06) + (0.01 \text{ x } 0.1) = 0.011 \qquad (2)$$

Although this value would vary somewhat as a function of elevation mask angle and user location, selection of 0.01 for this probability is believed to be sufficiently accurate for our purpose.

Table 2. Conditional Probability of HPL Distribution

Number of SVs Operating[1]	**24**	**23**	**22**	**21**
Pr {No of SVs Operating}[2]	0.70	0.24	0.05	0.01
Pr {0.5 nmi < HPL < 2 nmi}}[3]	0	0.03	0.06	0.10
Pr {0.4 nmi < HPL < 1 nmi}}[3]	0.01	0.05	0.07	0.13

* Notes

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1. The probabilities of fewer than 21 satellites operating in the constellation are neglected.
2. Based on Reference [10].
3. Typical values with an elevation mask angle of 5 degree and no barometric altimeter aiding assumed. In case barometric altimeter aiding is used, our estimate of P2 would be even smaller than the value given in Equation (Eq.) (2).

Pr {missed detection of position error exceeding 0.5 nmi | 0.5 nmi < HPL < 2, satellite failure} (P3)

Two factors have been considered in obtaining this probability.

- As was discussed earlier, calculation of HPL assumes that a failure is always on the most-difficult-to-detect satellite. In reality, however, a failure is equally likely to occur among the satellites used for navigation. This is taken into account in our estimation of the above probability.
- The rate of GPS satellite failure discussed earlier applies to any satellite failure that results in a pseudorange error larger than 150 m. On the other hand, in order to cause a position error larger than 0.5 nmi, the pseudorange error has to be significantly larger than 150 m. Figure 5 shows a typical distribution of the range bias error upon

occurrence of a missed detection of a position error exceeding 0.5 nmi. The figure, obtained via simulation, shows, for example, that the range error magnitude has to be larger than 500 m (this value depends on the geometry) before it has any chance to cause an undetected position error exceeding 0.5 nmi. That is, a significant fraction of time, a major service failure will not result in a range bias error large enough to cause a user position error exceeding 0.5 nmi. This is also taken into account in our analysis below.

In order to estimate the probability, P3, above, first note that P3 can be expressed as

$$\text{P3} = \sum_i (P_a * P_b) \tag{3}$$

where

P_a = P [missed detection of a position error exceeding 0.5 nmi | x_i < Range Error < x_{i+1}, 0.5 nmi < HPL < 2 nmi]

P_b = P [x_i < Range Error < x_{i+1} (causing a pseudorange error > 150 m)],

x_0 = 150 m, and x_i's can be arbitrarily chosen such that $x_{i-1} < x_i < x_{i+1}$

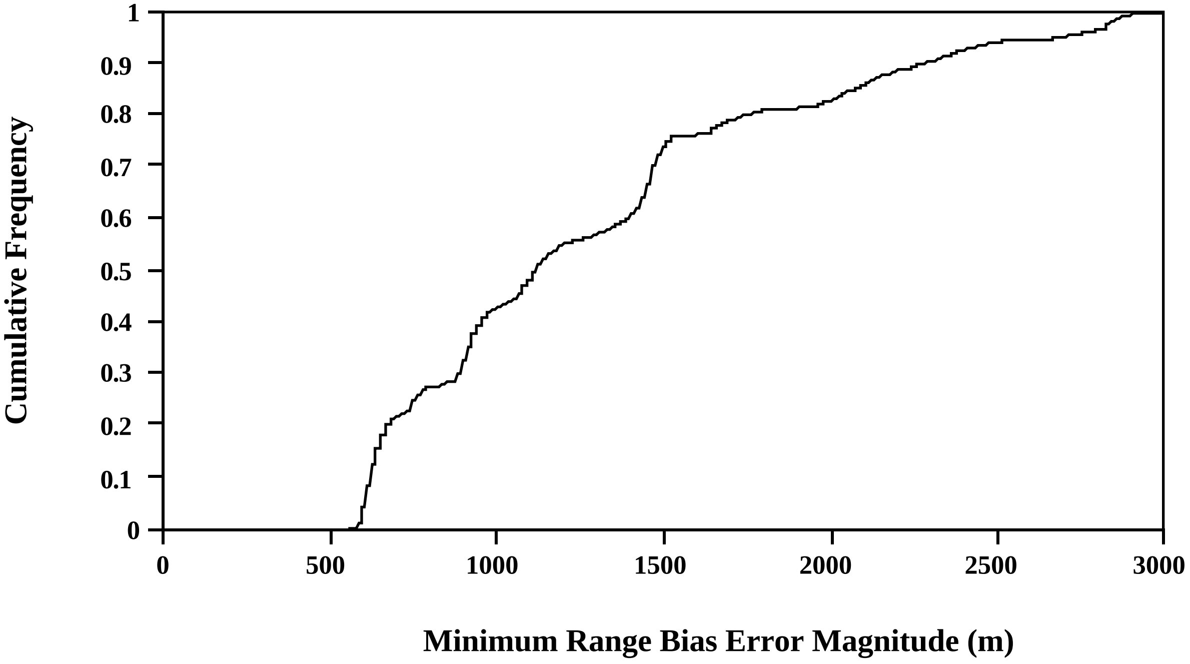

Figure 5. Typical Cumulative Frequency Versus Minimum Range Bias Error Magnitude Causing Missed Detection of 0.5 nmi Position Error

First, probability P_a, is obtained via simulation as a function of the range bias error magnitude, with the above two considerations. This is shown in Figure 6 for four different ranges of HPL values. The plot for the case of HPL between 0.5 nmi and 2 nmi is used to calculate P3. The plots for other cases will be used later.

For probability P_b, no data exists. However, for the purpose of our analysis, an assumption is made that the conditional probability distribution of the range bias error magnitude upon a major service failure is as shown in Figure 7. Considering that many advanced features have been put in place for the GPS Control Segment including such features as Satellite Zapping (SATZAP) [11], it is believed that the assumption is more than sufficiently conservative.

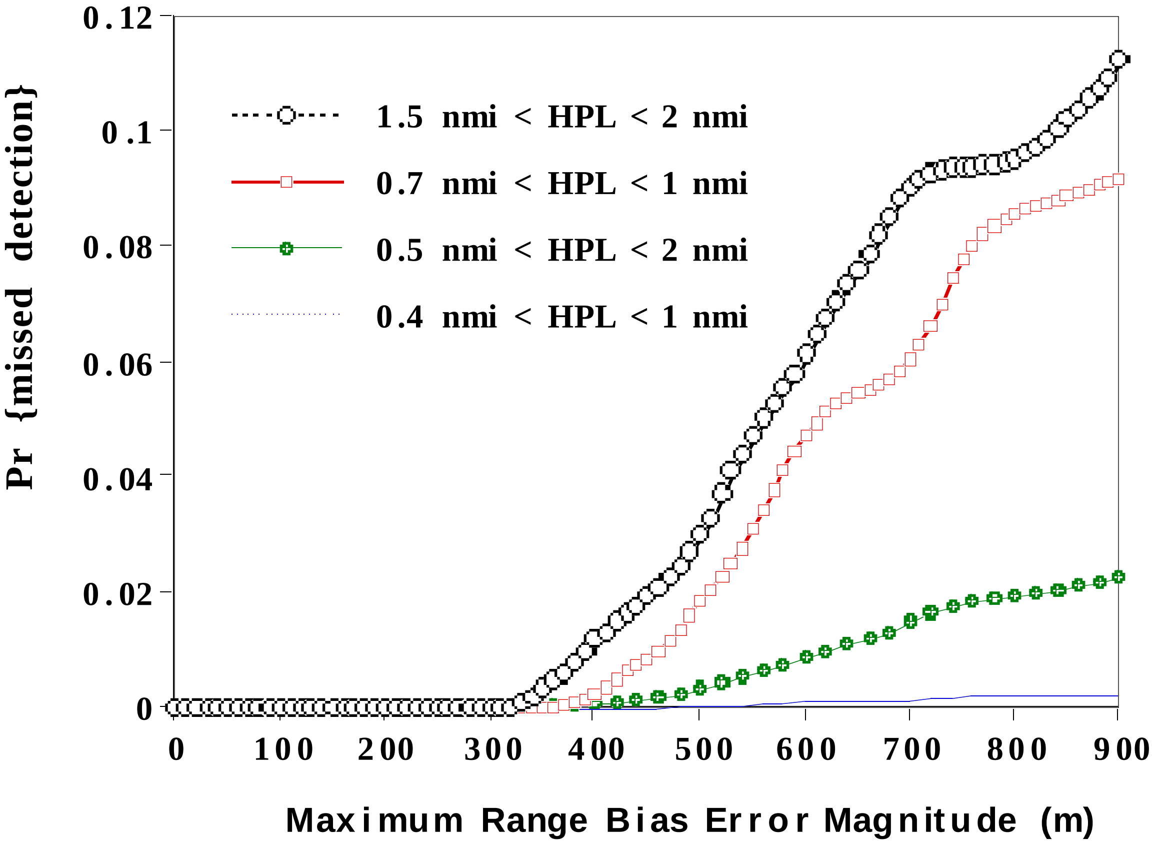

Figure 6. Average Pr { missed detection | Maximum Range Bias Error Magnitude } Assuming Failures are Equally Likely among SVs in view

In order to obtain probability P3, the probability of P_a in Figure 6 is averaged for each of the five range error intervals shown in Figure 7. By summing up the products of the two probabilities over the entire five intervals, we get

$$P3 = 0.003 \tag{4}$$

Estimation of the overall probability. The above estimations can be summarized as follows:

- P1 = Pr {satellite failure per hour} = 2.85 x 10^{-5}
- P2 = Pr {0.5 nmi < HPL < 2 nmi} = 0.01

P3 = Pr {missed detection of position error exceeding 0.5 nmi target level of protection | 0.5 nmi < HPL < 2, GPS satellite failure} = 0.003

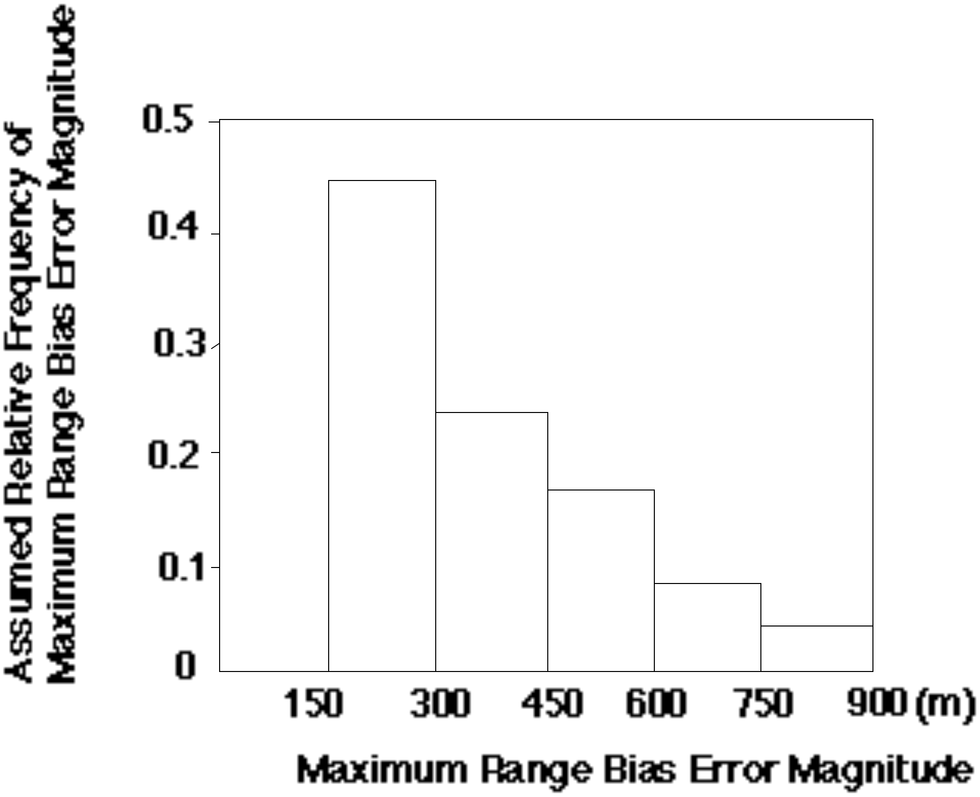

Figure 7. Assumed Relative Frequency of Maximum Range Bias Error Magnitude Upon Occurrence of a Major GPS Failure

From the above estimates,

Pr {the overall rate of occurrence of a GPS failure per hour causing an undetected position error exceeding a target level of protection of 0.5 nmi}

$$\begin{aligned} &= P1 * P2 * P3 \\ &\leq (2.85 \times 10^{-5})\ (0.01)\ (0.003) \\ &= 9 \times 10^{-10} \end{aligned} \tag{5}$$

The above probability is more than acceptably small. However, there is one issue on which not everybody may agree. That is the question of whether one may include Pr {0.5 nmi < HPL < 2 nmi} in calculating the above probability. Some may argue that any probability of this kind, namely, a probability of an event that may lead into a safety compromising situation, should be acceptably small even in the worst situation ***known to the receiver***.

According to this argument, because HPL information at a given time is always known to the user receiver, it cannot be considered a random event at any given time, and, therefore, cannot be factored into this probability. On the other hand, constellation states, failed satellites, and range bias error on the failed satellites are factors not known to the receiver at a given instant of operation, and, therefore, the probability of an undesirable event can be obtained as an average over these events.

Because the HPL constraint in this en route phase of flight is between 0.5 nmi and 2 nmi, the worst set of geometries are those with HPL close to 2 nmi. For this worst set, geometries with HPL between 1.5 nmi and 2 nmi are used. Using the data for the probability of missed detection for the worst set, shown in Figure 6, and the same approach as in Eq. (3), one can obtain

Pr {missed detection of position error exceeding 0.5 nmi | HPL close to 2 nmi, satellite failure}

$$= 0.02 \tag{6}$$

Then,

Pr {the overall rate of occurrence of a GPS failure causing an undetected position error exceeding a target level of protection of 0.5 nmi per hour | HPL close to 2 nmi}

$$\leq (2.85 \times 10^{-5} / \text{hr})\ (0.02) = 6 \times 10^{-7} \tag{7}$$

It should be remembered that this probability applies only for the worst acceptable geometries, which may occur significantly less than 1 percent of the time. Then this probability may be considered to be small enough to be acceptable. Moreover, there are two additional reasons to believe these estimates are sufficiently conservative.

- It was assumed in our simulation that a range bias error of a given magnitude starts from a relatively small value and increases up to a certain value and a missed detection along the way was counted as a missed detection in our simulation. However, a missed detection actually occurs only for a relatively small range of bias error magnitudes between the normal region and the detection region (See Figure 3). In the case of a ramp-type error, for example, this means that the duration over which a large error remains undetected would be relatively short depending on the error growth rate. If the error grows quickly, then it will be detected quickly; if it grows slowly, then there may be more time to react by some other means. In case of a random bias-type error, the likelihood of the error being in that small range of values causing a missed detection would be significantly less than 1.
- Our estimation assumes that all of the major service failures would result in an error that can be detected only by RAIM, that is, an error that slowly creeps in. However, it is commonly understood that the frequency of a step-type failure is significant relative to the frequency of a slowly drifting error. Such step-type failures are more readily detectable by the airborne equipment.

3.2 Proposed Fix Displacement Tolerance for Terminal Phase of Flight

For the terminal phase of flight, a target protection level of horizontal position error of 0.4 nmi is selected. This value is chosen as an intermediate value between 0.5 nmi chosen for the en route phase and 0.3 nmi to be chosen for the nonprecision approach phase of flight. Similar to the case of the en route phase case, an analysis may be performed as follows.

First, it is noted that, depending on the HPL determined as a function of the user-to-satellite geometry, the user aircraft in the terminal phase of flight will be in one of the following three situations:

- HPL is larger than 1 nmi
- HPL is between 0.4 nmi and 1 nmi
- HPL is smaller than 0.4 nmi

The first case poses no problem because RAIM will be declared unavailable in the terminal phase of flight. The second case needs to be carefully examined. One the other hand, the probability of the third case is negligible compared to the probability of the second case for the same reason stated before in Section 3.1. Therefore, the rate of occurrence of an undetected position error exceeding the target level of protection of 0.4 nmi is governed by the product of the following probabilities:

- Pr {GPS satellite failure per hour} (P1)
- Pr {0.4 nmi < HPL < 1 nmi} (P4)

- Pr {missed detection of position error exceeding 0.4 nmi | 0.4 nmi < HPL < 1 nmi, GPS satellite failure} (P5)

Each of these probabilities is evaluated below.

Pr {satellite failure per hour}

As was derived in Eq. (1),

Pr {satellite failure per hour} = 2.85×10^{-5}

Pr { 0.4 nmi < HPL < 1 nmi} (P4)

Using the information in Table 2, the overall probability of having an HPL between 0.4 nmi and 1 nmi is derived as

Pr { 0.4 nmi < HPL < 1 nmi}

$$= (0.7 \times 0.01) + (0.24 \times 0.05) + (0.05 \times 0.07) + (0.01 \times 0.13)$$

$$= 0.024 \qquad (8)$$

Pr {missed detection of position error exceeding 0.4 nmi | 0.4 nmi < HPL < 1, satellite failure} (P5)

As was done for the en route case, this probability can be estimated taking into account that a failure is equally likely to occur among the satellites being used for navigation and that a significant fraction of time, a satellite failure will not result in a range bias error magnitude large enough to cause a user position error exceeding 0.4 nmi.

As Eq. (3) was used to derive P3 for the en route case, P5 is given by

$$\text{P5} = \sum_i (P_a * P_b) \qquad (9)$$

where P_a, in this case, is

P_a = P [missed detection of a position error exceeding 0.4 nmi

| x_i < Range Error < x_{i+1}, 0.4 nmi < HPL < 1 nmi]

Probability P_a obtained in the same manner as before is plotted in Figure 6 (case of HPL between 0.4 nmi and 1 nmi) and is used to calculate P5. For probability P_b, the same assumption is made as before shown in Figure 7. P5 is then derived as

$$\text{P5} = 0.0004 \qquad (10)$$

Estimation of the overall probabilities. The above estimations can be summarized as follows:

- P1 = Pr {satellite failure per hour} = 2.85×10^{-5}

- P4 = Pr {0.4 nmi < HPL < 1 nmi} = 0.024
- P5 = Pr {missed detection | satellite failure} = 0.0004

From the above estimates,

Pr {the overall rate of occurrence of the undesirable event per hour that a GPS failure causing an undetected position error exceeding a target level of protection of 0.4 nmi}

$$\begin{aligned} &= P1 * P4 * P5 \\ &\leq (2.85 \text{ x } 10^{-5})\ (0.024)\ (0.0004) \\ &= 3 \text{ x } 10^{-10} \end{aligned} \quad (11)$$

This probability is acceptably small as was the case for the en route phase of flight.

On the other hand, the conditional rate of occurrence of the undesirable event for the worst situation known to the receiver, that is, for a geometry with an HPL close to 1 nmi, can be estimated. Figure 6 shows probability of missed detection for HPL between 0.7 nmi and 1 nmi. Using this data and the same approach as in Eq. (3), we can obtain

Pr {missed detection of position error exceeding 0.4 nmi | HPL close to 1 nmi, satellite failure}

$$= 0.014 \quad (12)$$

Then,

Pr {the overall rate of occurrence of a GPS failure per hour causing an undetected position error exceeding a target level of protection of 0.4 nmi | HPL close to 1 nmi}

$$\begin{aligned} &\leq (2.85 \text{ x } 10^{-5})\ (0.014) \\ &= 4 \text{ x } 10^{-7} \end{aligned} \quad (13)$$

If it is remembered that this probability applies only for the worst acceptable geometry in the terminal phase of flight, this probability may still be considered to be sufficiently small to be acceptable.

The two reasons stated earlier for believing that the estimates for the en route case were conservative also apply here for the terminal phase of flight. One is that a missed detection occurs for a relatively small range of bias error magnitudes between the normal and detection regions. The other reason is that a significant portion of the major GPS service failures are of a step type, which would be more readily detectable by the airborne equipment than slowly drifting errors that only RAIM can detect.

3.3 Proposed Fix Displacement Tolerance for Nonprecision Approach Phase of Flight

For the nonprecision approach phase of flight, a FDT of 0.3 nmi would be appropriate for nonprecision approach. Because 0.3 nmi is the integrity limit (i.e., horizontal alert limit) for nonprecision approach phase of flight, we can say that the probability of undetected integrity failure is even less than the values obtained above for the en route and terminal phase cases. For this reason, a FDT of 0.3 nmi is recommended for the nonprecision approach phase of flight.

3.4 Considerations for the Cross-Track Tolerance (XTT)

In order to consider the ATT, only the navigation system error must be considered. However, to establish the relationship between total system performance and the XTT and between total system performance and the width of OCAs, it is necessary to consider the flight technical tolerance (FTT). As we have seen, the technique customarily applied is combining the FTT with the other system components of the error budget by RSS. However, in the case of satellite navigation, techniques including direct addition have been applied in an effort to assure that the initial criteria were conservative. This is extremely conservative, and, since the model for the navigation system tolerance and total system tolerance is probabilistic, it is considered appropriate to include a probabilistic value for the FTT.

The question now arises of what values of FTT are appropriate for each phase of flight. The values used in the development of the initial TERPS criteria [1] and the proposal for PANS-OPS [2] utilized FSD in some cases as the basis for establishing FTT, again an effort to assure conservative initial criteria. The FSD values for TSO-C129 GPS receivers are: ±5 nmi, en route; ±1 nmi, terminal; and ±.3 nmi, approach [5, 12]. The FSD values for GPS/WAAS receivers are: ±2.0 nmi, en route; ±1.0 nmi, terminal; and ±0.3 nmi, or ±2 deg, whichever is smaller [13]. We note here that for TSO-C129 GPS receivers , the values of allowed flight technical error (FTE) are 1 nmi for en route (which is from FAA test results), with an FSD of ±5 nmi, and the value for FTT for VOR in PANS-OPS [3] is ±2.5 deg, with an FSD of ±10 deg. These are identified as 95 percent values. These FTT values are about 20–25 percent of FSD, compared to the use of 100 percent of FSD for the FTT used in TERPS [1] and proposed PANS-OPS criteria [2]. This significant increase in percentage is further indication of the deliberately conservative application for the initial GPS criteria.

The actual values for FTT must come from extensive analysis of flight test data. However, the flight tests that address this are either just completed or in progress and the FAA has not released the data. The data reduction will likely be completed later this year. In the meantime, we will draw some specific conclusions. First, any system, when flown by trained pilots, that resulted in an FSD, 5 percent, would be unacceptable and never implemented because of high workload and an unacceptable number of cases in which pilots could not complete approaches. Second, from results with previous systems, a value of about 30 percent FSD would be a reasonable choice for FTT used in planning and analytical study. And finally, smaller FSD values reduce FTE until the scale is so small that it induces unacceptable continuing workload, and a system that could not maintain one-third FSD, 95 percent, would very likely be considered to induce an unacceptable workload.

Since FTE is independent of the navigation system errors, combining the FTT with other components of system tolerance using RSS is considered satisfactory. The combined distribution tends strongly to be normal but with slightly thickened tails. An investigation of this is contained in the manual for the ICAO Collision Risk Model [14]. Although the work was addressed to Instrument Landing System (ILS), it illustrates that combining different distributions in this manner produces appropriate values for application to total system tolerance. We therefore use this in developing the results given in this paper.

Section 4

Summary and Recommendation

As use of GPS becomes widespread, a critical issue has been raised, namely, how to derive fix displacement tolerance (FDT) for GPS. In the existing criteria for FDTs and obstacle clearance areas (OCAs), there is not full consistency in either the techniques or values used in forming the relationship between system performance and the values of FDT and OCAs. In particular, while TERPS criteria [1] and proposed PANS-OPS criteria [2] were previously derived on the basis of Receiver Autonomous Integrity Monitoring (RAIM) alert limits, the criteria were in some cases too conservative to achieve the potential capabilities of GPS. This analysis examined the relationship of GPS system performance and the FDT and proposed a method for developing the appropriate relationship.

Ideally, one would start with a target level of safety and calculate the smallest allowable FDT that would satisfy the target level of safety for each phase of flight. However, such a baseline analysis would require data that has not yet been collected. For this reason, it was found to be most logical to select target values of FDT that would be operationally meaningful and then performs the analysis relative to these values. For the target values of FDT, 0.5 nmi was selected for the user in en route phase of flight because it is the smallest FDT defined for the current ground-based navaids (i.e., along track for a DME fix) in en route phase. The value of 0.3 nmi was selected for the user in nonprecision approach phase of flight for three reasons:

- It is adequately small to support current nonprecision approach FDTs provided by the existing ground based navaids
- It is the integrity limit (i.e., horizontal alert limit) for the nonprecision approach phase of flight
- It is unlikely that use of a smaller value would be of any advantage for nonprecision approaches.

For terminal phase of flight, 0.4 nmi was selected, a value between 0.3 nmi and 0.5 nmi.

As summarized in Table 3, this analysis estimated two different probabilities of a GPS integrity failure occurring and causing a missed detection of a horizontal position error exceeding the target values of FDT. One is the overall average rate of occurrence of such an event without a prior knowledge about the HPL of the user in the given phase of flight. This probability turns out to be less than 10^{-9}/hr for all three cases with different phases of flight. The other is the overall rate of occurrence of such an event in the worst situation known to the receiver, namely, for a geometry with an HPL close to the Horizontal Alert Limit (HAL) for the given phase of flight. This probability turns out to be on the order of 10^{-7}/hr for all cases with different phases of flight. The authors believe that both probabilities are acceptably small.

There are three additional factors that lead the authors to believe that these are acceptable probabilities. One is the adequate safety margin that exists with the primary and secondary OCAs, especially in the en route phase of flight. Another factor is that a missed detection occurs only for bias error magnitudes within a relatively small range of values. Still another reason is that a step-type failure, which is believed to occur more frequently than slowly drifting errors, would be more readily detectable by most receivers.

For these reasons stated above, the authors recommend that the target values of FDTs selected and analyzed be used as the along-track tolerances for the respective phases of flight. For the cross track tolerance, a flight technical tolerance (FTT) needs to be added. Regarding the question of how FTT should be added, it is considered appropriate to add an FTT value by root-sum-square. Regarding the question of what values of FTT would be appropriate for what phases of flight, the FAA has been doing extensive analyses and is expected to release the data some time in 1998. Our analysis could also be refined as we collect more data and gain fresh insights.

One final note is that the analysis in this paper assumed that the integrity is provided by RAIM, such as the case of stand-alone GPS with a TSO-C129 receiver. In case of GPS/WAAS, the receiver may dynamically switch for integrity between the WAAS signal and RAIM in real time, depending on whether the WAAS signal can provide integrity at the time, and the integrity limits (i.e., horizontal alert limits) are identical with the RAIM integrity limits for a TSO-C129 receiver. Therefore, the same criteria would apply as an upper bound for GPS/WAAS as for stand-alone GPS. Because the target values of FDT selected are all equal to or smaller than the FDTs provided by the current ground-based navaids for all three phases, GPS user equipped with a TSO-C129 receiver have compatible accuracy and integrity to fly over existing airways.

Table 3—Summary of the Analysis

Phase of Flight (Integrity alert limit)	**En Route (2 nmi)**	**Terminal (1 nmi)**	**Nonprecision Approach (0.3 nmi)**
Recommended along-track tolerance	0.5 nmi	0.4 nmi	0.3 nmi
Overall average probability of event E*	9 x 10^{-10} /hr	3 x 10^{-10} /hr	< 10^{-10}/hr
Probability of event E* when HPL is close to the RAIM integrity limit	6 x 10^{-7} /hr	4 x 10^{-7} /hr	< 10^{-7} /hr

* Event E: An event in which failure occurs and causes a missed detection of position error exceeding the recommended tolerance.

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Appendix A

Distribution of Horizontal Protection Level (HPL)

An analysis has been performed to determine how often Horizontal Protection Level (HPL) would exceed 0.5 nmi when seven satellites are in view. All geometries with seven satellites in view in each constellation are counted; the results are tabulated below in Figure A-2.

Table A-1. Conditions for the Analysis

Mask angle	5 deg
Sampling time	5 min (over 24 hr)
User locations	Five locations in CONUS: • Atlanta • San Fransisco • Dallas/Fort Worth • Chicago - O'Hare • New York - JFK
Constellations	Four different GPS constellations: • Optimized 24-satellite constellation with no SV removed • Average 23-satellite constellation (Optimized 24 w/o SV #1) • Average 22-satellite constellation (Optimized 24 w/o SV #2 and #15) • Average 21-satellite constellation (Optimized 24 w/o SV #3, #20, and #22)

Table A-2. Number of Geometries in Different Groups of HPL Values (In Case Seven Satelllites Are in View)

	HPL < 0.3 nmi	0.3 nmi ≤ HPL < 0.4 nmi	0.4 nmi ≤ HPL < 0.5 nmi	0.5 nmi ≤ HPL < 1 nmi	HPL ≥ 1 nmi
24 SV constellation	610	10	5	0	0
23 SV constellation	576	37	9	0	0
22 SV constellation	605	49	13	1	0
21 SV constellation	445	39	7	0	0
Total cases (percent)	2236 (92.9 %)	135 (5.6 %)	34 (1.4 %)	1 (< 0.1 %)	0

Glossary

ATT	along-track tolerance
CRM	Collision Risk Model
deg	degree(s)
DME	Distance Measuring Equipment
Eq.	Equation
EUROCAE	European Organization for Civil Aviation Equipment
FAA	Federal Aviation Administration
FDT	Fix Displacement Tolerance
FSD	full scale deflection
FTE	flight technical error
FTT	flight technical tolerance
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
HAL	Horizontal Alert Limit
HPL	Horizontal Protection Level
ICAO	International Civil Aviation Organization
ILS	Instrument Landing System
m	meter(s)
NAS	National Airspace System
NAVAID	Navigational Aid
nmi	nautical mile(s)
OCA	obstacle clearance area
OCP	Obstacle Clearance Panel
PANS-OPS	Procedures for Air Navigation Services–Operations

RAIM	Receiver Autonomous Integrity Monitoring
RNAV	Area Navigation
RSS	root sum square
SATZAP	Satellite Zapping
SC	Special Committee
SPS	Standard Positioning Service
SV	Satellite Vehicle
TERPS	Terminal Instrument Procedures
TSO	Technical Standard Order
VHF	very high frequency
VOR	VHF Omnidirectional Range
WAAS	Wide Area Augmentation System
XTT	cross-track tolerance