SPECTRUM SHAPING USING CODE-HOPPING CDMA

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ABSTRACT

Spectral shaping is one of the primary technical challenges in spectrally constrained multiple access applications. In this paper, we introduce an efficient orthogonal CDMA-based spectral shaping technique. In our approach, termed Code-Hopping CDMA, users are allowed to hop between available spreading codes in an orthogonal fashion with different frequencies. We show that code hoping patterns can be chosen such that the power spectral density of the resulting signal would approximate a predetermined desired shape. We will demonstrate our approach through two examples and analyze the impact of the hopping pattern on the system capacity.

1. INTRODUCTION

Multiple access (MA) schemes are used to simultaneously share available bandwidth in communication systems. In conventional MA applications a finite and contiguous amount of spectrum which is designated for a particular service is allocated to multiple users. Therefore, the use of designated radio spectrum is unconstrained and can be utilized in many ways to achieve maximum efficiency. Time division multiple access (TDMA), frequency division multiple access (FDMA), and code division multiple access (CDMA) are the three major MA techniques. In recent years growing demand on available radio spectrum, as the fundamental shared resource of wireless communication systems, has forced many applications to use spectrum which is partially in use by other services or by legacy applications. Ultra wide band (UWB) communication systems [1], which share their spectrum with many services, an example of such applications where minimizing the impact of interference or co-existence with weaker narrow band systems is one of the main challenges. Another example is the modernized GPS system whose signal must share the L1 and L2 bands with existing commercial C/A Code in the center of the bands [2, 3]. Additionally, a recent guideline from the FCC Spectrum Policy Task Force [4], which recommends maximum flexibility in spectrum use by both licensed and unlicensed users, has motivated extensive research activities [5] in the area of dynamic resource allocation.

One of the primary technical challenges in flexible radio spectrum allocation is shaping the signal's spectrum to limit mutual interference with existing licensed users. The simplest approach to interference mitigation is to filter the interfering signal to suppress spectral content in certain bands. This will result in removing useful information along with the interference. A more efficient approach is to shape the spectrum at the transmitter. Spectral shaping reduces interference to the existing signals while preserving information signal and potentially enables us to have higher gains in other bands.

The above applications require design of MA schemes for non-contiguous radio spectrum. In these spectrally constrained MA applications, conventional TDMA and CDMA are generally not considered since all the users have to use the whole bandwidth for transmission. The most intuitive and most frequently used approach is based on FDMA or its more efficient version orthogonal frequency division multiple access (OFDMA) [6] where the spectrum shaping is achieved by turning off a set of carriers at the frequencies occupied by existing licensed users or at which strong narrow-band interference is present. Aside from well studied advantages and disadvantages of orthogonal frequency division multiplexing (OFDM), there are several drawbacks in applying OFDM to spectrum shaping. The main issue is spectral leakage that occurs when the existing signals or narrowband interferences are not centered at one of the OFDM carriers. Depending on the power of the existing signal, spectral leakage could potentially impact a considerable number of OFDM carriers, forcing us to turn off a large number of carriers.

In this paper, we introduce a novel orthogonal CDMA-based spectral shaping technique with potentially higher spectral efficiency. In our approach, which we refer to as Code-Hopping CDMA (CH-CDMA), users are allowed to hop between available spreading codes in an orthogonal fashion. We show that code hoping patterns can be chosen such that the power spectral density (PSD) of the signal approximates a predetermined desired shape. We will demonstrate our approach through a couple of examples and analyze the impact of the hopping pattern on the system capacity.

2. SPECTRAL SHAPING USING CH-CDMA

Our orthogonal CDMA-based approach to the constrained spectrum sharing problem allows users to hop among a set of orthogonal spreading sequences in a predetermined pattern. The approach is based on the simple observation that in orthogonal CDMA systems the spectral content of orthogonal spreading codes, such as Walsh-Hadamard (WH) codes, are often ignored despite the fact that they are well characterized in both frequency and time domain. In fact, orthogonal CDMA systems usually scramble the chips after spreading, which results in flattening the PSD of the signal. For constrained MA, however, the available spectrum is not contiguous and the goal is to shape the spectrum such that the transmitted signals would have sufficient spectrum separation from certain bands. A simple constrained MA algorithm is to choose a spreading code that provides the maximum spectral separation [3]. Generalizing this approach, we use more than one spreading code to spread the information bits. Moreover, we use different codes with different proportions. In other words, we *hop* among available codes with pre-determined probabilities. The time-average autocorrelation function (ACF) of the resulting signal will then be a weighted average of the ACFs of the codes, where the weights are the same as the proportions corresponding to each code. Furthermore, the PSD of the code-hopping signal will also be a weighted average of the PSDs of the codes. The desired spectra can then be realized by choosing the appropriate proportions in the CH-CDMA scheme.

2.1 Signal Model

Consider an orthogonal CDMA system with N orthogonal spreading sequences $\{q_j(t)\}_{j=1}^{N}$. The transmitted signal for user k can be expressed as

$$s_k(t) = \sum_{i=-\infty}^{\infty} a_i q_{k_i}(t - iT_{\mathrm{b}}) \tag{1}$$

where a_i is the ith information bit and T_b is the bit duration. For simplicity rectangular pulse shape is assumed. Unlike conventional orthogonal CDMA, in CH-CDMA every user utilizes all the codes with the same predetermined frequency while maintaining orthogonality. That is, no two users share the same code at the same time. Therefore, the spreading sequence $q_{k_i}(t)$, can be considered as a random variable where

$$q_{k_i} \in Q = \{q_1(t), q_2(t), \cdots, q_N(t)\},$$

and the probability of the jth code being used is p_j for all users sharing the same constrained bandwidth.

2.2 ACF and PSD of the CH-CDMA Signal

Let the binary information symbols, $a_i \in \{\pm 1\}$ be independent and identically distributed (i.i.d.). The ACF of the CH-CDMA signal can then be expressed as

$$\begin{aligned} R_{s_k}(t;\tau) &= E\left[s_k(t+\tau)s_k(t)\right] \\ &= \sum_{i=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} E\left[a_i a_l\right] E[q_{k_i}(t+\tau-iT_{\mathrm{b}})q_{k_l}(t-lT_{\mathrm{b}})] \\ &= \sum_{i=-\infty}^{\infty}\sum_{j=1}^{N} p_j q_j(t+\tau-iT_{\mathrm{b}})q_j(t-iT_{\mathrm{b}}) \end{aligned} \tag{2}$$

where the second expectation over all spreading sequences is given by

$$\begin{aligned} &E[q_{k_i}(t+\tau-iT_{\mathrm{b}})q_{k_i}(t-iT_{\mathrm{b}})] \\ &\quad = \sum_{j=1}^{N} p_j q_j(t+\tau-iT_{\mathrm{b}})q_j(t-iT_{\mathrm{b}}) \end{aligned} \tag{3}$$

Assuming cyclostationarity for the signal, the average ACF can be expressed as the weighted sum of the ACFs of the spreading codes [7],

$$R_{s_k}(\tau) = \sum_{j=1}^{N} p_j R_{q_j}(\tau) \tag{4}$$

The Fourier transform of the ACF yields the signal's power spectrum:

$$S_k(f) = \sum_{j=1}^{N} p_j S_{q_j}(f) \tag{5}$$

Given a set of orthogonal spreading functions we can then solve for the weights $\{p_j\}$ such that the mean-squared error between the desired PSD, $D(f)$, and $S_k(f)$ is minimized over a specific frequency region, $\Re$:

$$\min\ \xi_{LS} = \int_{\Re}[D(f) - S_k(f)]^2\, d\omega \tag{6}$$

subject to $p_j \geq 0$ and $\sum p_j = 1$ for all j.

2.3 CH-CDMA Capacity

An orthogonal CDMA system utilizing N orthogonal codes can support N users/channels. In the CH-CDMA, however, the number of users that can orthogonally share the CDMA channels is a function of the bias in code probabilities/weights. This is expected since any constraint on bandwidth resources should negatively impact the system capacity. In general, the maximum number of users that can share the CH-CDMA channels is given by

$$n_{\max} = \left\lfloor \frac{1}{\max_j\{p_j\}} \right\rfloor \tag{7}$$

In the extreme case, a maximum of N users can be achieved when the weights are uniformly distributed (i.e. $p_j = 1/N$ for all j) and only one user can be supported when $\max\{p_j\} > 0.5$. It is interesting to note that a uniform weight distribution corresponds to the unconstraint case where the chip sequence is completely random and the PSD is flat.

3. CH-CDMA DESIGN EXAMPLES

The following two design examples provide insight into CH-CDMA. The first example uses length-4 WH codes to demonstrate some characteristics of the ACF and PSD of CH-CDMA as a function of weights. The second example uses length-32 WH codes to determine weights corresponding to a bandstop-shaped PSD.

3.1 Example 1: Length-4 WH Codes

The four WH codes $\{1\ 1\ 1\ 1, 1\ \text{-}1\ 1\ \text{-}1, 1\ 1\ \text{-}1\ \text{-}1, 1\ \text{-}1\ \text{-}1\ 1\}$ generate distinct PSDs and ACFs, as shown by the dashed lines in Figures 1 and 2. Let *P* be the code probability vector of length-4. The first entry in *P* corresponds to the first WH code, the second entry to the second code, and so forth. In this example, suppose the desired PSD has a highpass shape with a null at and near DC. The first code clearly is not useful since most of its power is concentrated at or near DC. Therefore, we assign a zero weight to the first code. The third code also has a considerable amount of power near DC. We choose to assign a low weight (0.1) to the third code. The second and forth code, however, both have very good spectral separation from low frequencies. Therefore, we examine two CH-CDMA codes with the following *P* vectors:

$$P1 = [0.0\ 0.6\ 0.1\ 0.3]$$

and

$$P2 = [0.0\ 0.3\ 0.1\ 0.6]$$

Where *P1* emphasizes the second code while *P2* assigns the highest code probability to the forth code. PSDs and ACFs of both codes are depicted in Figures 1 and 2 by solid lines. Although both codes have very good spectral separation from the stop band, *P2* seems to have slightly less power in the stop band and also less variation at the passband. In some applications it is equally important to analyze the ACF properties of the codes. From Figure 2, the advantages of the ACF of the *P2* code lie in its lower side lobes, while its drawback is that *P1*'s ACF has a narrower main lobe.

For this design example, we used an ad-hoc approach to demonstrate the variability of the power spectrum with respect to the CH-CDMA code probabilities. A more systematic design approach is presented in the next example.

3.2 Example 2: Bandstop-Shaped PSD

In this example we use length-32 WH codes in CH-CDMA to design a bandstop-shaped PSD. A least-squares approach is used to determine the code weights that minimize the mean-squared error between $D(f)$ and $S_k(f)$. The mean-squared error measure is given by

$$\xi_{LS} = \int_{\Re} \left[\gamma D(f) - S_k(f)\right]^2 df \qquad (8)$$

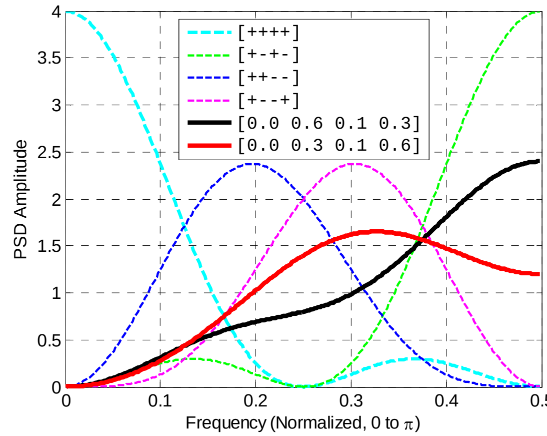

Figure 1: PSD's of the four length-4 WH codes and two CH-CDMA codes.

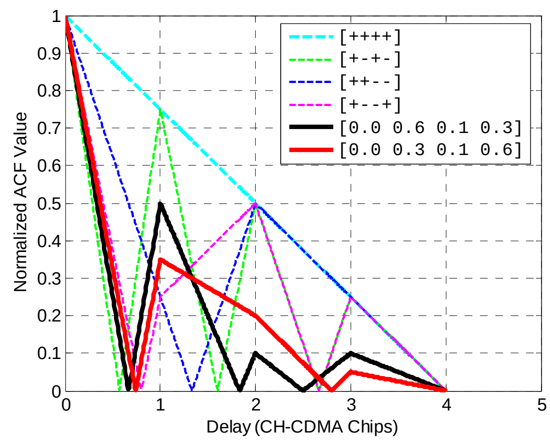

Figure 2: ACF's of the four length-4 WH codes and two CH-CDMA codes.

The scaling factor γ is given by $\gamma = \dfrac{S_k(f_0)}{D(f_0)}$, where f_0 is the minimum frequency value for the maximum value of $D(f)$ [8]. Let W and $g(f)$ be the $N \times N$ WH matrix and Fourier transform of the pulse shaping function, respectively. Then, the PSD of the CH-CDMA signal can be expressed as

$$S_k(f) = P^T W^T e e^H W \left|g(f)\right|^2 \qquad (9)$$

Where $e = [1\ \ e^{-j2\pi f}\ \ e^{-j4\pi f} \cdots e^{-j(N-1)2\pi f}]^T$. Substituting (9) into (8), the error measure can be written as

$$\xi_{LS} = P^T X P \qquad (10)$$

where

$$X = \int_{\Re} W^T \left| e_0 e_0^H \frac{D(f)}{D(f_0)} \left|g(f_0)\right|^2 - e e^H \left|g(f)\right|^2 \right|^2 W df \qquad (11)$$

and $e_0 = [1 \;\; e^{-j2\pi f_0} \;\; e^{-j4\pi f_0} \cdots e^{-j(N-1)2\pi f_0}]^T$. We use constrained nonlinear optimization techniques to minimize the objective function in (10) with respect to *P*, subject to $\sum p_j = 1$ and $p_j \geq 0$ for all *j*. We employed the MATLAB function "fmincon" which uses a sequential quadratic programming (SQP) method to solve for *P*. In this example, length-32 WH codes and a bandstop-shaped desired PSD were used and the pulse shape function was set to 1 across all frequencies. The resulting PSD, scaled so that the mean value in the passband equals the desired value, is shown in Figure 3. The corresponding ACF is shown in Figure 4 and the resulting *P* vector is:

```
[0.0342  0.0317    0        0      0
 0       0.0683    0.0800   0      0.0802
 0       0.0601    0.0795   0      0.0638
 0       0.0370    0.0574   0      0
 0.0523  0.0515    0.0189   0      0.0394
 0       0         0.0699   0      0.0507
 0.0814  0.0438].
```

Given $p_{j\max} = 0.0814$, (7) suggests that the maximum number of users is 12. However, quantizing *P* will impact both the number of users and the signal's PSD. Studying this tradeoff is a subject of our future studies.

4. CONCLUSION AND FUTURE STUDIES

In this paper, we have introduced a novel CDMA-based spectral shaping technique. Our proposed CH-CDMA approach is based on using multiple spreading codes, where each code is used with a different probability. We showed that code probabilities determine the signal's spectral shape. The system capacity as a function of code probabilities was also analyzed.

Our future studies include analyzing the performance of various spreading code sets relative to different desired PSD shapes. Another important issue that needs to be addressed is the quantization effect on our system performance. Finally, we plan to study our approach in the context of different applications, such as UWB.

5. ACKNOWLEDGMENT

The author is grateful to *Michael L Dickens* for writing the simulation code and for helpful discussions.

6. REFERENCES

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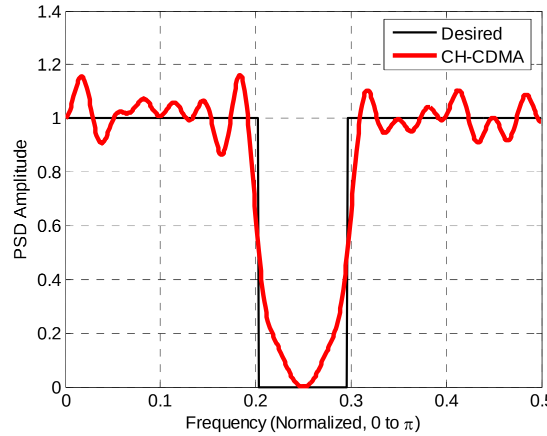

Figure 3: Bandstop-shaped PSD using length-32 WH codes.

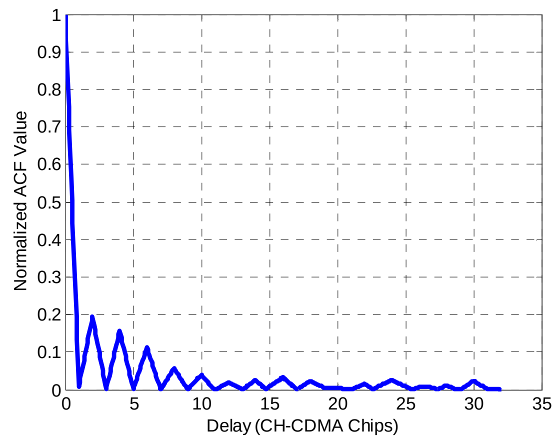

Figure 4: ACF of bandstop-shaped PSD using length-32 WH codes.

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