ACQUIRING MULTIPLE SPREAD SPECTRUM SIGNALS IN A MULTIANTENNA AD HOC NETWORK

Robert M. Taylor Jr.

The MITRE Corporation 7515 Colshire Drive McLean, VA 22102 rtaylor@mitre.org

ABSTRACT

In this study we investigate acquisition (code offset and channel state estimation) of multiple direct sequence spread spectrum DSSS signals in a multi-antenna ad hoc network. We derive the form of the optimal acquisition algorithm and show that it is NP-complete. We then derive a tractable suboptimal approach where we use a successive multistage cancellation approach in estimating the initial code offsets and channel matrix. Finally, we show probability of error curves for the optimal and suboptimal delay estimators.

1. INTRODUCTION

Mobile ad hoc networks (MANET) employing packet radio communications are rapidly becoming a target platform for a number of mobile communications applications. Conventional ad hoc networks employ single antenna nodes for both transmission and reception and suffer from the inability to resolve colliding packets. Medium access control (MAC) layer protocols have been designed to help avoid collisions. However, as the number of users and data packets increase, collisions become inevitable and requests for retransmission quickly limit the overall system capacity. Single antenna receivers employing direct sequence spread spectrum (DSSS) signaling can take advantage of the spread signal and use the different delays (code offsets, lags) to help discriminate packets arriving in the same time window. In packet radio communications, all users in the network use the same spreading sequence and therefore can only be discriminated based on their delays. Estimation of these different delays is the job of the acquisition stage. For the single antenna case, near simultaneous packet arrivals will prevent successful demodulation. Other problems such as overloading (number of signals greater than spreading length) and the near-far condition (one transmitter very close to receiver and drowns out signals further away) will cause the single antenna receiver to fail.

Multiple antenna receivers (adaptive arrays) are able to exploit the spatial domain to overcome many of these time domain problems. Adaptive arrays for MANET were shown to improve throughput performance of a slotted-ALOHA based network [1, 2] and a carrier sense multiple access (CSMA) based network [3, 4]. These authors used adaptive beam and null steering methods which assume a known fixed and calibrated antenna array.

The goal of this work is to derive estimators of the time delays and channel matrices for the acquisition stage of a multi-antenna system receiving multiple DSSS signals. To this end we build on the work of Dlugos et al. [5] who derived the maximum likelihood estimator and computed the acquisition performance for a single DSSS signal impinging on an adaptive array in a frequency flat fading environment. We consider the case of multiple transmitted signals and show that the maximum likelihood acquisition algorithm is NP-complete. We propose an efficient suboptimal procedure for esimating the time delays and channel matrix assuming the number of sources is known.

The rest of this paper is organized as follows. We describe the signal model in Section 2. We derive optimal and suboptimal algorithms for acquistion in Section 3. We demonstrate the performance of the proposed acquisition algorithm in Section 4.

2. SIGNAL MODEL

For analysis and simulation purposes, we work with signals that have already been downconverted and digitized. The digital baseband DSSS signal emitted from transmitter i sampled at N_s samples per chip at N_c chips per bit for an N_b-bit block is written $\forall n = 0, ..., N_c N_s N_b - 1$ as:

$$s_i[n] = p_i[n] b_i[\left\lfloor \frac{n}{N_s N_c} \right\rfloor] \tag{1}$$

where $\lfloor x \rfloor = floor(x)$, $p_i[n]$ is the filtered and sampled spreading code sequence for transmitter i and $b_i[\lfloor \frac{n}{N_s N_c} \rfloor]$ is the data bit sequence ($b_i \in \{-1, 1\}$) for BPSK modulation.

In keeping with the standard practice in ad hoc network systems, we assume all packets have the same spreading sequence. Different packets arrive with different delays. We assume that the delay of a given packet is constant across all the receiving antenna elements. This assumption breaks down if the antenna spacing is large and the signal bandwidth is sufficiently high. The mobile radio channel is modeled as a wide sense stationary uncorrelated scattering (WS-

SUS) frequency flat slow fading channel. The received signal at sensor j is then written as:

$$x_j[n] = \sum_{i=0}^{n_T-1} s_i[n-\lambda_i] A_{ij} e^{j\phi_{ij}} + w_j[n] \tag{2}$$

where $s_i[n-\lambda_i]$ is the ith source signal delayed by λ_i and n_T is the number of transmitted (source) signals. The variables A_{ij} and ϕ_{ij} are the amplitude and phase with respect to transmitter i and receiver j. Rewriting (2) in matrix-vector form yields $\forall n = 0, 1, ..., N_c N_s N_b - 1$

$$\mathbf{x}[n] = \mathbf{A}\mathbf{s}[n, \boldsymbol{\lambda}] + \mathbf{w}[n] \tag{3}$$

where $\mathbf{x}[n] \in \mathcal{C}^{n_R \times 1}$ is the zero-mean observation vector collected across all n_R sensors at time sample n, $\mathbf{s}[n] \in \mathcal{R}^{n_T \times 1}$ are the hidden zero-mean source signals containing the waveforms at time n from each of n_T transmitters, $\mathbf{A} \in \mathcal{C}^{n_R \times n_T}$ is the complex-valued mixing matrix modeling random amplitude and phase values for each transmitter-receiver pair, $\mathbf{w}[n] \in \mathcal{C}^{n_R \times 1}$ is the zero-mean complex Gaussian thermal receiver noise at sample n, and $\boldsymbol{\lambda}$ is the vector of circular shift time delays written as

$$\boldsymbol{\lambda} = [\lambda_0, \lambda_1, \cdots, \lambda_{n_T-1}]^T \tag{4}$$

where λ_i is an integer taking values in $0 \leq \lambda_i \leq N_c N_s$.

We collect the first N samples (snapshots) across all receivers in (3) and stack the snapshots into the matrix $\mathbf{X} = [\mathbf{x}[0], \mathbf{x}[1], \cdots, \mathbf{x}[N_p-1]]$. Doing likewise for $\mathbf{s}[n-\boldsymbol{\lambda}]$ and $\mathbf{w}[n]$, we can write the received signal model as

$$\mathbf{X} = \mathbf{A}\mathbf{S}(\boldsymbol{\lambda}) + \mathbf{W} \ . \tag{5}$$

As a preprocessing step for the algorithms to be described later, we whiten the observation data by applying a linear transformation $\mathbf{V}$ to yield

$$\mathbf{Z} = \mathbf{V}\mathbf{X} = \mathbf{H}\mathbf{S}(\boldsymbol{\lambda}) + \tilde{\mathbf{W}} \tag{6}$$

where

$$\mathbf{H} = \mathbf{V}\mathbf{A} = [\mathbf{h}_0, \mathbf{h}_1, \cdots, \mathbf{h}_{n_T-1}] \in \mathcal{C}^{n_R \times n_T} \tag{7}$$

is the whitened mixing matrix, $\mathbf{Z}$ is the whitened observation matrix, and $\tilde{\mathbf{W}} = \mathbf{V}\mathbf{W}$ is the new observation noise matrix. The whitening transformation matrix is formed as

$$\mathbf{V} = \mathbf{E}\mathbf{D}^{-1/2}\mathbf{E}^H \tag{8}$$

where $\mathbf{E}$ and $\mathbf{D}$ are computed from the eigen decomposition of the observation covariance matrix as $\mathcal{E}[\mathbf{X}\mathbf{X}^H] = \mathbf{E}\mathbf{D}\mathbf{E}^H$. Whitening causes the observed data matrix to become uncorrelated and have unit power in each dimension. We also assume that our source signals are uncorrelated and have unit power. Thus, we can write

$$\mathcal{E}[\mathbf{Z}\mathbf{Z}^H] = \mathbf{I}_{n_R \times n_R} \quad \mathcal{E}[\mathbf{S}\mathbf{S}^H] = \mathbf{I}_{n_T \times n_T} \ . \tag{9}$$

The source signals, in actual fact, will almost never all be equal to unit power. The different signal amplitudes will be contained within the $\mathbf{H}$ matrix in (7) and therefore unattainable without explicit knowledge of the $\mathbf{A}$ matrix values in (5).

3. MULTIPLE SIGNAL ACQUISITION

As mentioned previously, the objective in the acquisition stage is to estimate the time delays (or code offsets) for every source signal as well as the channel matrix (or channel state information). We can estimate the channel matrix $\mathbf{H}$ with respect to the whitened observation as in (6), or we can estimate the array manifold matrix $\mathbf{A}$ with respect to the original observation. We opt for the former to simplify the analysis.

Fortunately, both data and control packets in a MANET contain initial known bit patterns called preambles to aid in the acquisition process. These known bit patterns for the different sources causes the $\mathbf{S}(\boldsymbol{\lambda})$ matrix in (5) to take the form

$$\mathbf{S}(\boldsymbol{\lambda}) = \begin{bmatrix} \mathbf{s}_0^T \\ \mathbf{s}_1^T \\ \vdots \\ \mathbf{s}_{n_T-1}^T \end{bmatrix} = \begin{bmatrix} \mathbf{p}^T(\lambda_0) \\ \mathbf{p}^T(\lambda_1) \\ \vdots \\ \mathbf{p}^T(\lambda_{n_T-1}) \end{bmatrix} = \mathbf{P}(\boldsymbol{\lambda}) \tag{10}$$

where $\mathbf{P}(\boldsymbol{\lambda}) \in \mathcal{R}^{n_T \times N_p}$, N_p is the number of samples in the preamble, $\mathbf{p}$ is the known spreading sequence covering the length of the preamble, and

$$\mathbf{p}^T(\lambda_k) = [p[((n-\lambda_k))_{N_p}], p[((n-\lambda_k+1))_{N_p}], ..., \\ p[((n-\lambda_k+N_p-1))_{N_p}]] \tag{11}$$

where $((x))_N$ means x modulo N. The whitened observation model for the acquisition stage is therefore written

$$\mathbf{Z} = \mathbf{H}\mathbf{P}(\boldsymbol{\lambda}) + \mathbf{Y} \tag{12}$$

where now $\mathbf{Z} \in \mathcal{C}^{n_R \times N_p}$ and is a complex Gaussian matrix with pdf $\mathbf{Z} \sim \mathcal{CN}(\mathbf{H}\mathbf{P}(\boldsymbol{\lambda}), \mathbf{I})$. Note that the covariance matrix equals the $n_R \times n_R$ identity matrix for the whitened data case.

3.1. Optimal Acquisition

The optimal estimator for acquiring the time delays of the source signals and the mixing matrix which contains all the channel state information is obtained through the maximum likelihood estimator (MLE) given as:

$$\begin{bmatrix} \hat{\boldsymbol{\lambda}} & \hat{\mathbf{H}} \end{bmatrix} = \arg\max_{\boldsymbol{\lambda}, \mathbf{H}} p(\mathbf{Z}|\boldsymbol{\lambda}, \mathbf{H}) \ . \tag{13}$$

The MLE is optimal in the sense that, under the signal model given in (3), it is the minimum variance unbiased estimator (MVUE). It achieves the Cramer-Rao lower bound and contains the minimum variance among the class of all unbiased estimators. To derive the MLE, we first write out the pdf for the whitened data matrix $\mathbf{Z}$ calculated from (6) and modeled as a zero-mean complex Gaussian random matrix in (12)

$$p(\mathbf{Z}|\boldsymbol{\lambda},\mathbf{H}) \propto etr(-(\mathbf{Z}-\mathbf{H}\mathbf{P}(\boldsymbol{\lambda}))(\mathbf{Z}-\mathbf{H}\mathbf{P}(\boldsymbol{\lambda}))^H) \quad (14)$$

where $etr(\cdot) = \exp(tr[\cdot])$. The approach here is analogous to the derivation of the single transmitter case in [5]. If we add and subtract the term $\mathbf{Z}\mathbf{P}^T(\boldsymbol{\lambda})(\mathbf{P}(\boldsymbol{\lambda})\mathbf{P}^T(\boldsymbol{\lambda}))^{-1}\mathbf{P}(\boldsymbol{\lambda})\mathbf{Z}^H$ within the trace operator we can manipulate the expression to be

$$\begin{aligned} & tr[\mathbf{Z}\mathbf{Z}^H - 2\Re\{\mathbf{Z}\mathbf{P}^T(\boldsymbol{\lambda})\mathbf{H}^H\} + \mathbf{H}\mathbf{P}(\boldsymbol{\lambda})\mathbf{P}^T(\boldsymbol{\lambda})\mathbf{H}^H] \\ & = tr[\mathbf{Z}(\mathbf{I}-\mathbf{P}^T(\boldsymbol{\lambda})(\mathbf{P}(\boldsymbol{\lambda})\mathbf{P}^T(\boldsymbol{\lambda}))^{-1}\mathbf{P}(\boldsymbol{\lambda}))\mathbf{Z}^H + \\ & \quad (\mathbf{H}-\mathbf{Z}\mathbf{P}^T(\boldsymbol{\lambda})(\mathbf{P}(\boldsymbol{\lambda})\mathbf{P}^T(\boldsymbol{\lambda}))^{-1})\mathbf{P}(\boldsymbol{\lambda})\mathbf{P}^T(\boldsymbol{\lambda}) \times \\ & \quad (\mathbf{H}-\mathbf{Z}\mathbf{P}^T(\boldsymbol{\lambda})(\mathbf{P}(\boldsymbol{\lambda})\mathbf{P}^T(\boldsymbol{\lambda}))^{-1})^H] \end{aligned} \quad (15)$$

Since both terms within the trace are semidefinite, the trace is minimized when the second term is set to zero. This allows us to solve for the channel matrix MLE (which is a function of the yet unknown time delay vector) as

$$\hat{\mathbf{H}}^{MLE} = \mathbf{Z}\mathbf{P}^T(\boldsymbol{\lambda})(\mathbf{P}(\boldsymbol{\lambda})\mathbf{P}^T(\boldsymbol{\lambda}))^{-1} \ . \quad (16)$$

The time delays estimate can be computed independent of the channel matrix estimate and then plugged into (16) to solve for the channel parameters. The MLE from (13) then becomes

$$\left[\hat{\boldsymbol{\lambda}}\ \hat{\mathbf{H}}\right] = \arg\max_{\mathbf{H}} \left[\max_{\boldsymbol{\lambda}} p(\mathbf{Z}|\boldsymbol{\lambda},\mathbf{H})\right] \ . \quad (17)$$

From the first term in the trace, we can solve for the time delays estimate as:

$$\hat{\boldsymbol{\lambda}} = \arg\max_{\boldsymbol{\lambda}} tr[\mathbf{P}(\boldsymbol{\lambda})\mathbf{Z}^H\mathbf{Z}\mathbf{P}^T(\boldsymbol{\lambda})(\mathbf{P}(\boldsymbol{\lambda})\mathbf{P}^T(\boldsymbol{\lambda}))^{-1}] \quad (18)$$

The time delays estimation problem is a combinatorial optimization problem since the time delays are positive integers $0 \leq \lambda_i < N_c N_s \ \ \forall i = 0, 1, ..., n_p - 1$.

3.2. Suboptimal Acquisition

The estimate for the channel matrix in (16) is easily computable once the time delays estimate is found. However, the optimal solution to the time delays estimate involves discrete optimization methods. Since (18) is an NP-complete problem, and therefore too computationally complex for real-time acquisition, we are interested in suboptimal approaches to this problem. One method for approximating $\hat{\boldsymbol{\lambda}}$ that is robust to close delay spacings is to use successive multistage cancellation (SMC). In this approach we estimate one source (or equivalently one time delay) at a time, estimate the channel vector corresponding to that source, reconstruct the individual received signal, and then subtract that reconstructed signal from the received mixture.

We start by assigning $\mathbf{Z}^{(0)} = \mathbf{Z}$ and $\mathbf{Y}^{(0)} = \mathbf{Y}$. We then proceed to show the steps within a given stage of the SMC acquisition approach.

3.2.1. Estimating Time Delays

We first note that when (18) is reduced to the $n_p = 1$ case that $\mathbf{P}(\boldsymbol{\lambda}) = \mathbf{p}^T(\lambda_i)$ and

$$(\mathbf{P}(\boldsymbol{\lambda})\mathbf{P}^T(\boldsymbol{\lambda}))^{-1} = (\mathbf{p}^T(\lambda_i)\mathbf{p}(\lambda_i))^{-1} = \frac{1}{\|\mathbf{p}\|^2} \quad (19)$$

This allows us to write the time delay estimate for the i^{th} source from (18) as

$$\begin{aligned} \hat{\lambda}_i &= \arg\max_{\lambda_i} \frac{1}{\|\mathbf{p}\|^2}\mathbf{p}^T(\lambda_i)[\mathbf{Z}^{(i)}]^H\mathbf{Z}^{(i)}\mathbf{p}(\lambda_i) \\ &= \arg\max_{\lambda_i} \left\|\mathbf{Z}^{(i)}\mathbf{p}(\lambda_i)\right\|^2 \end{aligned} \quad (20)$$

where $\|\cdot\|$ denotes the L_2 norm. The parameter estimate $\hat{\lambda}_i$ in (20) can be efficiently computed using circular convolution via discrete Fourier transforms. The vector $\hat{\boldsymbol{\lambda}}$ up through stage i is formed as

$$\hat{\boldsymbol{\lambda}}^{(i)} = [\lambda_0, \lambda_1, ..., \lambda_i] \quad (21)$$

3.2.2. Estimating the Channel Matrix

Once the i^{th} time delay estimate is computed, we solve (16) for the $n_p = 1$ case which gives

$$\hat{\mathbf{h}}_i = \frac{1}{\|\mathbf{p}\|^2}\mathbf{Z}\mathbf{p}(\lambda_i) \ . \quad (22)$$

We collect the channel vectors in (22) into the partial channel matrix estimate as

$$\hat{\mathbf{H}}^{(i)} = \left[\hat{\mathbf{h}}_0, \hat{\mathbf{h}}_1, ..., \hat{\mathbf{h}}_i\right] \ . \quad (23)$$

Given the i^{th} partial channel matrix estimate in (23), we can reconstruct the signal subspace

$$\hat{\mathbf{X}}^{(i)} = [\mathbf{Y}^{(i)}]^{-1}\hat{\mathbf{H}}^{(i)}\hat{\mathbf{S}}^{(i)} \quad (24)$$

where

$$\hat{\mathbf{S}}^{(i)} = \Re\{[\hat{\mathbf{H}}^{(i)}]^{-1}\mathbf{Z}^{(i)}\} \ . \quad (25)$$

Subtracting (24) from the original observation matrix $\mathbf{X}$ in (5) gives the stage i residual observation matrix

$$\mathbf{X}_r^{(i)} = \mathbf{X} - \hat{\mathbf{X}}^{(i)} \ . \quad (26)$$

We then reassign the new whitened observation matrix to be

$$\mathbf{Z}^{(i+1)} = \mathbf{V}^{(i+1)}\mathbf{X}_r^{(i)} \tag{27}$$

where

$$\mathbf{V}^{(i+1)} = \mathbf{E}_r\mathbf{D}_r^{-1/2}\mathbf{E}_r^H \tag{28}$$

is the new transformation matrix formed from $E[\mathbf{X}_r^{(i)}[\mathbf{X}_r^{(i)}]^H] = \mathbf{E}_r\mathbf{D}_r\mathbf{E}_r^H$. We then increment the stage counter $i = i + 1$ and go to (20). Once all the source signals have been found, we have $\hat{\boldsymbol{\lambda}}$ and we can plug this into (16) to obtain a more accurate estimate of the overall channel matrix. We summarize the complete algorithm here.

Suboptimal Acquisition Algorithm

1. Select the first N samples from (2) across all receivers.
2. Whiten the data according to (6) and (8).
3. Initialize $i = 0$, set $\mathbf{Z}^{(0)} = \mathbf{Z}$, $\mathbf{V}^{(0)} = \mathbf{V}$, and store known preamble spreading sequence parameterized by circular delay shift λ_i as $\boldsymbol{\Sigma}(\lambda_i)$
4. Estimate time delay (code offset) of strongest available signal using (20)
5. Estimate channel vector corresponding to that time delay estimate using (22) and stack channel vector estimate into the partial channel matrix estimate as in (23).
6. Compute residual observation matrix (26) using (25), (23), and (28).
7. Update the whitened observation matrix according to (27) and the whitening transformation matrix according to (28).
8. Increment stage counter $i = i + 1$ and go to step 4 if $i < n_T$.
9. Assign $\hat{\boldsymbol{\lambda}} = \hat{\boldsymbol{\lambda}}^{(n_T)}$. Plug $\hat{\boldsymbol{\lambda}}$ into (16) to obtain channel matrix estimate.

4. SIMULATION RESULTS

We used four transmitters and six receive elements to create a channel matrix with magnitude one in each element but with a different random phase each trial. We randomize the bits, channel matrix, and receiver noise at each trial. The signal delay spacing between the four transmitters was held fixed at 30 samples. Our spreading code is an upsampled and filtered Barker code with 11 chips per bit and we sample at 10 samples per chip.

Fig. 1 shows the average probability of error for the optimal delays estimator from (18) and suboptimal delays estimator from (20)-(28) estimating using 200 Monte Carlo trials. We plot the average probability of error P_E as a function of the post-despread SNR per bit $\frac{E_b}{N_0}$ (assuming a uniform distribution of received powers). The successive multistage cancellation approach for estimating the delays vector in (4) is computationally efficient and performs comparably well to the intractable optimal estimator.

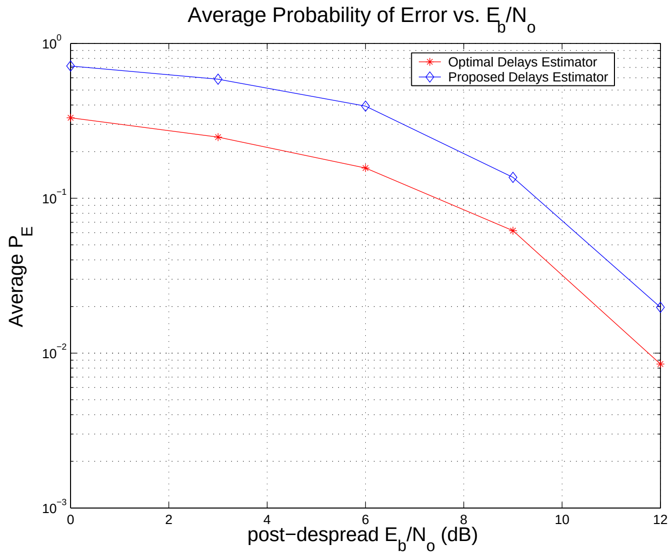

Fig. 1. Performance comparison of the proposed suboptimal acquisition algorithm to the optimal NP-complete delays estimator.

5. REFERENCES

[1] J. Ward and R.T. Compton, Jr., “Improving the Performance of a Slotted ALOHA Packet Radio Network with an Adaptive Array,” *IEEE Transactions on Communications*, vol. 40, pp. 292-300, Feb. 1992.

[2] J. Ward and R.T. Compton, Jr., “High Throughput Slotted ALOHA Packet Radio Networks with Adaptive Arrays,” *IEEE Transactions on Communications*, vol. 41, pp. 460-470, March 1993.

[3] A. Sugihara, K. Enomoto, and I. Sasase, “Throughput Performance of a Slotted Nonpersistent CSMA with an Adaptive Array,” *IEEE PIMRC*, 1995, pp. 633-637.

[4] Y. Chen, J. Li, and K. Liu, “Performance of Multi-hop Packet Radio Networks with Mobile Adaptive Antennas,” *IEEE VTC*, 2000, pp. 1318-1322.

[5] D. M. Dlugos and R. A. Scholtz, “Acquisition of Spread Spectrum Signals by an Adaptive Array,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, No. 8, August 1989.