Code Tracking Performance for Novel Unambiguous M-Code Time Discriminators

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BIOGRAPHY

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ABSTRACT

The conventional early/late gate time discriminator for the M-Code signal has multiple zeros, which can produce ambiguous or false lock positions. This paper introduces two new classes of unambiguous (i.e., no false lock positions) discriminators that have a number of very attractive features.

INTRODUCTION

Utilization of the GPS navigation system requires TOA (time-of-arrival) measurements at the receiver from several satellites. An essential procedure in TOA estimation is the tracking of epochs in the received signals from each satellite. Separate tracking loops are implemented for each satellite-receiver path. Each tracking loop utilizes a time discriminator which, ideally, produces an error signal proportional to the difference between the timing of the received code signal and the replica code signal generated by the tracking loop. In fact, all time discriminators are linear only over a limited region. The conventional early/late gate time discriminator for the M-Code signal has multiple zeroes which, under non-ideal conditions, can produce "ambiguous" or false lock positions. These false lock positions produce a bias in the estimated TOA. This paper evaluates M-Code tracking loop performance using "unambiguous" normalized time discriminator techniques. Normalization is used to keep the time discriminator slope for small errors independent of signal level. Such normalization keeps the loop gain, and thus the loop noise bandwidth, from fluctuating with signal level at high signal-to-noise ratio(SNR).

The two types of unambiguous normalized discriminators considered are the multiple gate delay (MGD) and the partial sideband (PSB) classes. The multiple gate delay [1,2] uses multiple, appropriately weighted early and late gates and the partial sideband discriminator [3] uses weighted combinations of the upper and lower sidebands of the M-Code signal. The bump-jump discriminator [4] is a special case of the MGD discriminator that uses only four gates (early, late, very early, and very late) plus a jumping algorithm. The BPSK-Like Technique [5,6,7] is close to a special case of the PSB discriminator where the parameter β (to be defined) is set equal to zero. In the next section, these will be discussed in more detail.

DISCRIMINATOR DESCRIPTIONS

In order to discuss the unambiguous discriminators, let us define the following: r(t) = received signal complex envelope plus noise after front-end filtering (total bandwidth = 24 MHz), s(t) = stored replica of the transmitted binary offset carrier(BOC) signal, $z_U(t)$ = signal plus noise after filtering by the upper-sideband (0 to 12 MHz) brickwall filter and $z_L(t)$ = signal plus noise after filtering by the lower sideband (-12 MHz to 0) brickwall filter. In general, r(t) can be written as

$$r(t) = \hat{s}(t) + n(t)\ , \qquad (1)$$

where $\hat{s}(t)$ is the filtered spreading sequence and n(t) is the complex envelope of the received noise (to simplify notation we do not discuss the data modulation, which does not affect our analysis). The stored replica has no noise and has $\hat{s}(t)$ replaced by $s(t)$, the unfiltered spreading sequence.

The class of normalized PSB discriminators uses a linear combination of the upper and lower sidebands of the received signal, to obtain modified upper and lower signals. These are

$$y_U(t) = z_U(t) + \beta z_L(t)\,, \tag{2}$$

$$y_L(t) = \beta z_U(t) + z_L(t)\,, \tag{3}$$

where $0 \leq \beta \leq 1$. For $\beta = 0$, the combination signals reduce to the pure upper and lower sideband signals. For $\beta = 1$, the discriminator becomes the conventional non-coherent, normalized early/late gate discriminator.

Next, define the outputs of the crosscorrelation of the different classes of signal with the replica. The crosscorrelation of the total received signal with the replica at the end of the k^{th} integrate and dump is

$$R(\tau) = \int_{t_k}^{T_o + t_k} dt\, r(t)\, s^*(t+\tau)\,, \tag{4}$$

and the crosscorrelations with the upper and lower sidebands of the received signal are

$$R_U(\tau) = \int_{t_k}^{t_k + T_o} dt\, y_U(t)\, s^*(t+\tau)\,, \tag{5}$$

$$R_L(\tau) = \int_{t_k}^{t_k + T_o} dt\, y_L(t)\, s^*(t+\tau)\,, \tag{6}$$

where T_o is the integration time of the correlators. Then, in terms of these quantities, we can specify the error output of the non-coherent, normalized PSB4 discriminator as

$$D_{PSB4}(\tau) = \frac{R_U^2\left(\tau + \frac{\Delta}{2}\right) - R_U^2\left(\tau - \frac{\Delta}{2}\right) + R_L^2\left(\tau + \frac{\Delta}{2}\right) - R_L^2\left(\tau - \frac{\Delta}{2}\right)}{R_U^2\left(\tau + \frac{\Delta}{2}\right) + R_U^2\left(\tau - \frac{\Delta}{2}\right) + R_L^2\left(\tau + \frac{\Delta}{2}\right) + R_L^2\left(\tau - \frac{\Delta}{2}\right)} \tag{7}$$

where Δ is the gate spacing and the parameter β is used to control the shape of $D_{PSB4}(\tau)$. Implementation of this discriminator requires two pairs of early/late gates in addition to the single sideband filters. Another class of PSB discriminators considered is the PSB2 class which uses only a single pair of early/late gates. Either (but not both) the upper or lower sidebands in Equation (7) are used in the PSB2 class(e.g., the first two terms in the numerator and denominator of Equation (7) or the last two terms in the numerator and denominator of Equation (7)).

The error output of the non-coherent normalized multiple gate delay (MGD) discriminator of order N is given by

$$D_{MGD}(\tau) = \frac{\sum_{n=1}^{N} a_n \left[R^2\left(\tau - \left(m - \frac{1}{2}\right)\Delta\right) - R^2\left(\tau + \left(m - \frac{1}{2}\right)\Delta\right) \right]}{\sum_{n=1}^{N} a_n \left[R^2\left(\tau - \left(m - \frac{1}{2}\right)\Delta\right) + R^2\left(\tau + \left(m - \frac{1}{2}\right)\Delta\right) \right]}, \tag{8}$$

where the coefficients a_n are used to shape the response of the discriminator. Note that for $N = 1$, the result in Equation (8) reduces to the conventional normalized, non-coherent early/late gate discriminator.

The gate spacing Δ and the parameter β can be used to control the shape of the PSB discriminator and the gate spacing Δ and the coefficients a_n can be used to control the MGD discriminator shape. We find that each discriminator type can have two fundamental shapes: either $D(\tau)$ is relatively monotonic, but has a shallow slope at $\tau = 0$,or $D(\tau)$ is quite bumpy, but with a steep slope at $\tau = 0$. The bumpy type has a smaller tracking error, but a longer convergence time when compared with the smooth class. Some typical results for the noise-free MGD and PSB4 discriminators are plotted versus delay normalized by the M-Code chip length T (48.875 ns) in Figures 1 and 2, and the parameters used to generate these results are shown in Table 1. Note that in the noise-free limit, the PSB4 and PSB2 discriminators are identical, so the plot in Figure 2 also applies to the PSB2 discriminator.

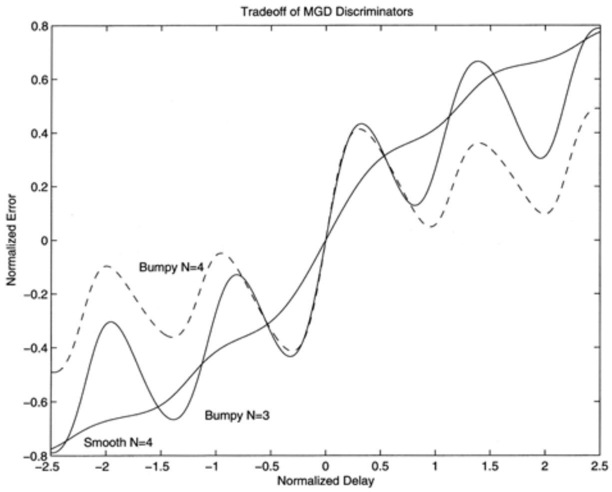

Figure 1. Trade-off of MGD Discriminators

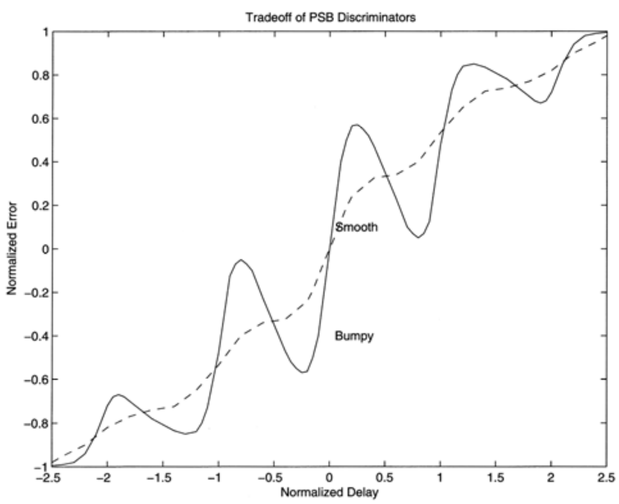

Figure 2. Trade-off of PSB Discriminators

Table 1. Discriminator Parameters

Type	N	Coefficient Vector	Δ(ns)	β
Bumpy MGD	3	[1 1 1]	29.3	N/A
Smooth MGD	4	[1 1.25 1.5 1.75]	25.7	N/A
Bumpy MGD	4	[1 1.125 1.25 1.375]	9.78	N/A
Bumpy PSB2	N/A	N/A	145	0.28
Smooth PSB4	N/A	N/A	142	0.05
Bumpy PSB4	N/A	N/A	145	0.28

Note that all the discriminators have the property that the only zero occurs at the correct delay ($\tau = 0$), so the multiple ambiguities inherent in the conventional early/late gate discriminator have been removed.

ASYMPTOTIC TRACKING ERROR

Although the simulations to be presented later use a second-order discrete time tracking loop, it is useful to use a first-order loop to calculate the asymptotic tracking error. Suppose the correct signal delay is $\tau = 0$ and the estimate of the delay at the time step n is $\tau(n)$. Then, the estimate at time step (n + 1) is

$$\tau(n+1) = \tau(n) - K_L D[\tau(n)], \qquad (9)$$

where D is the discriminator error function in Equations (7) and (8), and K_L is the loop gain given by [8]

$$K_L = \frac{4B_L T_o}{D_o'(0)}, \qquad (10)$$

where B_L is the one-sided loop bandwidth, T_o is the integration time of the correlator and $D_o'(0)$ is the time derivative of the noise-free error function at $\tau = 0$. When $\tau(n)$ is close to the correct delay, we can approximate $D(\tau) \simeq D_o'(0)\tau + \delta D(\tau)$ so that Equation (9) becomes

$$\tau(n+1) = \tau(n)(1 - 4B_L T_o) - K_L \delta D(n), \qquad (11)$$

If we use the fact that $\delta D(n)$ is statistically independent of $\tau(n)$, then it is readily seen by squaring both sides of Equation (11) and taking the expectation that

$$\sigma_\tau^2(n+1) = \sigma_\tau^2(n)(1 - 4B_L T_o)^2 + K_L^2 E(\delta D^2), \quad (12)$$

where $\sigma_\tau^2 = E(\tau^2)$ and $E(\ldots)$ denotes an expectation.

Furthermore, as $n \to \infty$, $\sigma_\tau^2(n+1) = \sigma_\tau^2(n)$ so we obtain for the variance of the delay estimation error

$$\sigma_\tau^2 = \frac{K_L^2 E(\delta D^2)}{8B_L T_o(1 - 2B_L T_o)}. \qquad (13)$$

Finally, if we substitute for K_L and $E(\delta D^2)$, we obtain for the asymptotic estimation error

$$\frac{\sigma_\tau}{T} = \frac{\Gamma}{\left(\frac{C}{N_o B_L}\right)^{1/2} \left(1 - 2B_L T_o\right)^{1/2}}, \qquad (14)$$

where C/N_0 is the carrier-to-noise ratio, T = M-Code chip duration (48.875 ns), and Γ is a function of the signal and noise correlation functions that is too lengthy to be presented here. However, numerical values for Γ have been obtained and are presented in Table 2, for both types of discriminators studied here and for the BPSK-like and subcarrier phase cancellation (SCPC) techniques presented in Reference 7. The variance of the delay tracking error has also been obtained [3] for a second-order discrete tracking loop, and is close to the results shown above for the first-order loop.

From the results in Table 2, we see that both the bumpy MGD and PSB discriminators introduced here can achieve nearly the same asymptotic error as the conventional, non-coherent early/late gate discriminator, but without the ambiguities inherent in the latter. In fact, the bumpy N = 4 MGD discriminators is just as accurate as the non-coherent early/late gate discriminator.

It is also evident from Table 2 that the BPSK-like and the Sub-Carrier Phase Cancellation (SCPC) discriminators cannot compete with the MGD and PSB classes for M-Code tracking applications. Therefore, we will not include these types in our simulations to follow.

Table 2. Sensitivity Parameter Γ

Type of Discriminator	Γ	SNR Penalty Relative to Early/Late Gate (dB)
Early/Late Gate	0.377	0
Bumpy MGD, N = 3	0.53	2.96
Smooth MGD, N = 4	1.51	12.9
Bumpy MGD, N = 4	0.383	0.14
Bumpy PSB2	0.7	5.35
Smooth PSB4	1.2	10.05
Bumpy PSB4	0.54	3.1
SCPC, Ref. 7	2.71	17.1
BPSK-Like, Ref. 7	2.98	18.0

SIMULATED RESULTS

The behavior of the MGD and PSB trackers was simulated using a second-order, discrete-time loop as described by Stephens and Thomas [9]. We first obtained the steady state rms tracking error by running the simulation for 10^5 iterations (one iteration per correlator integration time T_o) for the cases when $B_L T_o = 0.01$ and $B_L T_o = 0.1$ and for varying values of signal-to-noise ratio SNR, where $SNR = (C/N_o)T_o$. We found [3] that the SNR penalties for the various discriminator types are consistent with the analytical results presented in Table 2.

At sufficiently low input SNR, the loop will lose lock at some particular time instant. To evaluate loss of lock, the SNR was decreased in 0.1 dB increments until the loop unlocked as defined by the presence of an error greater than 500 ns (It was found that when this condition exists, the loop "ran away"). Table 3 presents the SNRs at which the loop unlocked for the different discriminator types considered. Observe that all of the discriminators unlock at a slightly higher level than the conventional early/late gate for $B_L T_o = 0.01$, but the bumpy MGD for N = 4 is actually better than the conventional early/late gate for $B_L T_o = 0.1$.

Table 3. Loss of Lock SNR Thresholds (dB)

Type of Discriminator	$B_L T_o = 0.01$	$B_L T_o = 0.1$
Early/Late	-4.2	4.9
Smooth MGD, N = 4	-3.4	4.9
Bumpy MGD, N = 4	-3.4	3.5
Smooth PSB4	-2.9	4.1
Bumpy PSB4	-2.9	4.1

The response of a code-tracking loop to different dynamic time-offset conditions and different SNRs is of considerable interest. Here we explore the response to a step Δτ in the time offset when the loop is closed, although future efforts will explore the impacts of velocity, acceleration, jerk, etc.

Calculations were performed for SNRs of both 12 dB and 25 dB, but only the 25 dB results will be presented here. Also, two different loop bandwidths were considered; these are $B_L T_o = 0.01$ and $B_L T_o = 0.1$. For all simulations, the iteration period was equal to $T_o = 20$ ms.

Step inputs errors of 14.8, 47, 153, and 186 ns were considered. Because the M-Code chip period T = 48.875, these step inputs correspond to step errors of 0.3T, 0.96T, 3.13T, and 3.8T, respectively. Typical responses for the smooth and bumpy MGD (N = 4) discriminator are shown in Figure 3 for ten different noise realizations, when SNR = 25 dB and $B_L T_o$ = 0.01. As expected, the smooth discriminator produces a faster convergence than the bumpy one.

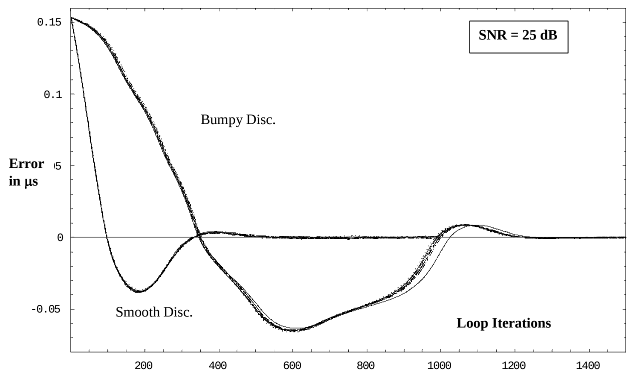

Figure 3. Settling Time of MGD (N = 4) Discriminator for Step-Input Error of 153 ns

A summary of the settling times for the smooth and bumpy MGD (N = 4) discriminators is presented in Figure 4, from which it is evident that when the bumpy discriminator is used in the tracking loop, its settling time is approximately three times as large as for the smooth discriminator, except for small step errors ($\Delta\tau << T$) in which case they behave similarly. Similar results were obtained using the smooth and bumpy classes of PSB discriminator.

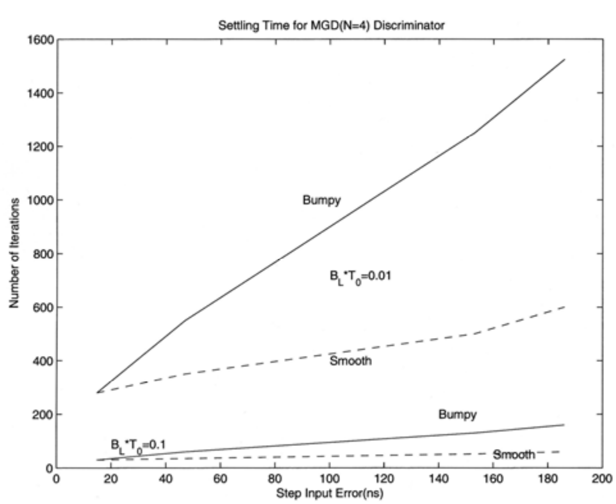

Figure 4. Settling Time for MGD (N = 4) Discriminator

Although not shown, the bumpy MGD (N = 3) has [2], a response time that is approximately 0.65 of the bumpy MGD (N = 4) response time. Thus, because the bumpy MGD (N = 3) uses two fewer correlators, converges faster, but has a 2.82 dB penalty in SNR (relative to bumpy MGD (N = 4)), it represents a good compromise solution.

CONCLUSIONS AND RECOMMENDATIONS

We have demonstrated that the MGD (N = 4) bumpy discriminator provides unambiguous tracking with virtually no SNR penalty relative to the highly-ambiguous, non-coherent early/late gate tracker. The bumpy PSB4 and the bumpy MGD (N = 3) also provide unambiguous tracking, but with a 3 dB SNR penalty. However, comparisons of step responses of loops using the MGD and PSB bumpy discriminators with loops using the MGD and PSB smooth discriminators reveal that for large steps, the settling time for the bumpy discriminators is up to three times that for the smooth discriminators. For sufficiently small steps, the settling time is the same.

The MGD and PSB time discriminators provide a range of options. These allow a trade-off between SNR penalties and settling times for large step responses.

The performance calculations presented assumed no linear distortion in the system. It is important to compare the effects of linear distortion on tracking performance for the conventional and the unambiguous time discriminators, considering all relevant sources of distortion, such as multipath, ionospheric distortion, and receiver mismatch. This is the subject of an ongoing study.

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