# A STUDY OF THE PERFORMANCE PROPERTIES OF SMALL ANTENNAS

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**ABSTRACT: Electrically small antennas are of interest in a variety of applications, particularly where the antennas must be unobtrusive. The performance properties considered in the design of small antennas typically include impedance, radiation efficiency, pattern shape, polarization and particularly operating bandwidth and quality factor (*Q*). In this paper, we compare the performance properties of several fundamental small antenna designs as a function of overall height and size (*ka*). Antennas studied here include the multi-arm folded helix, the matched disk loaded dipole, the matched spherical-cap dipole and the multi-arm spherical resonator. The antennas are compared to determine which configuration offers the best performance in terms of pattern shape, radiation efficiency, and ½-power and 2:1 VSWR bandwidths.**

## 1. Introduction

Optimization of the performance properties of electrically small antennas has been given considerable attention in recent years. Performance optimization goals generally include achieving a good impedance match (low VSWR), high radiation efficiency and low $Q$ or wide operating bandwidth. In this optimization process, the small antenna's $Q$ and bandwidth properties are typically compared against well defined fundamental limits.

Design approaches used in achieving the optimization goals stated above vary and include techniques such capacity or top-hat loading [1]-[2], the use of multiple folded arms in wire monopole or dipole antennas [2]-[3], inductive loading in wire antennas (an increase in conductor length) [4], and more recently the use of Metamaterials [5]-[7] and multi-arm coupled structures [8]-[10].

It is well known that any electrically small antenna can be impedance matched at any single frequency using an external matching network comprised of reactive matching components. One challenge in using an external matching network with an electrically small antenna is that the loss resistance within the matching network often exceeds the radiation resistance of the antenna, resulting in low overall efficiency. In some cases, the reduction in mismatch loss achieved with the impedance match exceeds the increase in loss due to the matching components and better overall performance is achieved relative to the isolated mismatched antenna.

Impedance matching can also be accomplished within the antenna structure using a number of techniques that include but are not limited to the use of multiple folded arms [11]-[13] and shunt or parallel matching stubs [14]-[15]. These techniques are often more efficient than impedance matching the small antenna using an external matching network. Many electrically small antenna designs have been described which exhibit low VSWR and high radiation efficiency [8]-[9], [12]-[13], [15], [16]-[19].

The remaining challenge in designing a small antenna is optimizing the operating bandwidth, which is often characterized using the antenna's $Q$. Recent efforts in this area have demonstrated that small antennas can be designed to achieve a $Q$ that closely approaches fundamental limits.

The objective of this work is to examine and compare the relative performance properties of a number of fundamental small antenna designs which include the folded spherical helix [11]-[12], the folded cylindrical helix, the disk-loaded dipole [20], the spherical-cap dipole [21]-[22] and the spherical resonator antenna [8]. The folded spherical helix is a single resonant antenna that exhibits a $Q$ within 1.5 times the lower bound. The disk-loaded dipole and spherical-cap dipole are well known designs that are often assumed to be optimum approaches in achieving the lowest possible $Q$ with a small antenna. The spherical resonator is of interest because it exhibits closely spaced resonances within its defined VSWR bandwidth and may offer wider bandwidths than can be achieved with small antennas that exhibit a single resonance.

The performance properties studied include impedance, radiation efficiency, pattern and polarization performance, $Q$ and bandwidth.

## 2. Fundamental Limitations of Small Antennas

Most of what follows in this section has been presented in previous journal and/or conference publications. It is presented here for completeness of the work and as a necessary background discussion for the relative performance comparisons that are to be made between the different small antennas.

An antenna is considered to be electrically small as a function of its overall size or occupied volume relative to the wavelength. A small antenna is one where $ka \leq 0.5$, where $k$ is the free space wavenumber, $2\pi/\lambda$, and $a$ is the radius of an imaginary sphere circumscribing the maximum dimensions of the antenna.

The frequency dependent impedance of the electrically small antenna is given by $Z(\omega) = R(\omega) + j\,X(\omega)$, where $\omega$ is the radian frequency $2\pi f$, $f$ is the frequency in Hz, $R(\omega)$ is the antenna's total feed point resistance (including both radiation and loss terms) and $X(\omega)$ is the antenna's feed point reactance. The bandwidth and $Q$ of the small antenna are defined at radian frequency $\omega_0$, where the antenna is either naturally self-resonant or tuned to resonance with a lossless series reactance. In most instances, the tuned or self-resonant small antenna exhibits a single resonance within its defined VSWR bandwidth. The maximum extent of the defined bandwidth is typically limited to values of VSWR less than 5.828 (½-power).

The bandwidth of the small antenna is often characterized by its $Q$ because matched VSWR bandwidth and $Q$ have been shown to be inversely related and a lower bound on the minimum achievable $Q$ is defined and well known. No electrically small antenna will exhibit a $Q$ less than the lower bound.

The exact $Q$ of an electrically small antenna that is tuned or made to be self-resonant at a frequency $\omega_0$, is defined as [23]-[24]

$$Q(\omega_0) = \frac{\omega_0 |W|}{P} \tag{1}$$

where $W$ is internal energy and $P$ is the total power accepted by the antenna. If the tuned small antenna exhibits a single resonance within its defined VSWR bandwidth, its $Q$ can be accurately approximated from its impedance properties using [24]

$$Q(\omega_0) \approx Q_Z(\omega_0) = \frac{\omega_0}{2\,R(\omega_0)} \sqrt{R'(\omega_0)^2 + \left( X'(\omega_0) + \frac{|X(\omega_0)|}{\omega_0} \right)^2} \tag{2}$$

where $R'(\omega_0)$ and $X'(\omega_0)$ are the frequency derivatives of resistance and reactance, respectively. The lower bound on $Q$ is given by [25]

$$Q_{lb} = \eta_r \left( \frac{1}{(ka)^3} + \frac{1}{ka} \right) \tag{3}$$

where $\eta_r$ is the antenna's radiation efficiency.

To characterize the antenna's bandwidth, it is necessary to define bandwidth in a manner such that $Q$ and bandwidth are inversely related. The definition of bandwidth suitable for this purpose is fractional matched VSWR bandwidth, $FBW_V(\omega_0)$, where the VSWR of the tuned antenna is determined using a characteristic impedance, $Z_{CH}$, equal to the antenna's feed point resistance, $R(\omega_0)$. Fractional matched VSWR bandwidth is given by

$$FBW_V(\omega_0) = \frac{\omega_+ - \omega_-}{\omega_0} \tag{4}$$

where $\omega_+$ and $\omega_-$ are the frequencies above and below $\omega_0$, respectively, where the VSWR is equal to any arbitrary value denoted by $s$. Fractional matched VSWR bandwidth and $Q$ are related as [24]

$$Q(\omega_0) \approx \frac{2\sqrt{\beta}}{FBW_V(\omega_0)}, \qquad \sqrt{\beta} = \frac{s-1}{2\sqrt{s}} \le 1 \tag{5}$$

The approximations in Eqs. (2) and (5) were derived in [24] under the assumptions that the tuned antenna exhibits a single resonance within its defined VSWR bandwidth and that the ½-power matched VSWR bandwidth (s = 5.828) is not too large.

The minimum achievable $Q$ of a tuned or matched small antenna exhibiting a single resonance is limited by the lower bound defined in Eq. (3). Since $Q$ and matched VSWR bandwidth are inversely related, the matched VSWR bandwidth of the small antenna exhibiting a single resonance will not be greater than that predicted by the inverse of $Q$. Using Eqs. (3) and (5), an upper bound on the fractional matched VSWR bandwidth can be written as

$$FBW_{Vub} = \frac{1}{\eta_r} \frac{(ka)^3}{1+(ka)^2} \frac{s-1}{\sqrt{s}} \tag{6}$$

## 3. The Small Antennas

In this section, the basic configurations and performance properties of the small antennas are presented. These antennas include the folded spherical helix, the folded cylindrical helix, the disk-loaded dipole, the spherical-cap dipole and the multi-arm spherical resonator. Performance properties considered include the antenna's impedance, radiation efficiency, pattern characteristics (including polarization), VSWR bandwidth and *Q*.

### 3.1 The Folded Spherical Helix

The first antenna considered is the 4-arm folded spherical helix [11]-[12] because it was specifically designed to exhibit a good impedance match in a 50Ω system, high radiation efficiency and a low *Q*. The folded spherical helix exhibits a single resonance and has a *Q* that is within 1.5 times the lower bound at a value of $ka \approx 0.263$. The folded spherical helix is considered the baseline antenna for this study. Other antennas compared to the folded spherical helix will be designed to have the same height, same value of *ka* or operate at (or very near) the same frequency. Here, the design objective is to operate all of the antennas as close to 300 MHz as possible.

The folded spherical helix, its impedance, matched VSWR ($Z_{CH}$ = 57.8Ω) and radiation patterns are presented in Fig. 1. Numerical results are obtained using the NEC4 engine of EZNEC/4 Pro [26]. Copper conductor loss is included in all of the numerical simulations. The antenna is operated as a dipole (no ground plane), has a single feed point and has an overall height and diameter of 8.36 cm. It has a conductor diameter of 2.6 mm. It is self-resonant at a frequency of 299.8 MHz.

The *Q* of the folded spherical helix, calculated using Eq. (2), is 84.78, which is approximately 1.5 times the lower bound of 57.39. Its radiation efficiency is 97.4%. Its ½-power and 2:1 VSWR bandwidths are 2.37% and 0.87%, respectively. The radiation pattern illustrates that the folded spherical helix primarily operates as an electric dipole with minimal cross-polarization.

### 3.2 The Folded Cylindrical Helix

The 4-arm folded cylindrical helix is designed based on the same principles as the folded spherical helix, with the exception that the conductors are wound on the outside volume of an imaginary cylinder rather than a sphere. In this instance, the cylindrical helix has the same overall height and diameter as the spherical helix, 8.36 cm. The conductor diameter is also the same, 2.6 mm. To maintain nearly the same operating frequency as the folded spherical helix, the total conductor length in each arm had to be adjusted. Since the cylindrical helix occupies a greater physical volume than the spherical helix, it is expected to have greater bandwidth and lower *Q*. However, since it occupies less of the spherical volume defined by the value of *ka*, its *Q* will not approach the lower bound as closely as that of the spherical helix. The cylindrical folded helix is self-resonant at 301.1 MHz, with a value of $ka = 0.373$.

The folded cylindrical helix, its impedance, matched VSWR ($Z_{CH}$ = 83.6Ω) and radiation patterns are presented in Fig. 2. Its resonant resistance is greater than that of the folded cylindrical helix and it exhibits a VSWR less than 2.0 in a 50Ω system. The *Q* of the folded spherical helix is 53.5, which is approximately 2.5 times the lower bound of 21.35. Its radiation efficiency is 98.3%. Its ½-power and 2:1 VSWR bandwidths are 3.75% and 1.33%, respectively, both greater than those of the spherical helix. Similar to the folded spherical helix, the folded cylindrical helix primarily operates as an electric dipole with minimal cross-polarization.

### 3.3 The Disk-Loaded Dipole

Top, capacitive or disk-loading a straight-wire dipole or monopole antenna is a well known technique for size reduction [20]. A wire-grid and solid disk version of a disk-loaded dipole are depicted in Fig. 3. The wire grid version is modeled using EZNEC while the solid disk version is modeled using Microwave Studio [27]. Each of these antennas has the same overall height and diameter as the folded spherical helix, 8.36 cm. These antennas have the same value of $ka$ as the folded cylindrical helix.

The impedances of these antennas are presented in Fig. 4. As expected, the impedances of these antennas are similar. The solid top-disk version of the antenna has less capacitive reactance since there is more capacitance between the upper and lower disks. Without tuning, neither is resonant near 300 MHz. The wire-grid top-hat version of the antenna is considered further since it can be easily modeled using EZNEC and the results can be more directly compared to the antennas constructed using only wires.

To tune the disk-loaded dipole for self-resonance near 300 MHz, a helical coil dipole is placed between the upper and lower plates as illustrated in Fig. 5(a). To then impedance match the antenna to 50Ω, a shunt or parallel stub is placed across the feed point as illustrated in Fig. 5(b).

The impedance, matched VSWR ($Z_{CH}$ = 50Ω) and radiation patterns of the matched disk-loaded dipole are presented in Fig. 6. The antenna is matched at a frequency of 294.4 MHz, where $ka$ = 0.365. The $Q$ of the disk loaded dipole is 48.0, which is approximately 2.14 times the lower bound of 22.27. Its radiation efficiency is 95.5%. Its ½-power and 2:1 VSWR bandwidths are 3.04% and 1.44%, respectively, both greater than those of the spherical helix, but slightly less than those of the cylindrical helix. The matched disk-loaded dipole operates as an electric dipole with minimal cross-polarization.

### 3.4 The Spherical-Cap Dipole

The spherical-cap dipole is also a top-loaded antenna similar to the disk-loaded dipole. With this design, the top-loading structure maintains a spherical shape so that the entire antenna structure fits within the spherical volume defined by the value of $ka$. In [21], Wheeler described the spherical-cap dipole in these terms: "*the best simple utilization of a spherical volume is achieved by spherical caps covering about one-half the surface, tuned by a distributed inductor.*" The distributed inductor Wheeler refers to is the helical coil dipole that was used to tune the disk-loaded dipole as described in the previous section. In [22], Lopez modeled an untuned spherical cap dipole and achieved a $Q$ that was within 1.75 times the lower bound. Lopez concluded that this was the lower achievable bound for electric antennas. We see that the folded spherical helix more closely approaches the lower bound, being within approximately 1.5 times its value. A theoretical design approaching within 1.5 times the lower bound was also presented by Stuart and Pidwerbetsky in [7]. As discussed by Thal in [28], the lower achievable limit for electric dipoles is 1.5 times the lower bound.

Here, a wire grid version of the spherical-cap dipole, having an overall diameter of 8.36 cm is considered. The matched spherical-cap dipole, its impedance, matched VSWR ($Z_{CH}$ = 50Ω) and radiation patterns are presented in Fig. 7. The antenna is matched at a frequency of 293.6 MHz, where $ka$ = 0.26. The $Q$ of the spherical-cap dipole is 95.29, which is approximately 1.68 times the lower bound of 56.86. Its radiation efficiency is 93.3%. Its ½-power and 2:1 VSWR bandwidths are 2.06% and 0.74%, respectively, both slightly less than those of the spherical helix. The spherical-cap dipole operates as an electric dipole with minimal cross-polarization.

### 3.5 The Spherical Resonator

The spherical resonator antenna was introduced by Stuart and Tran in [8] and [9]. It is comprised of a single-fed wire structure electromagnetically coupled to closely spaced identical wires as illustrated in Fig. 8, which depicts a 6-arm version of the antenna. At a frequency where strong coupling modes exist between the feed-arm and the parasitic arms, a tight (narrow band) coupling loop develops in the impedance of the antenna as illustrated in Fig. 9. In the region of this tight impedance loop, the frequency derivatives of the antenna's resistance and reactance are such that an accurate approximation of the antenna's exact $Q$ may not hold and the value of approximate $Q$ determined using Eq. (3) may be less than the lower bound [10]. When the spherical resonator is tuned and/or matched, the existence of the tight coupling loop results in closely spaced multiple resonances within the defined VSWR bandwidth. The expected advantage of these closely spaced multiple resonances is that greater bandwidth may be achieved with this antenna than can be achieved with small antennas that exhibit single resonances.

Following the approach in [8] and [9], the spherical resonator antenna considered here was designed without dielectric and scaled so that the impedance loop was centered near a frequency of 300 MHz. The resulting implementation of the spherical resonator considered here is a 6-arm version since it exhibits a resistance of approximately 50Ω near the center of the impedance loop. To operate at near 300 MHz, without dielectric loading, the overall dipole length of the antenna is 19.89 cm, a dimension substantially larger than the antennas considered in the previous sections. Since the antenna is not inherently matched to 50Ω, some method of impedance matching must be employed. Impedance matching can be achieved through tuning using an external matching network

[8], series inductors located at the feed point, or an increase in the feed-arm length using a meander-line technique. Here, the meander-line is chosen as the impedance matching technique.

The matched 6-arm spherical resonator, its impedance, matched VSWR ($Z_{CH}$ = 50Ω) and radiation patterns are presented in Fig. 10. The antenna is matched at a frequency of 298.2 MHz, where *ka* = 0.622. The estimated approximate *Q* of the spherical resonator is 1.64, which is substantially less than the lower bound of 5.66. This is an incorrect result since no small antenna can exhibit an exact *Q* that is less than the lower bound [10]. Additionally, it was shown in [10] that the exact *Q* does not as closely approach the lower bound as does the *Q* of the folded spherical helix. The antenna's radiation efficiency is 98%. Its ½-power and 2:1 VSWR bandwidths are 14.8% and 8.5%, respectively, both substantially greater than those of the spherical helix.

The 6-arm spherical resonator primarily operates as an electric dipole but there is a notable amount of power in the cross-polarized radiation pattern. For antennas that exhibit significant levels of cross-polarization, there must be a corresponding reduction in the lower bound on *Q* [12], [23], [25] and [29]. The lower bound on *Q* is reduced in direct proportion to the ratio of power contained in orthogonal polarizations, in which case Eq. (3) becomes [29]

$$Q_{lb} = \eta_r \left(\frac{1}{1+\gamma}\right)\left(\frac{1}{(ka)^3} + \frac{1}{ka}\right) \tag{7}$$

where $\gamma$ is the ratio of power between orthogonal polarizations and $\gamma$ is chosen such that $0 \le \gamma \le 1$. For example, if orthogonal polarizations have equal power, $\gamma = 1$ and the lower bound of Eq. (7) is ½ the lower bound of Eq. (3). If one polarization has twice the power of the other, $\gamma = 0.5$ and the lower bound of Eq. (7) is $^2/_3$ the lower bound of Eq. (3).

From Fig. 10, it is evident that the vertical polarization contains substantially more power than the horizontal polarization and the lower bound for the antenna is not significantly reduced. If the orthogonal polarizations shared equal power, the calculated lower bound of 5.66 would be reduced to 2.83, a value still greater than the calculated approximate *Q* of 1.64.

**3.6 Summary of Results**

A summary of the results discussed in this section is presented in Table 1. The folded spherical helix antenna exhibits a *Q* that most closely approaches the lower bound. The spherical resonator antenna exhibits the largest bandwidth but also has the highest value of *ka*. Given the difference in the values of *ka* between the spherical resonator and the other configurations, the obvious question to ask is how does the performance of each antenna compare when they are designed to have the same value of *ka*? This is considered in the next section.

**Table 1. Performance comparison of the folded spherical helix, the folded cylindrical helix, the matched disk-loaded dipole, the matched spherical-cap dipole and the spherical resonator antennas. All are designed to operate at or very near 300 MHz.**

| Antenna | Frequency (MHz) | Overall Height (cm) | *ka* | $\eta_r$ (%) | $Q_{lb}$ (includes loss) | $Q_Z$ | ½ -Power VSWR Bandwidth (%) |
|---|---|---|---|---|---|---|---|
| **Folded Spherical Helix** | 299.8 | 8.36 | 0.263 | 97.4 | 57.39 | 84.78 | 2.37 |
| **Cylindrical Folded Helix** | 301.1 | 8.36 | 0.373 | 98.3 | 21.35 | 53.5 | 3.75 |
| **Matched Disk Loaded Dipole** | 294.4 | 8.36 | 0.365 | 95.5 | 22.27 | 48.0 | 3.04 |
| **Matched Spherical Cap Dipole** | 296.3 | 8.36 | 0.26 | 93.3 | 56.86 | 95.29 | 2.06 |
| **Matched Spherical Resonator** | 298.2 | 19.89 | 0.622 | 98.0 | 5.66 | 1.64 | 14.8 |

## 4. The Small Antennas having the Same *ka* as the Spherical Resonator

In the previous section, a number of small antenna configurations were compared, where all were designed to operate at or very near 300 MHz. With the exception of the spherical resonator, all of the antennas had the same overall dipole length and, in some cases, the same value of *ka*. The spherical resonator exhibited the greatest bandwidth but it also had the highest value of *ka*. The question considered in this section is whether its greater bandwidth is a result of the higher value of *ka* (its larger volume) or whether the design itself offers more bandwidth when compared with the other antenna configurations modified to have the same value of *ka*?

### 4.1 The Matched Disk-Loaded Dipole Antenna

The first antenna considered is the matched disk-loaded dipole. The size of the antenna was adjusted to achieve a value of *ka* as close to 0.622 as possible, matching that of the spherical resonator. In this adjustment process, the physical dimensions were also set so as to maintain an operating frequency as close to 300 MHz as possible. In this case, the overall height of the disk-loaded dipole is 18 cm and the disk diameter is 8.4

cm. The antenna is impedance matched to 50Ω using a shunt-stub. A helical coil is not necessary for tuning.

The matched disk-loaded dipole, its impedance, matched VSWR ($Z_{CH}$ = 50Ω) and radiation patterns are presented in Fig. 11. The antenna is matched at a frequency of 304.5 MHz, where $ka$ = 0.634. The $Q$ of the antenna is 11.21, which is approximately 2.05 times the lower bound of 5.48. Its radiation efficiency is 99.5%. Its ½-power and 2:1 VSWR bandwidths are 18.25% and 6.34%, respectively. Its ½-power bandwidth is greater than that of the spherical resonator but its 2:1 VSWR bandwidth is less. The matched disk-loaded dipole operates as an electric dipole with minimal cross-polarization.

### 4.2 The Matched Spherical-Cap Dipole Antenna

As with the matched disk-loaded dipole, the size of the spherical-cap antenna was adjusted to achieve a value of $ka$ as close to 0.622 as possible and an operating frequency as close to 300 MHz as possible. In this case, the overall height of the spherical-cap dipole is 19.89 cm. The antenna is impedance matched to 50Ω using a shunt-stub. A helical coil is not necessary for tuning.

The matched spherical-cap dipole, its impedance, matched VSWR ($Z_{CH}$ = 50Ω) and radiation patterns are presented in Fig. 12. The antenna is matched at a frequency of 290.3 MHz, where $ka$ = 0.605. The $Q$ of the antenna is 11.92, which is approximately 2.02 times the lower bound of 5.89. Its radiation efficiency is 95.5%. Its ½-power and 2:1 VSWR bandwidths are 16.36% and 5.65%, respectively. Its ½-power bandwidth is greater than that of the spherical resonator but its 2:1 VSWR bandwidth is less. The matched spherical-cap dipole operates as an electric dipole with minimal cross-polarization.

### 4.3 The Folded Spherical Helix Antenna

Again, the size of the folded spherical helix antenna was adjusted to achieve a value of $ka$ as close to 0.622 as possible and an operating frequency as close to 300 MHz as possible. The overall height of the folded spherical helix 19.89 cm. The significant design variables with the folded spherical helix are the number of arms, which determines the resonant resistance, and the length of each arm, which determines the operating frequency. As the number of arms increases, and the arm length is adjusted to maintain the same operating frequency, the resonant resistance increases. The optimum configuration for the folded spherical helix in terms of impedance matching to 50Ω is a 2-arm spherical helix. An increase in the number of arms decreases the antenna $Q$ and increases the operating bandwidth. However, the resonant resistance increases and the antenna is no longer matched to 50Ω.

The 2-arm folded spherical helix, its impedance, matched VSWR ($Z_{CH}$ = 50Ω) and radiation patterns are presented in Fig. 13. The antenna is matched at a frequency of 298.2 MHz, where *ka* = 0.621. The *Q* of the antenna is 18.9, which is approximately 3.28 times the lower bound of 5.76. Its radiation efficiency is 99.3%. Its ½-power and 2:1 VSWR bandwidths are 12.43% and 4.37%, respectively. Its ½-power and 2:1 VSWR bandwidths are less than those of the spherical resonator. The folded spherical helix operates as an electric dipole, however, there a higher level of cross-polarization than with the 4-arm configuration designed to operate with $ka \approx 0.26$.

The other folded spherical helix configurations considered were the 4, 5 and 6-arm configurations. These offered a substantial decrease in *Q* and increase in bandwidth but the antennas are no longer well matched to 50Ω. A summary of their performance properties is presented in Table 2.

**Table 2. Summary of the performance properties of the multi-arm folded spherical helix antennas compared to the spherical resonator antenna.**

| Antenna | Frequency (MHz) | *ka* | Matching Impedance (Ω) | $Q_{lb}$[1] (includes loss) | $Q_Z$ | ½ -Power VSWR Bandwidth (%) | 2:1 VSWR Bandwidth (%) |
|---|---|---|---|---|---|---|---|
| **Matched Spherical Resonator** | 298.2 | 0.622 | 50 | 5.66 | 1.64 | 14.8 | 8.5 |
| **4-Arm Folded Spherical Helix** | 298.7 | 0.622 | 243.17 | 5.78 | 10.1 | 20.05 | 7.03 |
| **5-Arm Folded Spherical Helix** | 312 | 0.65 | 312.2 | 5.18 | 8.21 | 22.15 | 7.91 |
| **6-Arm Folded Spherical Helix** | 312 | 0.65 | 312 | 5.16 | 6.81 | 27.2 | 10.3 |

1. The stated lower bound includes the term for radiation efficiency but does not include the term that takes into account any power division between the vertical and horizontal polarizations. These configurations have some power delivered to the orthogonal polarization and the lower bound is slightly overstated.

## 4.4 Summary of Results

A summary and comparison of the results discussed in this section is presented in Table 3. The matched disk-loaded dipole exhibits the largest ½-power bandwidth. It also has the largest value of *ka*. The spherical resonator exhibits the largest 2:1 VSWR bandwidth.

**Table 3. Performance comparison of the the matched disk-loaded dipole, the matched spherical-cap dipole, the folded spherical helix and the spherical resonator antennas. All are designed to operate at or very near 300 MHz and have nearly equal values of *ka*.**

| Antenna | Frequency (MHz) | *ka* | $\eta_r$ (%) | $Q_{lb}$ (includes loss) | $Q_Z$ | ½ -Power VSWR Bandwidth (%) | 2:1 VSWR Bandwidth (%) |
|---|---|---|---|---|---|---|---|
| **Matched Spherical Resonator** | 298.2 | 0.622 | 98.0 | 5.66 | 1.64 | 14.8 | 8.5 |
| **Matched Disk Loaded Dipole** | 304.5 | 0.634 | 99.5 | 5.48 | 11.21 | 18.25 | 6.34 |
| **Matched Spherical Cap Dipole** | 290.3 | 0.605 | 95.5 | 5.89 | 11.92 | 16.36 | 5.65 |
| **2-Arm Folded Spherical Helix** | 298.2 | 0.621 | 99.3 | 5.76 | 18.9 | 12.43 | 4.37 |

## 5. The Small Antennas having a *ka* less than 0.5

In the previous section, several of the small antennas were compared having nearly the same value of *ka*. The values of *ka* ranged between 0.605 and 0.634, all above the electrically small antenna limit of $ka \leq 0.5$. Here, versions of each antenna were designed to operate near 300 MHz with values of $ka \approx 0.45$. The matched disk-loaded dipole, matched spherical-cap dipole and folded spherical helix were designed and similar in configuration to those described in the previous section. Their impedance, matched VSWR bandwidths and patterns are not presented here in figures.

To achieve a value of $ka \approx 0.45$ with the spherical resonator, an 8-arm configuration loaded with dielectric is necessary [8]. The dielectric material is Rogers RO4003. This configuration is depicted in Fig. 14(a) and was modeled using Microwave Studio. Its impedance and matched VSWR are presented in Fig. 14(b) and 14(c), respectively. In this case, the matched VSWR is determined by simply tuning the antenna with an external tuning inductor that is assumed to be lossless. A comparison of the performance of these antennas is presented in Table 4. In this particular case, the matched spherical-cap dipole exhibits the greatest bandwidth.

**Table 4. Performance comparison of the the matched disk-loaded dipole, the matched spherical-cap dipole, the folded spherical helix and the spherical resonator antennas. All are designed to operate at or very near 300 MHz and have values of** $ka \approx 0.45$**.**

| Antenna | Frequency (MHz) | $ka$ | $\eta_r$ (%) | $Q_{lb}$ (Lossless Antenna) | $Q_Z$ | ½ -Power VSWR Bandwidth (%) | 2:1 VSWR Bandwidth (%) |
|---|---|---|---|---|---|---|---|
| **8-Arm Matched Spherical Resonator** | 297.5 | 0.424 | 98.0 | 15.5 | 31.1 | 6.1 | 2.1 |
| **Matched Disk Loaded Dipole** | 300 | 0.45 | 99.5 | 13.2 | 26.2 | 7.4 | 2.6 |
| **Matched Spherical Cap Dipole** | 300 | 0.45 | 95.5 | 13.2 | 19.5 | 10 | 3.52 |
| **3-Arm Folded Spherical Helix** | 297.1 | 0.446 | 99.3 | 13.5 | 24.6 | 6.7 | 2.2 |

## 6. Discussion

The radiation properties of several fundamental electrically small antennas were compared where each had the same overall length, the same value of $ka$ or where they were designed to operate at the same frequency. For the small value of $ka \approx 0.26$, the folded spherical helix exhibits a $Q$ that most closely approaches the lower bound. It also offers the largest bandwidth at this value of $ka$.

The spherical resonator antenna was of interest because when tuned or matched, it exhibits multiple resonances within its defined VSWR bandwidth, which may possibly lead to achieving greater bandwidth than can be achieved with single resonance antennas. For nearly the same value of $ka$, the matched spherical-cap dipole exhibits the largest ½-power bandwidth but a slightly lower 2:1 VSWR bandwidth than the spherical resonator. No attempt was made here to optimize the bandwidth of any of these configurations. At a value of $ka \approx 0.45$, the matched spherical-cap dipole offers the largest bandwidth.

With both the folded spherical helix and the spherical resonator antennas, designing for an impedance match ($Z_{CH} = 50\Omega$) at a specific value of $ka$ can be a challenge. The design approach requires adjust of the number of arms, the arm length, and in some cases, may require dielectric loading. For example, with the folded spherical helix at value of $ka \approx 0.625$, a lower $Q$ and larger bandwidth than the other configurations can be achieved but the antennas are not well matched to $50\Omega$. Optimization of these designs is a topic of current research.

While the *Q* of the spherical-cap dipole may not closely approach the lower bound as desired at certain values of *ka*, it often may exhibit larger bandwidth than the other configurations. The design of the spherical-cap dipole is also quite straight-forward in that the resonant frequency can be adjusted using a helical coil dipole feed or an adjustment of the volume of the spherical-cap. Matching can be easily accomplished using a shunt stub, provided the resonant resistance is sufficiently lower than 50Ω.

In all cases, the specific value of *ka* may dictate the most suitable design approach from a physical perspective and it may also determine which design approach will provide the lowest *Q* and largest bandwidth.

## 7. References


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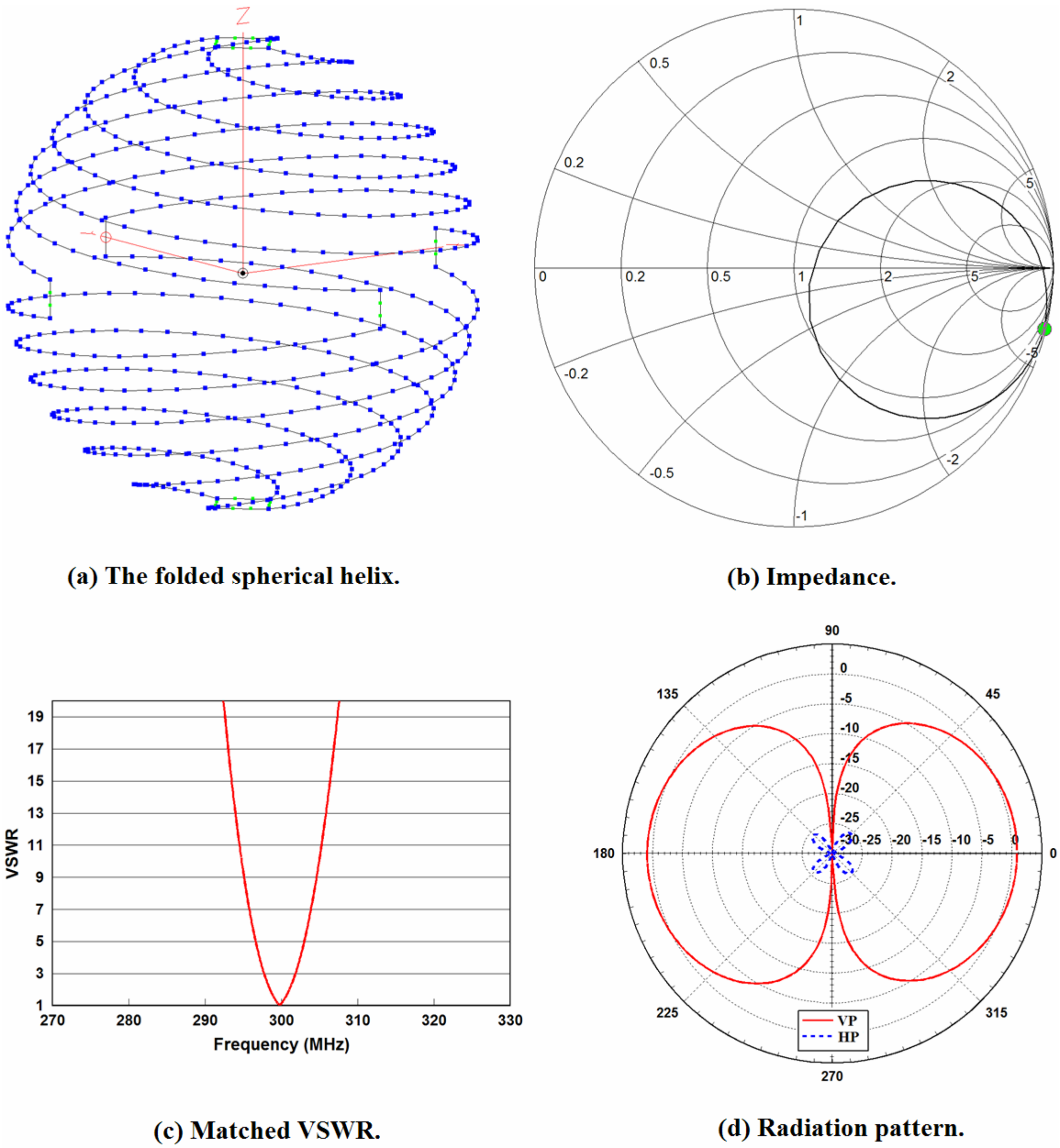


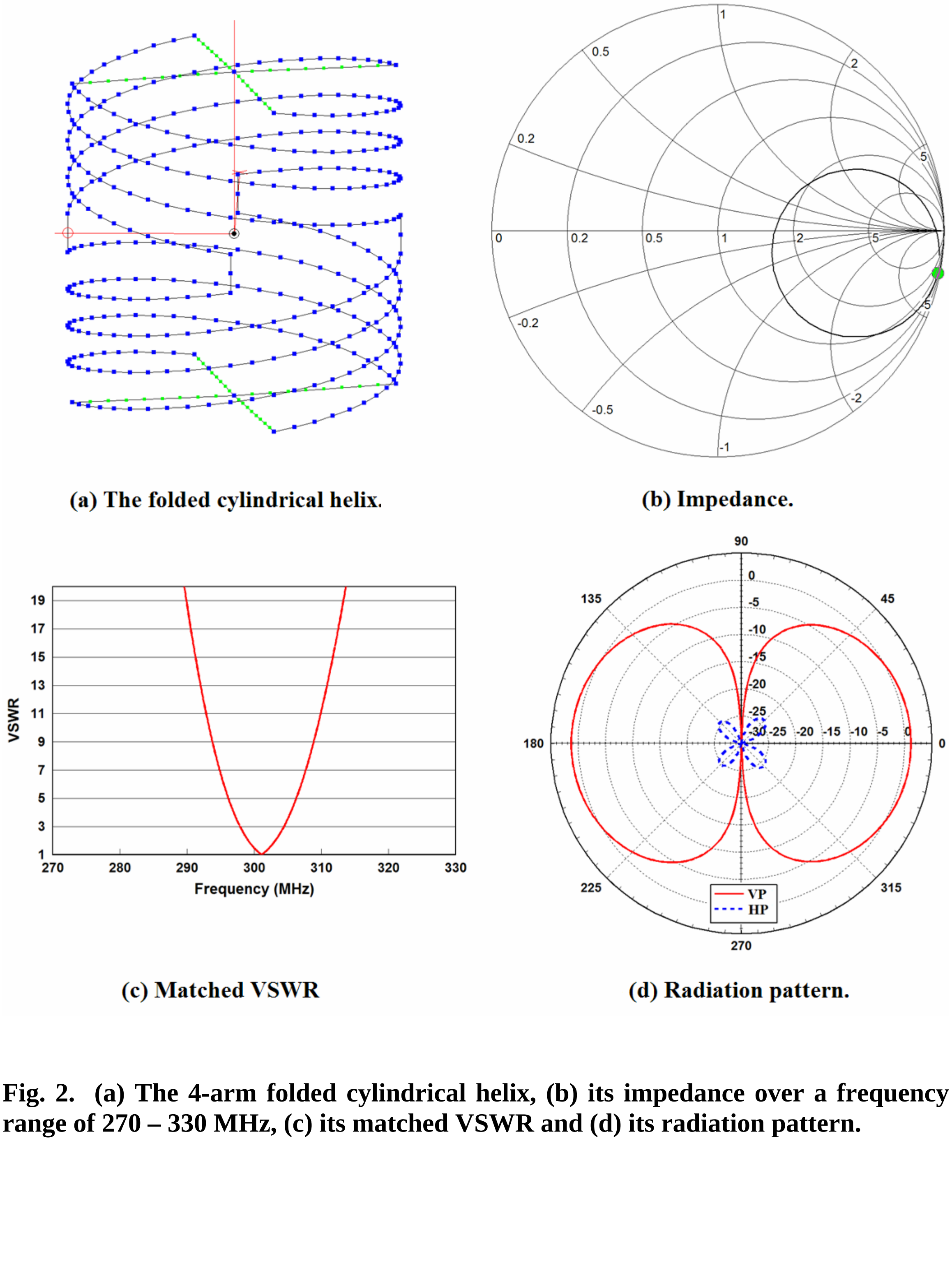

(a) The folded spherical helix.

(b) Impedance.

(c) Matched VSWR.

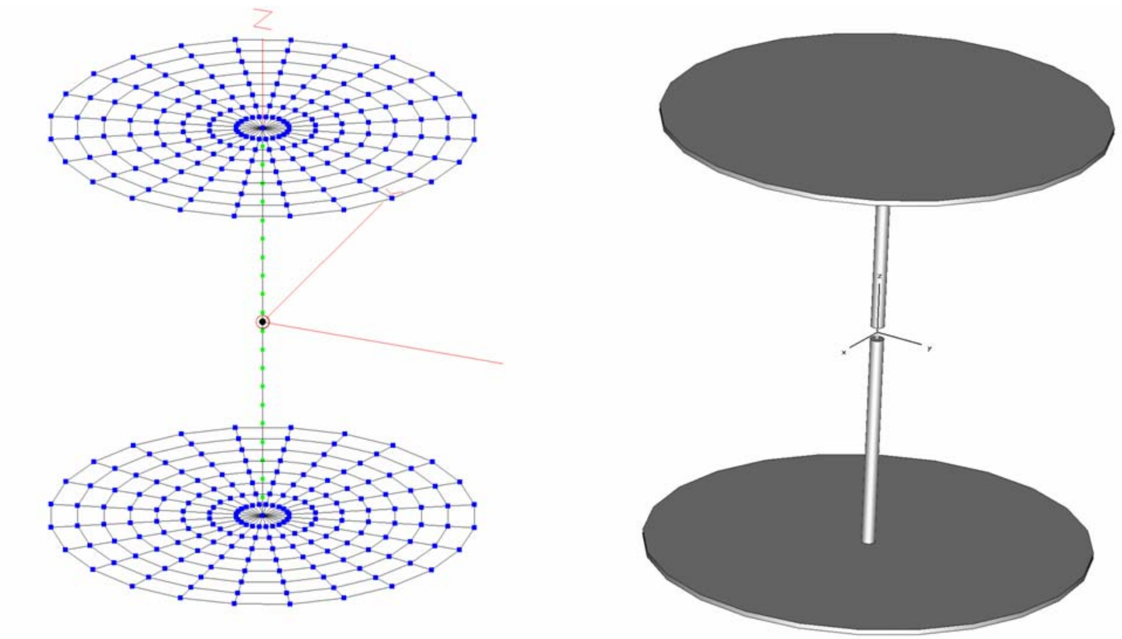
(d) Radiation pattern.

**Fig. 1. (a) The 4-arm folded spherical helix, (b) its impedance over a frequency range of 270 – 330 MHz, (c) its matched VSWR and (d) its radiation pattern.**




(a) The folded cylindrical helix. (b) Impedance. (c) Matched VSWR (d) Radiation pattern.

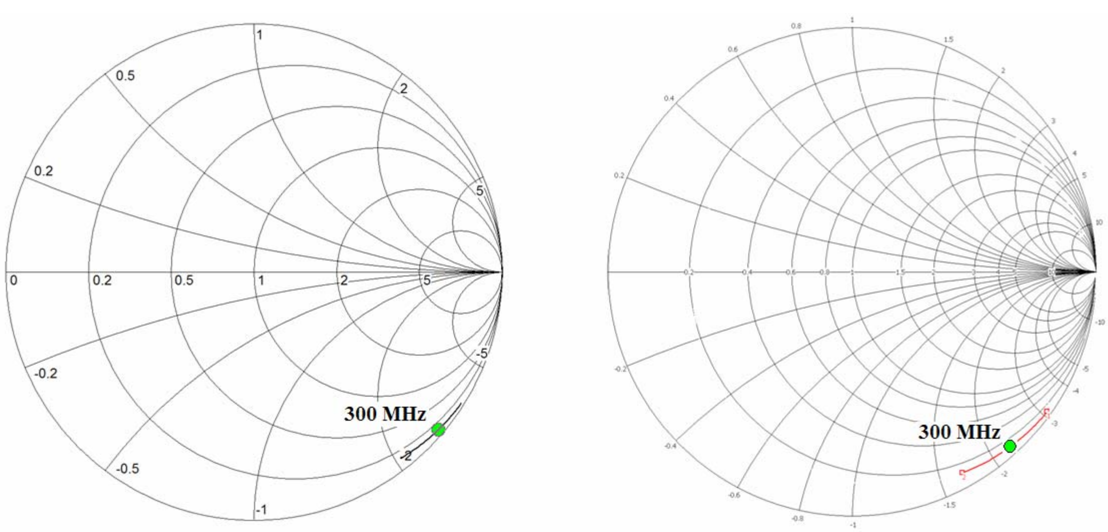
**Fig. 2. (a) The 4-arm folded cylindrical helix, (b) its impedance over a frequency range of 270 – 330 MHz, (c) its matched VSWR and (d) its radiation pattern.**



(a) The wire-gird disk-loaded dipole.

(b) The solid disk-load dipole.

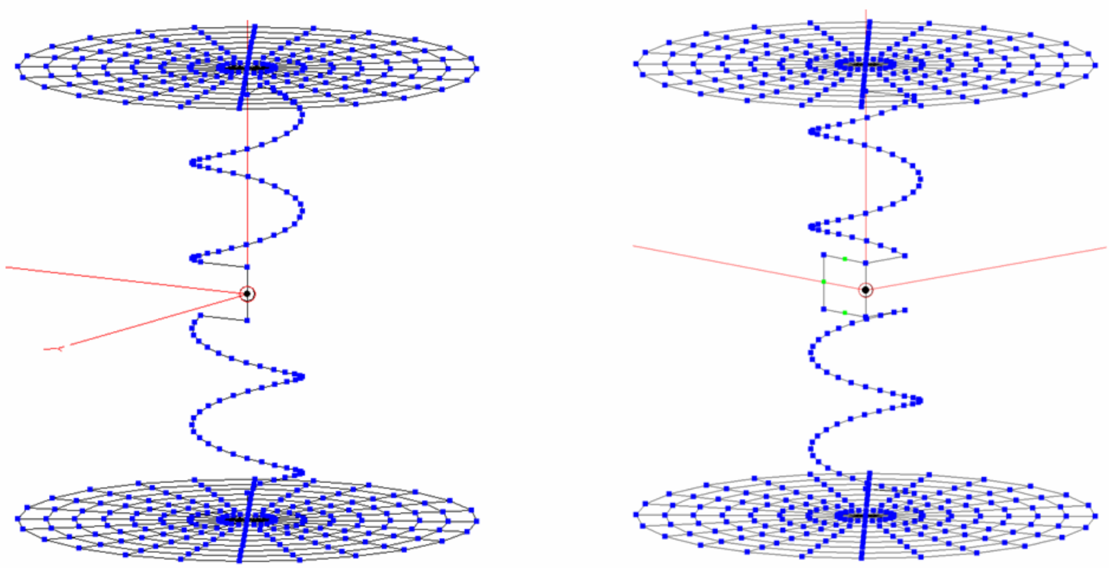
**Fig. 3. The wire-grid and solid disk-loaded loaded dipole antennas.**




(a) Impedance of the wire-grid disk loaded dipole.

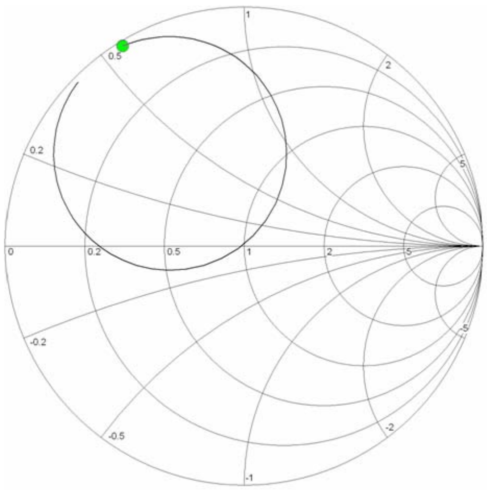
(b) Impedance of the solid disk-loaded dipole

**Fig. 4. Impedances of the wire-grid and solid disk-loaded loaded dipole antennas.**

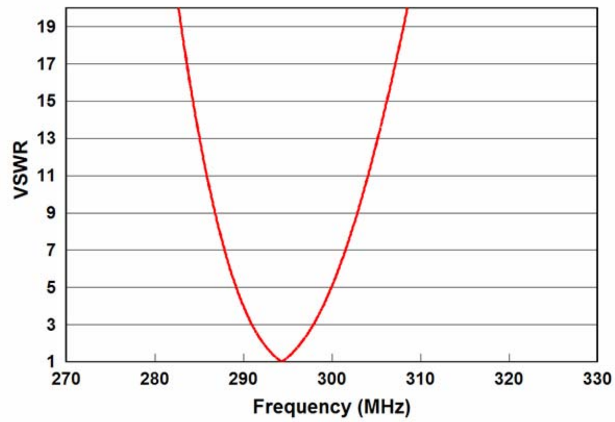


(a) Tuned disk-loaded dipole. (b) Matched disk-loaded dipole.

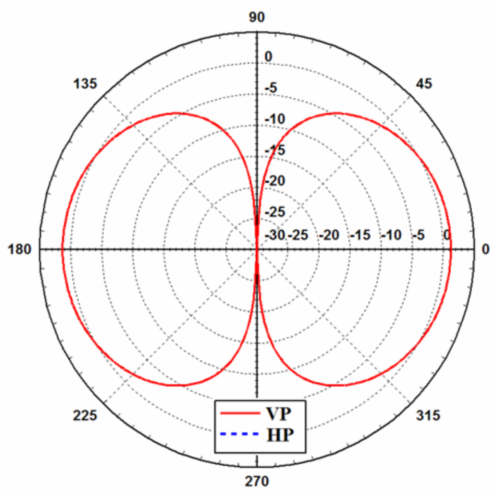
**Fig. 5. (a) The helical-coil tuned disk-loaded dipole. (b) The shunt stub matched disk-loaded dipole.**




(a) Impedance

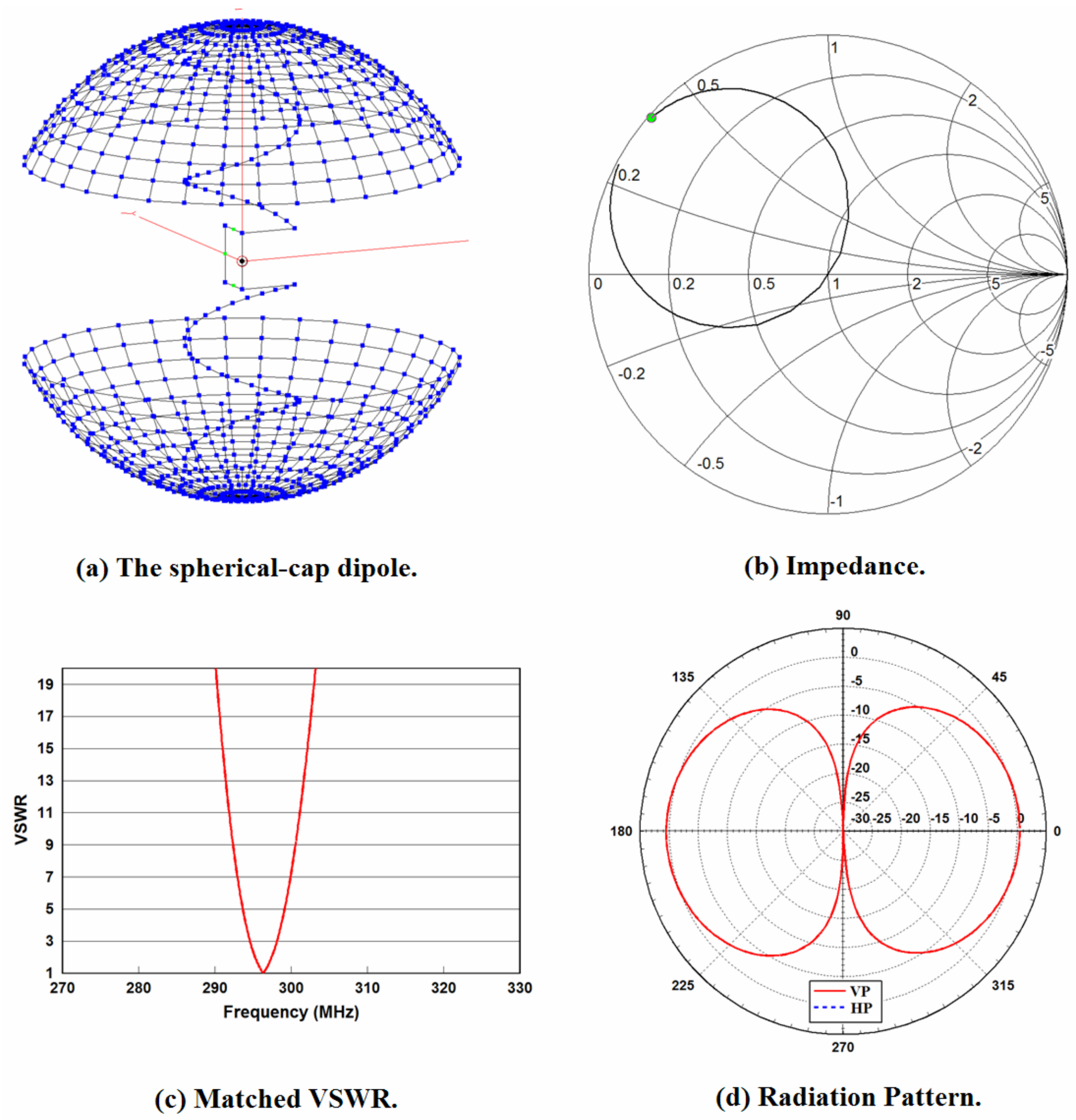



(b) Matched VSWR




(c) Radiation pattern.

**Fig. 6. Radiation properties of the matched disk-loaded dipole: (a) Impedance over a frequency range of 270 – 330 MHz, (b) Matched VSWR and (c) Radiation pattern.**




(a) The spherical-cap dipole.

(b) Impedance.

(c) Matched VSWR.

(d) Radiation Pattern.

**Fig. 7. (a) The spherical-cap dipole, (b) its impedance over a frequency range of 270 – 330 MHz, (c) its matched VSWR and (d) its radiation pattern.**

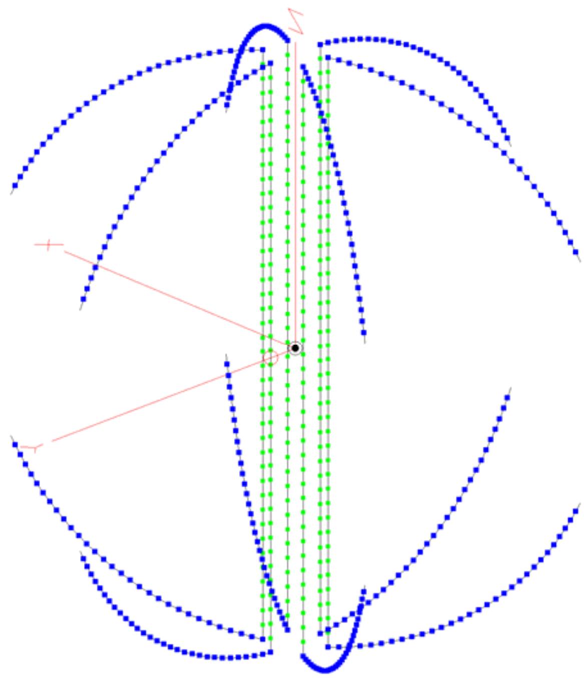


**Fig. 8. The 6-arm spherical resonator antenna.**

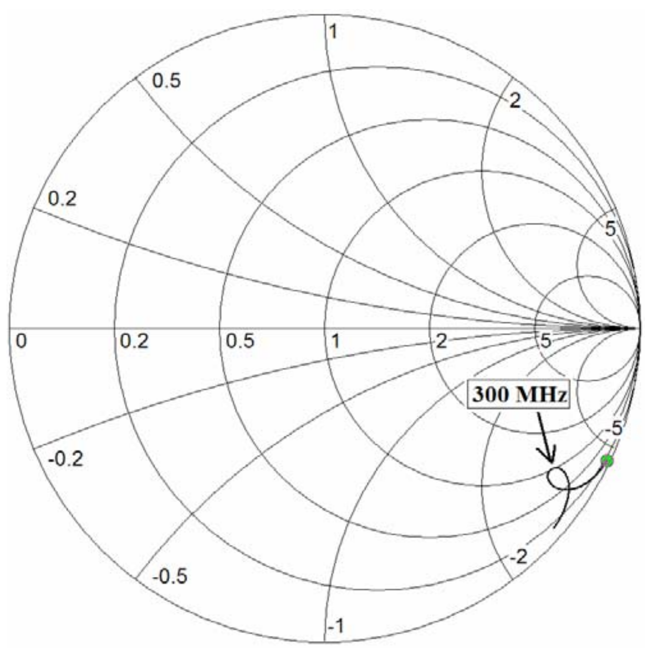
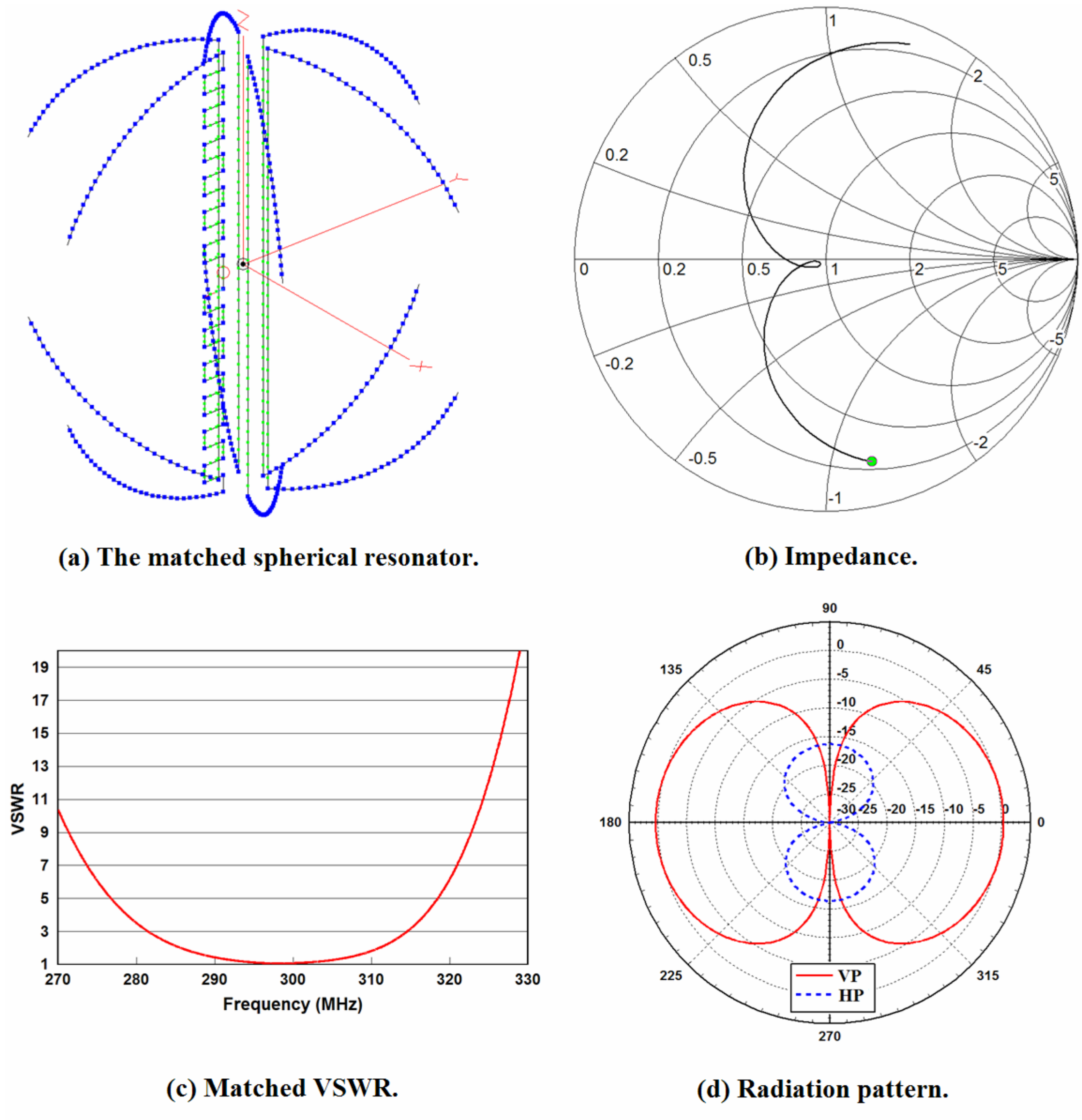



**Fig. 9. Impedance of the 6-arm spherical resonator antenna.**



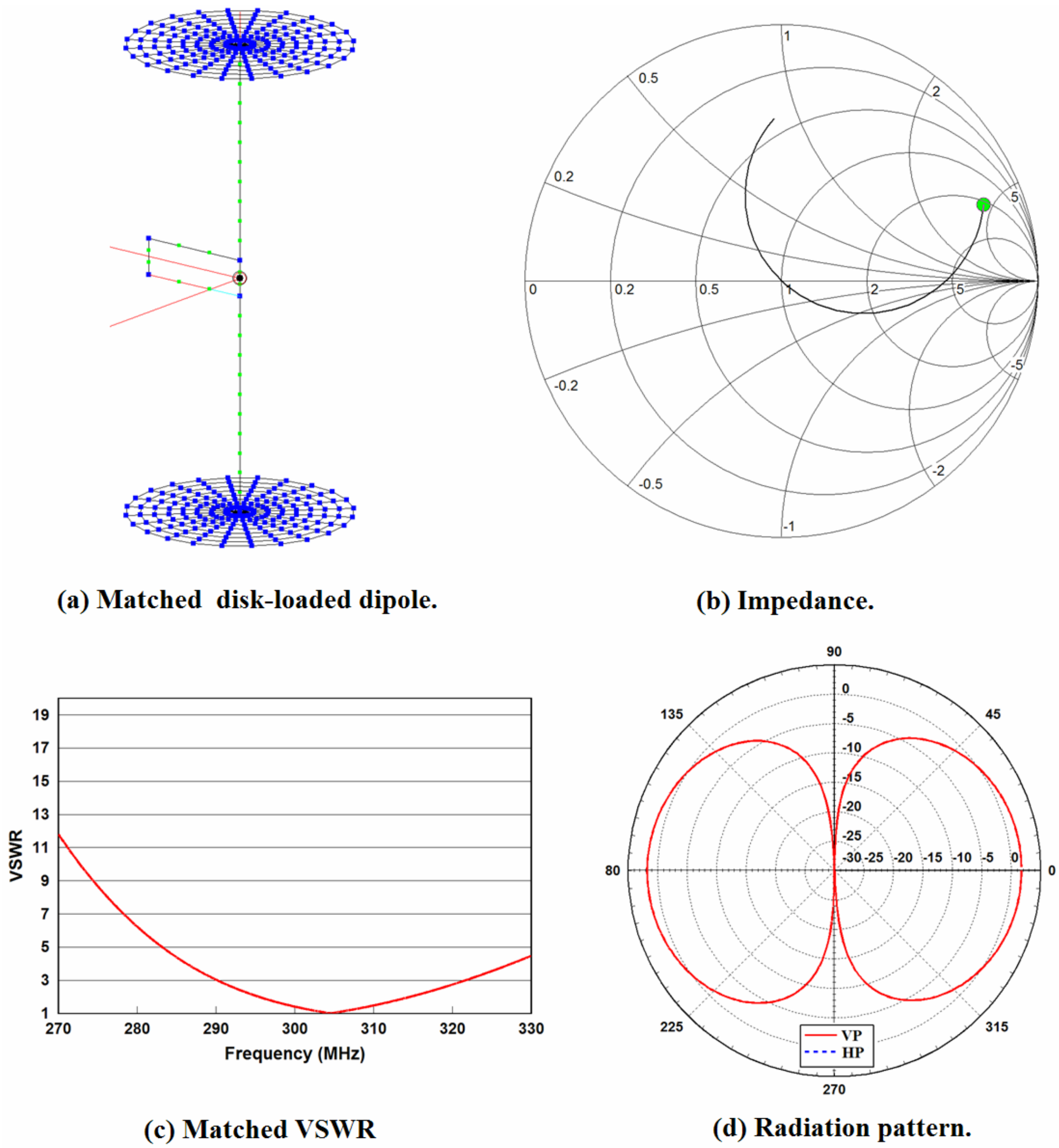

(a) The matched spherical resonator.

(b) Impedance.

(c) Matched VSWR.

(d) Radiation pattern.

**Fig. 10. (a) The matched spherical resonator antenna, (b) its impedance over a frequency range of 270 – 330 MHz, (c) its matched VSWR and (d) its radiation pattern.**



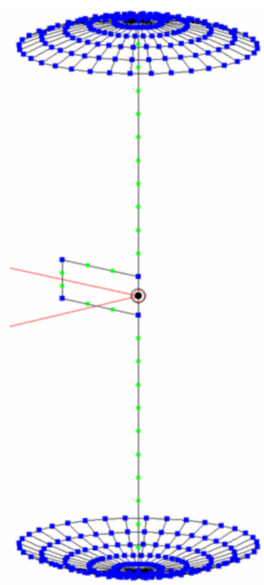

(a) Matched disk-loaded dipole.

(b) Impedance.

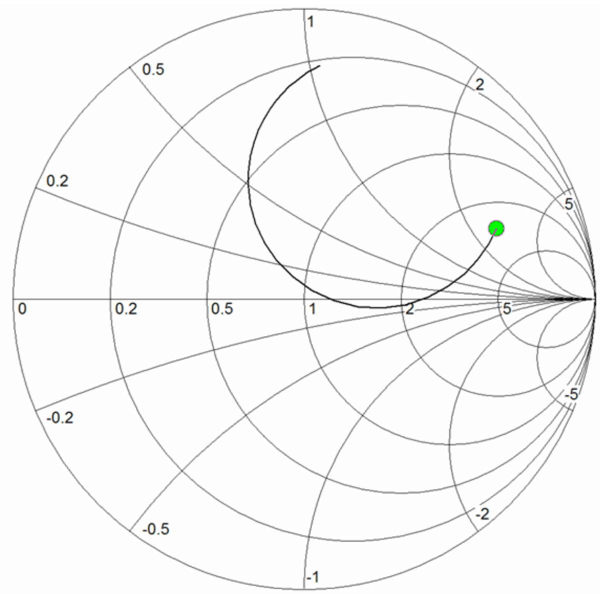
(c) Matched VSWR

(d) Radiation pattern.

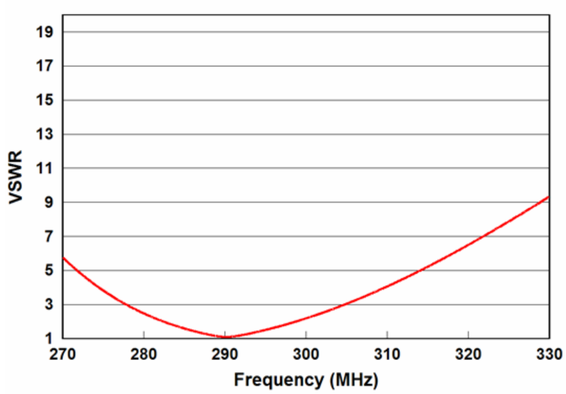
**Fig. 11. (a) The matched disk-load dipole having a *ka* = 0.634, (b) its impedance over a frequency range of 270 – 330 MHz, (c) its matched VSWR and (d) its radiation pattern.**



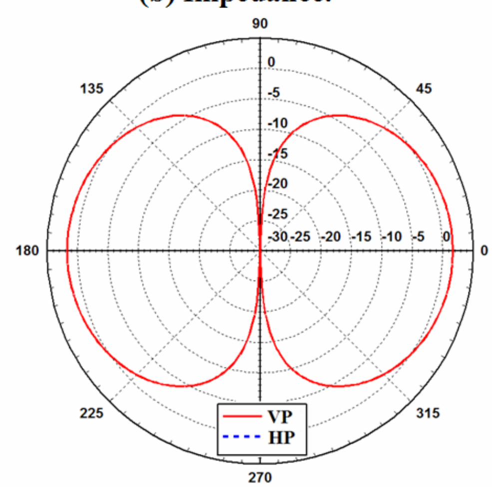
(a) Matched spherical-cap dipole.




(b) Impedance.

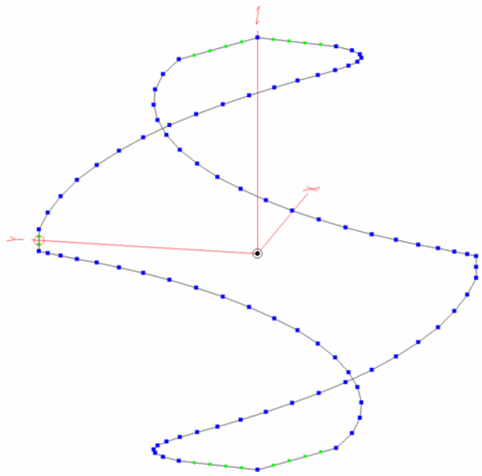



(c) Matched VSWR.

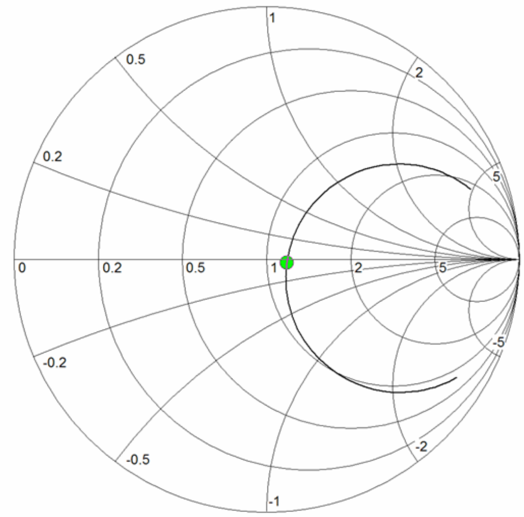



(d) Radiation pattern.

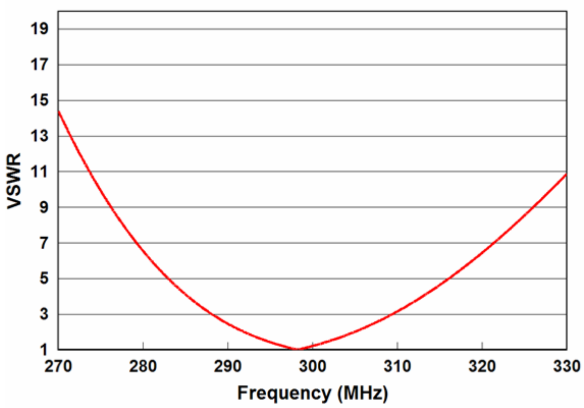
**Fig. 12. (a) The matched spherical-cap dipole having a *ka* = 0.605, (b) its impedance over a frequency range of 270 – 330 MHz, (c) its matched VSWR and (d) its radiation pattern.**



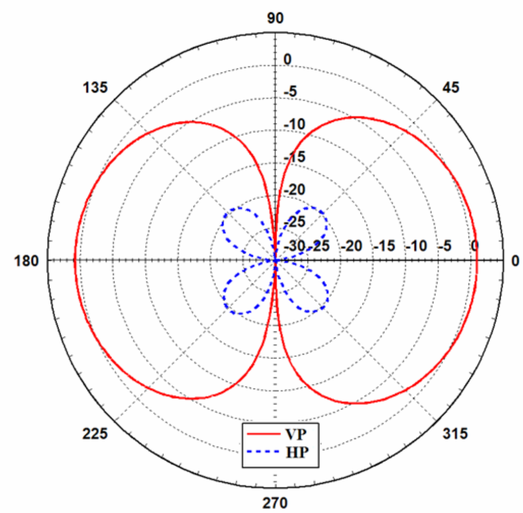
(a) 2-arm folded spherical helix.




(b) Impedance.

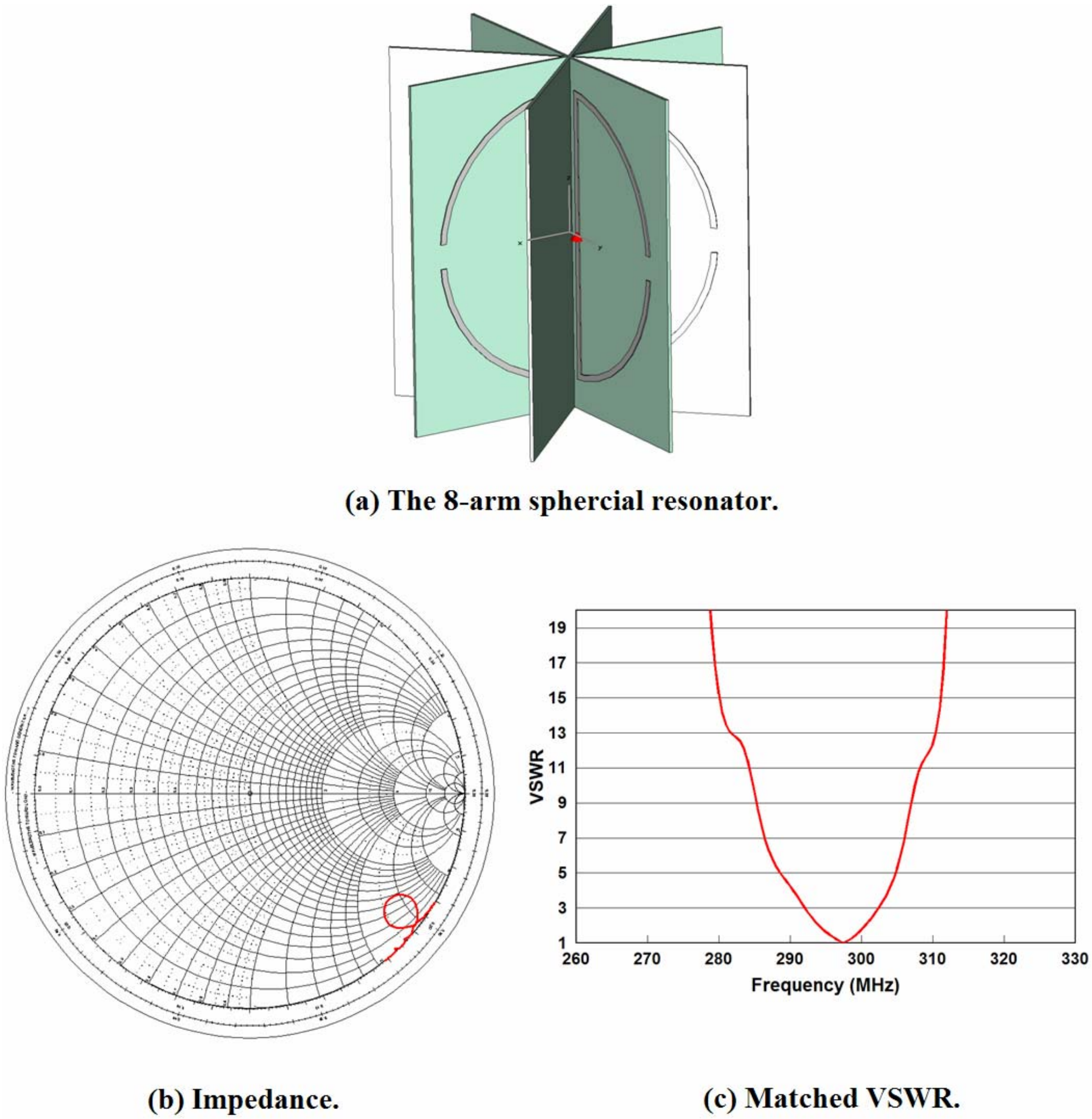



(c) Matched VSWR.




(d) Radiation Pattern

**Fig. 13. (a) The 2-arm folded spherical helix antenna having a *ka* = 0.621, (b) its impedance over a frequency range of 270 – 330 MHz, (c) its matched VSWR and (d) its radiation pattern.**




(a) The 8-arm sphercial resonator.

(b) Impedance.

(c) Matched VSWR.

**Fig. 14. (a) The 8-arm, dielectric loaded spherical resonator antenna having a *ka* = 0.428, (b) its impedance over a frequency range of 270 – 330 MHz, and (c) its matched VSWR.**