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# **ADS-B Surveillance Separation Error Sensitivity Analysis**

**Second Edition** 

Stanley R. Jones

August 2009



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**Second Edition** 

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August 2009

## Abstract

Two efforts are currently examining Automatic Dependent Surveillance-Broadcast (ADS-B) separation requirements: a Federal Aviation Administration (FAA) group, titled the Separation Standards Working Group (SSWG); and a joint RTCA/EUROCAE group termed the Requirements Focus Group (RFG). Although coordinated efforts, the SSWG emphasis is on particular issues of concern to the FAA. The different assumptions made by each of these groups are examined, and results are presented in relationship to the different Monopulse Secondary Surveillance Radar (MSSR) reference models used in each comparative assessment. Sensitivity of resulting ADS-B separation requirements to the assumptions and scenarios used in determining these requirements is quantified for both Global Positioning System (GPS) no-fault and fault conditions.

## **Executive Summary**

A modification to an International Civil Aviation Organization (ICAO) accepted collision risk model was initially used in a comparison to legacy radar surveillance to define requirements for Automatic Dependent Surveillance-Broadcast (ADS-B) support of Air Traffic Control (ATC) separation standards [1]. This model, termed Close Approach Probability (CAP) is the conditional probability that an aircraft pair actually over lap when the apparent distance between them is some given separation. Subsequent experience with this model indicated that a more flexible approach based on the normalized surveillance separation error probability isolated the surveillance requirements from other operational scenario risk factors treated by the usual fault tree risk assessment. When the ADS-B navigation source is in either a fault-free or fault condition, this is termed the Separation Error Probability (SEP) in this document. A weighted combination of these two conditions, termed the Separation Reduction Probability (SRP), is used here to indicate the long term likelihood that a controller will have to deal with a degraded surveillance related problem. Other documents use the term, Surveillance Separation Error (SSE) to refer to both these conditions [2]. The SSE (as well as the equivalent SEP) is defined as the probability that the separation error (apparent separation minus true separation) for two aircraft towards each other is greater than a given value [2].

Two efforts are currently examining ADS-B separation requirements: a Federal Aviation Administration (FAA) group, titled the Separation Standards Working Group (SSWG); and a joint RTCA/EUROCAE group termed the Requirements Focus Group (RFG) [2]. Although coordinated efforts, the SSWG emphasis is on particular issues of concern to the FAA. The following analyses recognize some of the different assumptions made by each of these groups and presents results in relationship to the different Monopulse Secondary Surveillance Radar (MSSR) reference models used in each comparative assessment. These results are given for the RFG selected MSSR reference ranges of 33 Nautical Miles (NM) for the terminal area, and 60 NM for the en route area. The RFG selected the 33 NM reference range on the basis that the assumed along-range MSSR error equals the cross-range error at this range. Since nominally half the terminal area traffic is typically within this range, and since SSWG simulated scenarios used both shorter and longer reference ranges, 33 NM is also used for the SSWG reference here. The en route 60 NM reference is considered a typical range in a multi-radar or mosaic environment.

Terminal area requirements for ADS-B to ADS-B separation, including latency and other time registration issues, and for ADS-B to MSSR separation, including registration and latency issues, have been examined for the RFG reference range of 33 NM. An ADS-B Navigation Accuracy Category for position (NACp) of 7 (i.e., estimated position uncertainty [EPU] of 0.1 NM) meets most ADS-B to ADS-B comparative accuracy needs, but NAC = 8 (i.e., EPU of 0.05 NM) is required for the uncompensated merge scenario and to off-set residual bias uncertainty effects for ADS-B to MSSR separations. A similar process defines requirements for en route separations except the reference MSSR separation error at 60 NM also includes possible residual bias errors. A NAC = 7 meets all En Route (E-R) scenarios. These requirements at a SEP = 0.05 (for both Terminal Area [TMA] and E-R) are independent of the SSWG or RFG reference MSSR models used, and are insensitive to reasonable assumptions made in time registration extrapolation of the ADS-B update.

Sensitivity of the 3 NM surveillance separation error to the Global Positioning System (GPS) fault condition behavior on nearby aircraft were then examined for an assumed NIC = 6 and various possible fault condition combinations. The RFG MSSR model is the primary reference assumed in this case since its representation of MSSR tail error behavior seems to better represents available measurements. Even under the reasonable worst case fault conditions, a NIC = 6 in the TMA, and a NIC = 5 in the E-R environment seem adequate to assure the comparative integrity of the position report.

# **Table of Contents**

1	Int	roduction	1-1
2	Bac	ckground	2-1
	2.1	MSSR Position Estimate Error Reference Models	2-1
	2.2	Surveillance Separation Error	2-2
3	AD	S-B to ADS-B No-Fault Surveillance Separation Error	3-1
	3.1	Terminal Area ADS-B to ADS-B No-fault SEP	3-1
	3.	1.1 For Along-Track Errors	3-4
	3.	1.2 For Cross-Track Errors	3-10
	3.2	En route Area ADS-B to ADS-B No-fault SEP	3-11
	3.2	2.1 Multi-MSSR En route Reference SEP	3-12
	3.2	2.2 En Route Area ADS-B to ADS-B No-fault SEP Examples	3-15
	3.3	Summary of ADS-B to ADS-B No-fault SEP Results	3-17
4	AD	S-B to MSSR No-Fault Surveillance Separation Error	4-1
	4.1	ADS-B to MSSR No-Fault SEP Reference Case	4-1
	4.2	ADS-B to MSSR No-Fault SEP Examples	4-3
	4.	2.1 Residual MSSR Azimuth Bias Uncertainty	4-8
	4.	2.2 Residual Time Bias Uncertainty	4-12
	4.3	Summary of ADS-B to MSSR No-Fault SEP Results	4-14
5	Fai	ult Condition SEP	5-1
	5.1	One Aircraft of a Pair in Fault Condition	5-1
	5.2	Both Aircraft of a Pair in Fault Condition	5-5
	5.3	En route Fault Condition	5-6
	5.4	ADS-B to MSSR Fault Condition Differences	5-6
	5.5	Summary of Fault Condition Results	5-6
6	Sui	mmary 6-1	
7	Lis	t of References	7-1
A	ppen	dix A Position Estimate Error Distributions and the Separation Error Probability	A-1
A	ppen	dix B MSSR Reference Sensitivity Examination	B-1
A	ppen	dix C ADS-B Fault Condition Separation Errors Compared with Two MSSR Error Models for Various References Range Assumptions	C-1

C.1 Introduction and MSSR Reference Models	C-1
C.2 ADS-B Separation Errors in GPS Fault Condition	C-5
C.3 Long Term Separation Reduction Probability	C-8
C.4 Summary of Results	C-12
Appendix D ADS-B Latency and Effects on Displayed Aircraft Separation Errors	<b>D-1</b>
D.1 Introduction	D-1
D.2 Analyses	D-1
D.2.1 Aircraft Movement During the Latency Interval	D-1
D.2.2 Effect of Asynchronous Latency on Separation	D-3
D.2.2.1 Relative Track Separation Considerations	D-3
D.3 Effect of Common Latency on Separation	D-5
D.4 Conclusions	D-7
D.5 References	D-8
Appendix E Glossary	E-1

# **List of Figures**

Figure 2-1. Cross-Range Separated MSSR pdfs, pm, Compared With Separation Error pdf, pse, Which is the Difference (or Convolution) of these Position Estimation Errors	2-3
Figure 3-1. Probability the MSSR-MSSR Cross-Range Surveillance Separation Error at R NM is Equal to or Greater Than Es, Compared With NACp = 7 and 8 No-Fault ADS-B to ADS-B Separation Error Probabilities (Without Extrapolation/Latency Effects)	3-2
Figure 3-2. Compensated Latency ADS-B to ADS-B Along-Track SEP, and Orthogonal Track SEP at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR	3-6
Figure 3-3. Uncompensated Latency ADS-B to ADS-B Along-Track SEP, and Orthogonal Track SEP at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR	3-8
Figure 3-4. NACp = 7 ADS-B to ADS-B SEP for Orthogonal Tracks (With and Without Latency Compensation) Compared With MSSR-MSSR SEP for Above Parameters	3-9
Figure 3-5. NAC 7 and NAC 8 Cross-Track Separation Error Probability at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR at R = 33 NM	3-11
Figure 3-6. Probability the MSSR-MSSR Cross-Range Surveillance Separation Error With Residual Bias Uncertainty at R = 60 NM is Equal to or Greater Than Es, Compared With NACp = 7 and 6 No-Fault ADS-B to ADS-B Separation Error Probabilities (Without Extrapolation/Latency Effects)	3-14
Figure 3-7. Compensated Latency ADS-B to ADS-B Along-Track SEP, and Orthogonal Track SEP at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR	3-15
Figure 3-8. Uncompensated Latency E-R ADS-B to ADS-B Along-track SEP, and Orthogonal Track SEP at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR	3-16
Figure 3-9. E-R Cross-Track Separation Error Probability at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR	3-17

Figure 4-1. ADS-B to MSSR SEP (With Residual Bias Error Uncertainty) Compared With ADS-B to ADS-B SEP for Same NACp = 7 Value. MSSR-MSSR SEPs for SSWG and RFG Models are also Shown for Reference	4-3
Figure 4-2. TMA NAC = 7 ADS-B to MSSR SEP at 0.05 Probability as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR (Compensated Latency)	4-5
Figure 4-3. TMA NAC = 8 ADS-B to MSSR SEP at 0.05 Probability as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR (Compensated Latency)	4-6
Figure 4-4. En Route NAC = 6 ADS-B to MSSR SEP at 0.05 Probability as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR (Compensated Latency)	4-7
Figure 4-5. En Route NAC = 7 ADS-B to MSSR SEP at 0.05 Probability as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR (Compensated Latency)	4-8
Figure 4-6. In-trail ADSB-MSSR (or Orthogonal ADSB Merge w/MSSR) SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR- MSSR and ADSB-ADSB SEPs for NACp = 7 and $\mu d = 0$	4-9
Figure 4-7. In-trail ADSB-MSSR (or Orthogonal ADSB Merge w/MSSR) SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR- MSSR and ADSB-ADSB SEPs for NACp = 8 and µd = 0.	4-10
Figure 4-8. Parallel-track ADSB-MSSR SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR-MSSR and ADSB-ADSB SEPs for NACp = 8	4-11
Figure 4-9. Orthogonal MSSR Merge w/ADSB SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR-MSSR and ADSB-ADSB SEPs for NACp = 8 and $\mu d = 0$	4-12
Figure 4-10. In-trail ADSB-MSSR (or Orthogonal ADSB Merge w/MSSR) SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR- MSSR and ADSB-ADSB SEPs for NACp = 8 and Indicated μd = 0.3	
sec Figure 4-11. Orthogonal MSSR Merge w/ADSB SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR-MSSR and ADSB-ADSB SEPs for NACp = 8 and $\mu d = 0.3$ sec	4-13 4-14
Figure 5-1. Probability the NIC = 6 Fault Condition ADS-B to ADS-B Separation Error is Greater Than or Equal to Es at Several Probabilities of Missed Detection of the Fault Bias Error Compared With the RFG MSSR- MSSR Along-Bange and Cross-Bange Surveillance Separation Error, at	
R = 33  NM	5-3

Figure 5-2. Long Term Probability the NIC = 6 SRP ADS-B to ADS-B Separation Error is Greater Than or Equal to Es Compared With the RFG MSSR- MSSR Along-Range and Cross-Range Surveillance Separation Error at R = 33  NM	5-5
Figure 5-3. Conditional Separation Error Probability vs. Separation Error for Different Combinations of NIC = 6 Fault Bias Errors on Each Aircraft at Time of Detection, Given that the Relative Geometry, Measurement Errors, and Satellite Solution Sets in Each Aircraft Produce these Independent Fault Condition Errors in Opposing Directions	5-7
Figure 5-4. Probability the Reasonable Worst Case Dependent Assumption NIC = 6 Fault Condition ADS-B to ADS-B Separation Error is Greater Than or Equal to Es at Several Probabilities of Missed Detection of the Fault Bias Error Compared With the RFG MSSR-MSSR Along-Range and Cross-Range Surveillance Separation Error at R = 33 NM (Fhe = 1.5 assumed)	5-8
Figure 5-5. Probability the NIC = 5 Fault Condition ADS-B to ADS-B Separation Error is Greater Than or Equal to Es at Several Probabilities of Missed Detection of the Fault Bias Error Compared With the MSSR-MSSR Along-range and Cross-range Surveillance Separation Error at R = 60 NM	5-9
Figure 5-6. Probability the NIC = 5 and Fhe = 1.5 SRP ADS-B to ADS-B Separation Error is Greater Than or Equal to Es at Several Probabilities of Missed Detection of the Fault Bias Error Compared With the MSSR-MSSR Along-range and Cross-range Surveillance Separation Error at R = 60 NM	5-10
Figure A-1. Cross-range Error Distributions for MSSR Relative Position Estimates on Two Targets Separated by Sa NM at Range, R NM	A-1
Figure A-2. Cross-range Separated MSSR pdfs, pm, Compared With Separation Error pdf, pse, Which is the Difference (or Convolution) of these Position Estimation Errors	A-2
Figure A-3. Derivation of the Cumulative Probability the Separation Error, $Cse \le y$ , When the Separation is Sa	A-3
Figure A-4. Order of Integration Changed in Computing the Cumulative Separation Error Probability When the Separation is Sa	A-3
Figure A-5. Normalization of the Separation Error pdf to the Separation, Sa, and Use of the Separation Error, Es, to Yield the Normalized Separation Error Probability, SEP	A-4
Figure A-6. Relationship of the Conditional CAP to the Separation Error Distribution, Given a Separation, So	A-4

Figure B-1. TMA Reference SSWG MSSR Total Cross-range 95% SEP Bounds vs. Range Compared With Cross-range and Along-range Position Only SEPs for s = 3 NM In-trail Separation	B-1
Figure B-2. Probability the En Route MSSR-MSSR Cross-range Surveillance Separation Error at R NM With Residual Bias Uncertainty Equal to Zero is Equal to or Greater Than Es, Compared With NACp = 7 and 6 No-fault ADS-B to ADS-B Separation Error Probabilities (Without Extrapolation / Latency Effects)	В-2
Figure B-3. En Route Reference SSWG MSSR Total Cross-range 95% SEP Bounds vs. Range Compared With Cross-range and Along-range Position Only SEPs for s = 5 NM In-trail Separation and Zero Relative Bias Errors (ACP = 0 and $\delta$ = 0 meters)	B-3
Figure B-4. Probability the En Route MSSR-MSSR Cross-range Surveillance Separation Error at R NM With Residual Bias Uncertainty ACP = 1 and $\delta$ = 60 Meters is Equal to or Greater Than Es, Compared With NACp = 7 and 6 No-fault ADS-B to ADS-B Separation Error Probabilities (Without Extrapolation / Latency Effects)	B-4
Figure B-5. En Route Reference SSWG MSSR Total Cross-range 95% SEP Bounds vs. Range Compared With Cross-range and Along-range Position Only SEPs for s = 5 NM In-trail Separation and Relative Bias Errors, ACP = $1 \delta = 60$ Meters	B-5
Figure B-6. Probability the En Route MSSR-MSSR Cross-range Surveillance Separation Error at R NM With Residual Bias Uncertainty ACP = 2 and $\delta$ = 120 Meters is Equal to or Greater Than Es, Compared With NACp = 7 and 6 No-fault ADS-B to ADS-B Separation Error Probabilities (Without Extrapolation / Latency Effects)	B-6
Figure B-7. En Route Reference SSWG MSSR Total Cross-range 95% SEP Bounds vs. Range Compared With Cross-range and Along-range Position Only SEPs for s = 5 NM In-trail Separation and Relative Bias Errors, ACP = $2 \delta = 120$ Meters	B-7
Figure B-8. Comparison of ADS-B NAC Values With RFG MSSR Reference Model Combined With Several Assumed Residual Azimuth Bias Uncertainty Values	B-9
Figure C-1. Comparison of RFG and SSWG Error Components at $R = 33$ NM	C-2
Figure C-2. Comparison of RFG and SSWG Along-Range and Cross-Range SSEs at R = 33 NM	C-3
Figure C-3. MSSR Cross-Range SSEs vs. RoA at SEP = $10^{-8}$ , $10^{-3}$ , and 0.05	C-4
Figure C-4. ADS-B SEP, Given a Fault With $Rc = 0.6$ NM, at pmd = 0.99, 0.5, and 0.001	C-5

Figure C-5. ADS-B Fault Condition SSEs for NIC = 6 and 7 Compared With RFG and SSWG MSSR SSEs at SEP = 0.05	C-6
Figure C-6. ADS-B Fault Condition SSEs for NIC = 6 and 7 Compared With RFG and SSWG MSSR SSEs at SEP = $10^{-3}$	C-7
Figure C-7. ADS-B Fault Condition SSEs for NIC = 6 and 7 Compared With RFG and SSWG MSSR SSEs at SEP = $10^{-8}$	C-8
Figure C-8. Long Term SRP for NIC = 6, NAC = 8 Compared With Reference MSSRs	C-9
Figure C-9. SRP = $0.05$ for NAC = 8, and NIC = 6 or 7 Compared With MSSRs	C-10
Figure C-10. SRP = $0.001$ for NIC = 6 and 7 Compared With Reference MSSRs	C-11
Figure C-11. SRP = $10^{-8}$ for NIC = 6 and 7 Compared With Reference MSSRs	C-12
Figure D-1. Along-Track Bias Error, d, for Uncompensated Delay, td at Speed, v	D-2
Figure D-2. Cross-track Displacement in Feet vs. Speed for 3 deg/sec Turn and Two Latency Values: td = 1 sec, and td = 2 sec	D-2
Figure D-3. Effects of Uncompensated Latency on Reported Separation, D, Compared With Actual Separation, S	D-4
Figure D-4. Reported/Actual Separation for Actual Separation, S = 3 NM, as a Function of Subtended Track Angle for Two Latency Bias Values	D-5
Figure D-5. Reported Separation / Actual Separation When Both Reports Are Delayed for Orthogonal Tracks ( $\alpha = 90 \text{ deg}$ )	D-6
Figure D-6. Reported Separation / Actual Separation When Both Reports are Delayed for Closing Off-set Tracks ( $\alpha = 180 \text{ deg}$ )	D-7
Table B-1. Summary of Sensitivity Results for Reference MSSR 95% Separation Error With Residual Bias Error and Dwell Time Latency	B-8

Error With Residual Bias Error and Dwell Time Latency

## **1** Introduction

A modification to an International Civil Aviation Organization (ICAO) accepted collision risk model was initially used in a comparison to legacy radar surveillance to define requirements for Automatic Dependent Surveillance-Broadcast (ADS-B) support of Air Traffic Control (ATC) separation standards [1]. This model, termed Close Approach Probability (CAP) is the conditional probability that an aircraft pair actually over lap when the apparent distance between them is some given separation. Subsequent experience with this model indicated that a more flexible approach based on the normalized surveillance separation error probability isolated the surveillance requirements from other operational scenario risk factors treated by the usual fault tree risk assessment. When the ADS-B navigation source is in either a fault-free or fault condition, this is termed the Separation Error Probability (SEP) in this document. A weighted combination of these two conditions, termed the Separation Reduction Probability (SRP), is used here to indicate the long term likelihood that a controller will have to deal with a degraded surveillance related problem. Other documents use the term, Surveillance Separation Error (SSE) to refer to both these conditions [2].<sup>1</sup> The SSE (as well as the equivalent SEP) is defined as the probability that the separation error (apparent separation minus true separation) for two aircraft towards each other is greater than a given value [2].

Two efforts are currently examining ADS-B separation requirements: a Federal Aviation Administration (FAA) group, titled the Separation Standards Working Group (SSWG); and a joint RTCA/EUROCAE group termed the Requirements Focus Group (RFG) [2]. Although coordinated efforts, the SSWG emphasis is on particular issues of concern to the FAA. The following analyses recognize some of the different assumptions made by each of these groups and presents results in relationship to the different Monopulse Secondary Surveillance Radar (MSSR) reference models used in each comparative assessment.<sup>2</sup> These results are given for the RFG selected MSSR reference ranges of 33 Nautical Miles (NM) for the terminal area, and 60 NM for the en route area. The RFG selected the 33 NM reference range on the basis that the assumed along-range MSSR error equals the cross-range error at this range. Since nominally half the terminal area traffic is typically within this range, and since SSWG simulated scenarios used both shorter and longer reference ranges, 33 NM is also used for the SSWG reference here. The en route 60 NM reference is considered a typical range in a multi-radar or mosaic environment.

A primary objective of the following examination is to quantify the sensitivity of resulting ADS-B separation requirements to the assumptions and scenarios used in determining these requirements. The next section reviews the two reference MSSR models and describes the SEP concept. Section 3 looks at the sensitivity of terminal area and en route fault-free ADS-B to ADS-B SEP to assumptions made in the analyses. ADS-B to MSSR no-fault SEP is treated in

<sup>&</sup>lt;sup>1</sup> The MathCAD programs used in most of the examples here were already in terms of SEP and SRP when the later term, SSE, was adopted.

<sup>&</sup>lt;sup>2</sup> The RFG azimuth error model includes a tail distribution based on recent data normalized to the EUROCONTROL accuracy standard. SSWG simulations to date have assumed a single Gaussian azimuth error distribution. Both groups use Gaussian distributed range errors, but with different standard deviations.

Section 4. Fault condition ADS-B to ADS-B SEP and SRP are reviewed in Section 5, and results are summarized in Section 6.

Details on the SEP derivation and its relationship to CAP are given in Appendix A. Sensitivity of the MSSR SEP model to practical dwell time differences for in-trail tracks, and the effect of residual bias uncertainty on the en route assumed reference are examined in Appendix B. Appendix C compares fault condition ADS-B separation error values with the reference MSSR models at different probability levels with the MSSR reference range as a variable parameter. A general treatment of latency effects on displayed separation for different separation scenarios is included as Appendix D.

A separate paper will describe ADS-B requirements for independent parallel approach monitoring. In this case, the probability of non-transgression zone penetration is also considered in determining acceptable Navigation Accuracy Category for position (NACp) and Navigation Integrity Category (NIC) values.

## 2 Background

As a baseline reference for a comparative assessment of ADS-B as a surveillance source, it is useful to determine the probability the MSSR surveillance separation error is equal to or greater than some value, Es. As described above, this is the Surveillance Separation Error, or in this document, the SEP. Alternative FAA SSWG and RFG MSSR models, and the SEP concept, are described in the following.

## 2.1 MSSR Position Estimate Error Reference Models

ADS-B surveillance separation errors are compared with separation errors associated with legacy MSSR cross-range and along-range position errors in the following. Two MSSR reference position estimate error models are used [3].<sup>3</sup> The SSWG model for the cross-range error probability density function (pdf) at the Terminal Area (TMA) reference range of R = 33 NM is a single Gaussian distribution and includes Common Digitizer-2 (CD-2) azimuth angle quantization of one Azimuth Change Point (ACP). The rms sum of the basic sensor azimuth standard deviation of 0.068 degrees and the one ACP quantization error of 0.025 degrees is 0.072 degrees. This produces a cross-range error of 0.04 NM at the selected range as shown below.

### AFS 450 MSSR Gaussian cross-range model w/ 1 ACP quantization for SSWG:

$$\sigma\phi := 0.068 \qquad R = 33 \qquad \sigma Mx(R) := \sqrt{\left(\sigma\phi \cdot \frac{\pi}{180} \cdot R\right)^2 + \left(\frac{360}{4096} \cdot \frac{\pi}{180} \cdot \frac{R}{\sqrt{12}}\right)^2} \qquad \sigma Mx(R) = 0.042$$

The cross-range SSWG error distribution pdf at range, R, is then:

$$p_{\text{MM}}(\zeta, \mathbf{R}) \coloneqq \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma M x(\mathbf{R})^2}} \cdot exp\left(\frac{-\zeta^2}{2 \cdot \sigma M x(\mathbf{R})^2}\right)$$

The SSWG assumed along-range standard deviation of 133 feet is based on BI-6 and transponder specifications, and includes either TMA or en route (E-R) CD-2 quantization of 1/64 or 1/8 NM.<sup>4</sup> For the TMA cases, this is:

Along-range MSSR errors for SSWG range error model 
$$\sigma r := 133$$
  $gt := 64$   $qe := 8$   $q := qt$   
SSWG quantized range error  $\sigma mr(q, \sigma r) := \sqrt{\left[\left(\frac{\sigma r}{6076}\right)^2 + \left(\frac{1}{q \cdot \sqrt{12}}\right)^2\right]}$   $q = 64$   $\sigma r = 133$   
 $\sigma mr(q, \sigma r) = 0.022$   
Note: SSWG sr = 136 ft w/o TMA quantization = 133 ft  $\sigma mr(q, \sigma r) \cdot 6076 = 135.794$   
 $\sigma Mr := \sigma mr(q, \sigma r)$   $\sigma Mr = 0.022$   
SMr(Mr)  $1.65 \cdot 2 | Mr^2 |$ 

 $<sup>^{3}</sup>$  This material, with some modifications, is provided in reference [11].

<sup>&</sup>lt;sup>4</sup> See Annex G in reference [11] for derivation of the standard deviation.

The RFG models of these MSSR position estimate error pdfs in the cross-range and along-range directions are:

**RFG MSSR Az Error Model**  $\sigma_1 := 0.054$   $\sigma_2 := 0.27$   $\alpha := 0.05$  **Redefine**  $\sigma$  **as cross-range error at:**  $R_{\text{W}} := 33$   $\sigma_{1x}(R) := \sigma_1 \cdot \frac{\pi}{180} \cdot R$   $\sigma_{1x}(R) = 0.031$  NM  $\sigma_{2x}(R) := \sigma_2 \cdot \frac{\pi}{180} \cdot R$   $\sigma_{2x}(R) = 0.156$  NM  $pn_1(\zeta, R) := \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma_{1x}(R)^2}} \cdot exp\left(\frac{-\zeta^2}{2 \cdot \sigma_{1x}(R)^2}\right)$   $pn_2(\zeta, R) := \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma_{2x}(R)^2}} \cdot exp\left(\frac{-\zeta^2}{2 \cdot \sigma_{2x}(R)^2}\right)$ 

The cross-range RFG error distribution pdf at range, R, is then:

$$pm(\zeta, R) \coloneqq (1 - \alpha) \cdot pn1(\zeta, R) + \alpha \cdot pn2(\zeta, R)$$

**RFG MSSR Range Error Model sd = 70 m**  $\sigma re := \frac{70}{1852}$   $\sigma re = 0.038$ **RFG along range error pdf**  $pmR(\zeta, \sigma re) := \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma re^2}} \cdot exp\left(\frac{-\zeta^2}{2 \cdot \sigma re^2}\right)$ 

## 2.2 Surveillance Separation Error

The separation error distribution for two time synchronized target position estimates separated by Sa NM in the cross-range direction is given by the convolution of the two target position error pdfs, pse, as illustrated in Figure 2-1. A somewhat unrealistic value of Sa = 0.2 NM is used in this example to better illustrate the separation error distribution characteristics relative to the position estimate errors for each target.

Convolution of MSSR pdfs, pm, separated by Sa is the MSSR-MSSR separation error pdf, pse:

pse(y,R) := 
$$\int_{-\infty}^{\infty} pm(\zeta,R) \cdot pm[\zeta - (Sa - y),R] d\zeta$$
  
 $\mathfrak{gl} := 0.054$ 
 $\mathfrak{gl} := 0.27$ 
 $\alpha := 0.05$ 
 $R = 33$ 
 $Sa = 0.2$   
 $\mathfrak{gl} := 0.054$ 
 $\mathfrak{gl} := 0.27$ 
 $\mathfrak{gl} := 0.27$ 
 $\mathfrak{gl} := 0.27$ 
 $\mathfrak{gl} := 0.05$ 
 $\mathfrak{gl} := 0.05$ 

#### Figure 2-1. Cross-Range Separated MSSR pdfs, pm, Compared With Separation Error pdf, pse, Which is the Difference (or Convolution) of these Position Estimation Errors

The cumulative distribution of pse is the probability the separation is less than or equal to some value, y, when the separation is Sa. If position estimates for both targets are made by the same radar, the MSSR separation error distribution is unbiased and essentially time synchronized. Appendix A shows a generalized formulation of surveillance separation errors results if the separation error is normalized to the separation, and a separation error, Es = Sa - y, is defined. The MSSR cross-range separation error probability is then conveniently expressed by interchanging the order of integration to give, Pse, the probability the separation error is equal to or greater than Es. For the RFG model this is:

$$\operatorname{Psec}_{-\infty}(\operatorname{Es}, R) := \int_{-\infty}^{\infty} \operatorname{pm}(y, R) \cdot \int_{-\infty}^{y} \operatorname{pm}(\zeta - \operatorname{Es}, R) \, d\zeta \, dy$$

And for the SSWG model separation error probability, PsS

$$PsS(Es,R) := \int_{-\infty}^{\infty} ps(y,R) \cdot \int_{-\infty}^{y} ps[(\xi - Es),R] d\xi dy$$

Similar expressions are given for the along-range SEP. Notice the similarity of this normalized separation error probability to the earlier used conditional CAP were the inter integration limits were +/- the aircraft width, and the probability was computed for a given apparent separation, Sa [1, 4].

Appendix A describes the relationships among position estimation errors, separation error probability, and CAP in more detail. Appendix B examines the effect that slightly different MSSR scanning beam dwell times on each aircraft has on the assumption made here of simultaneous position estimates. Appendix C shows how the MSSR reference range of applicability for both models affect the resulting comparisons.

## **3** ADS-B to ADS-B No-Fault Surveillance Separation Error

All of the above has assumed the MSSR is in a no-fault condition. The surveillance assessment for ADS-B uses the comparative separation error in a no-fault condition as well as the possibility of a Global Positioning System (GPS) navigation source fault condition producing misleading position data. Additionally, potential errors associated with asynchronous reception of ADS-B state vector updates from adjacent aircraft, and time delays onboard different aircraft are considered. All examples in the following are for TMA requirements; a similar process with an MSSR reference range of 60 NM, and including multi-sensor radar relative bias errors is used later for determination of E-R requirements. This section considers the no-fault condition SEP sensitivity to these errors as a function of the extrapolation time required to time synchronize the earlier received position update from one aircraft with the most recent reception from the other aircraft.

## 3.1 Terminal Area ADS-B to ADS-B No-fault SEP

The relationship between ADS-B position error standard deviations in meters and quantized NACp values is illustrated below for an assumed value of NACp = 7. Conversion of several NACp standard deviations from meters to NM is also shown for later reference.

### For no-fault ADS-B, select ADS-B Horizontal std dev (m)

$$\begin{split} &\text{NACp}(\sigma) \coloneqq \text{if}(\sigma \le 4.1, 10, \text{NACp}(\sigma)) \\ & \text{NACp}(\sigma) \coloneqq \text{if}(4.1 < \sigma \le 12.3, 9, \text{NACp}(\sigma)) \\ & \text{NACp}(\sigma) \coloneqq \text{if}(12.3 < \sigma \le 38, 8, \text{NACp}(\sigma)) \\ & \text{NACp}(\sigma) \coloneqq \text{if}(38 < \sigma \le 76, 7, \text{NACp}(\sigma)) \\ & \text{NACp}(\sigma) \coloneqq \text{if}(38 < \sigma \le 76, 7, \text{NACp}(\sigma)) \\ & \text{NACp}(\sigma) \coloneqq \text{if}(76 < \sigma \le 228, 6, \text{NACp}(\sigma)) \\ & \text{NACp}(\sigma) \coloneqq \text{if}(228 < \sigma \le 380, 5, \text{NACp}(\sigma)) \\ & \text{NACp}(\sigma) \coloneqq \text{if}(380 < \sigma \le 760, 4, \text{NACp}(\sigma)) \\ & \text{NACp}(\sigmagm) = 7 \qquad \sigmagm = 76 \end{split} \qquad \sigmags \coloneqq \frac{76}{1852} \qquad \sigmags \coloneqq \frac{228}{1852} \\ & \sigmags \vdash \frac{28}{1852} \\ & \sigmags \vdash \frac{28}{$$

In this case for NACp= 7, the no-fault one dimensional position error Gaussian distribution has a standard deviation,  $\sigma g = 0.041$  NM, and the pdf is pg, where:

**Cross-track ADS-B pdf:** 
$$\sigma g := \frac{\sigma g m}{1852}$$
  $\sigma g = 0.041$   $pg(y,\sigma g) := \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma g^2}} \cdot exp\left(\frac{-y^2}{2 \cdot \sigma g^2}\right)$ 

If ADS-B receptions on the ground from adjacent aircraft were synchronous as with radar adjacent target position estimates, the no-fault ADS-B SEP, Paa, could be directly calculated from  $\sigma g$  as shown below. Figure 3-1 compares this case for two NACp values with the two MSSR cross-range models at R = 33 NM. Notice the two MSSR separation errors are approximately the same at probabilities above about 0.02, and that the NACp = 7 SEP equals the SSWG cross-range SEP at this range. Since the two cross-range models have approximately equal 95 percent error bounds (or SEP = 0.05), the simpler single Gaussian model is used later as a sensitivity reference.



Separation error, Es NM

Figure 3-1. Probability the MSSR-MSSR Cross-Range Surveillance Separation Error at R NM is Equal to or Greater Than Es, Compared With NACp = 7 and 8 No-Fault ADS-B to ADS-B Separation Error Probabilities (Without Extrapolation/Latency Effects)

ADS-B position reports from different aircraft are not simultaneously received, however, so the earlier received report from one aircraft is assumed to be extrapolated (using the state vector instantaneous velocity) to the later time of reception of the adjacent aircraft position report for comparison with the reference MSSR. All the following assumes the separation error is based on extrapolation of ADS-B positions using the state vector velocity, but allows for velocity errors, the probability of a cross-track separation reduction due to a possible turn towards the adjacent aircraft during the asynchronous update window extrapolation time, and for uncertainty in the along-track position due to uncertainty in onboard aircraft latency before transmission of the navigation source determined position update. These latency related position error sources are assigned to the one aircraft of the pair that is being extrapolated. The net effect of time registration extrapolation is then to possibly increase either or both the cross-track and along-track position error uncertainty bounds relative to the basic ADS-B reported NACp value.

Sensitivity of the SEP to the above listed separation error degradation factors could be examined by appropriately increasing the basic NACp standard deviation to account for the degradation, and performing the convolution calculation shown in Figure 3-1. An equivalent, and more convenient approach, uses the fact that the separation error pdf for the convolution of two Gaussian position error pdfs is a Gaussian pdf with a standard deviation equal to the root sum squared (RSS) of the two position error standard deviations. The one-sided 95 percent SE bound is the cumulative probability, SE95, that 95 percent of the separation errors are no greater than 1.65 times the resulting separation error standard deviation,  $\sigma$ s. This is shown in the following normalized example where the position error standard deviation,  $\sigma a = \sigma b$ , are assumed to be unity.

$$\sigma a := 1$$
  $\sigma b := 1$   $\sigma s := \sqrt{\sigma a^2 + \sigma b^2}$   $SE95 := \int_{-\infty}^{1.65 \cdot \sigma s} pg(y, \sigma s) dy$   $SE95 = 0.95$ 

The equivalence of these two formulations of the separation error bound is illustrated below by application to the SSWG cross-range separation error shown in Figure 3-1 where the SEP for the one-sided 95 percent bound on the separation error, SMx = 0.1 NM, is shown to be Pse = 0.05 when Es = SMx:

# Relationship between the one sided 95% bound on SE and the 0.05 SEP value for the SSWG MSSR x-rng model

Std dev of x-rng error at R $R = 33$	$\sigma Mx(R) = 0.042$	
One-sided x-rng SE, SMx, at 95% prob	$\operatorname{SMx}(R) \coloneqq 1.65 \cdot \sqrt{2 \cdot (\sigma M x(R))^2}$	SMx(R) = 0.1
Value of SEP computation at SE, SMx	PsS(SMx(R), R) = 0.05	

This example also uses the fact that the RSS of two equal standard deviations is  $\sqrt{2}$  times the standard deviation. Similar relationships apply for SSWG along-range separation error probabilities with the 95 percent SE = 0.05 NM, and for both components of the RFG MSSR error models.

With this one-sided 95 percent SE concept, potential increases in the basic NACp value ADS-B position uncertainty due to extrapolation errors or unexpected turns can be plotted as a function of the asynchronous update extrapolation time.

## 3.1.1 For Along-Track Errors

Reference [5] shows the effect of latency on reported position errors is limited to along-track position errors for non-maneuvering aircraft. EUROCONTROL data collected on 1090ES ADS-B targets of opportunity [6] show onboard delay distributions over the equipped aircraft population are approximately Gaussian with a mean delay of  $\mu d = 0.3$  seconds (which can be compensated by the user if the value is known so that  $\mu d = 0$  for non-accelerating aircraft), and a standard deviation of  $\sigma d = 0.15$  seconds (which determines the along-track uncompensated position uncertainty error), as shown in reference [5]. For a random member of the aircraft population, the latency bias compensated along-track position error standard deviation is then  $\sigma T = 0.15 \text{ x spd (m/s)}$  where TMA speed = 320 kt, E-R speed = 600 kt, and final approach speed = 200 kt. Note that 1 kt is about 0.5 m/s, so 0.5 x v kts = v m/s. For terminal area speeds, the along track position uncertainty due to latency uncertainty, and bias error if the mean latency is uncompensated, are then:

$$\sigma d \coloneqq 0.15 \qquad v \coloneqq 320 \quad \text{kts} \qquad \sigma T(v, \sigma d) \coloneqq \sigma d \cdot \frac{0.5 \cdot v}{1852} \qquad \sigma T(v, \sigma d) = 0.013 \qquad \text{NM}$$
  
Along-trk mean bias error for each a/c  $\mu d \coloneqq 0.3 \qquad \mu T(v, \mu d) \coloneqq \mu d \cdot \frac{0.5 \cdot v}{1853} \qquad \mu T(v, \mu d) = 0.026 \quad \text{NM}$ 

Errors in the state vector velocity used in extrapolation also contribute to along-track position errors. Recent data collection in the CAPSTONE program with a Wide Area Augmentation System (WAAS) receiver indicated GPS derived velocity was so good that measurement accuracy was limited by aircraft plant noise [5]. As a general observation, very accurate estimates would be expected if the GPS velocity is derived from p-range Doppler measurements, or even if velocity is derived from successive GPS position estimates, a low variance in the estimate should still be obtained since relative position uncertainty from sample to sample should be low due to the long GPS de-correlation intervals. Indeed, velocity error measurements for a stationary receiver shown in Misra and Enge [7] are bounded by about 0.1 m/s. However, for some non-GPS navigation sources, any delay in velocity output may result in a velocity lag, or bias error for turning aircraft, but that is not considered in the ADS-B definition of NAC for velocity (NACv).

Thus, although these considerations suggest the NACv model of a random velocity error of, say +/-10 m/s at the 95 percent error limits for NACv = 1 is questionable in some cases, that model is conservatively assumed here to bound along-track and cross-track GPS velocity errors in the following. The NACv = 1 associated along-track time registered position uncertainty distribution due to this random model then has a standard distribution one-half the 95 percent bound, and the resulting position error standard deviation after extrapolation over an interval of t seconds is given by:

$$\sigma v := 5$$
 m/s  $\sigma t v(t, \sigma v) := \frac{\sigma v \cdot t}{1852}$  NM

Not

Note that 1 m/s is 2 kts

The resulting SEP due to along-track uncompensated position uncertainty with independent onboard latencies, and a random velocity error used in extrapolation of the basic NAC value, is then scenario dependent as described below. All examples use the fact that the mean difference in separate estimates is based on the difference in the means, and the resulting variance is the sum of the variances. Pair-wise separation scenario details are found in [8], reproduced here as Appendix D for reference. Error factors for each aircraft of the pair-wise encounter are enclosed in parentheses in the following. The general approach is to RSS the related standard deviations representing position uncertainty factors for each aircraft, and then RSS the results for each aircraft to yield the resulting separation error uncertainty standard deviation. The SEP at a probability of 0.05 is then 1.65 times this value as shown above. The effect of mean position errors for each aircraft on the separation error is the difference of these means projected along the separation direction of interest.

#### For in-trail ADSB-ADSB tracks

SE is convolution of position error pdfs, or std dev of SE is RSS of position error std devs. SEP at 0.05 prob is one sided 95% error bound or 1.65 time std dev of separation error

Latency errors affect both aircraft, but velocity errors only affect extrapolated aircraft. Note that latency bias errors for each intrail a/c subtract for relative separation error

$$SAn(\sigma gn, t, v, \sigma v, \sigma d) \coloneqq 1.65 \cdot \sqrt{|\sigma gn^2 + \sigma T(v, \sigma d)^2| + |\sigma gn^2 + \sigma T(v, \sigma d)^2 + \sigma tv(t, \sigma v)^2|}$$

#### For orthogonal merge ADSB-ADSB tracks

SE for 90 deg merge is only affected by latency and velocity errors of merging aircraft

$$\operatorname{SOn}(\operatorname{\sigmagn}, t, v, \operatorname{\sigmav}, \operatorname{\sigmad}, \mu d) \coloneqq 1.65 \cdot \sqrt{\operatorname{\sigmagn}^2 + 0} + \operatorname{\sigmagn}^2 + \operatorname{\sigmaT}(v, \operatorname{\sigmad})^2 + \operatorname{\sigmatv}(t, \operatorname{\sigmav})^2 + \mu T(v, \mu d)$$

Figure 3-2 compares these results as a function of the time registration extrapolation time with the reference SSWG MSSR errors at the 95% level for the assumed NACp = 7 and 8, the 10 m/s 95% velocity error, and compensated latency. This presentation of results can be interpreted in terms of the usual SEP plots by noting that the SSWG MSSR cross-range separation error, and NACp = 7 and 8 values (at t = 0) are the same as the corresponding Es values at SEP = 0.05 in Figure 3-1.

MSSR ref:  $\sigma Mr \cdot 6076 = 136$  q = 64  $SMr(\sigma Mr) = 0.052$   $\sigma \phi = 0.068$  R = 33  $\sigma Mx(R) = 0.042$ Op area: v = 320  $\sigma d = 0.15$   $\mu d = 0$   $\sigma v = 5$ ADS-B NAC:  $\sigma g 8 \cdot 1852 = 38.892$   $\sigma g 7 \cdot 1852 = 75.932$ 



Extrapolation time in sec

### Figure 3-2. Compensated Latency ADS-B to ADS-B Along-Track SEP, and Orthogonal Track SEP at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR

Notice also that an ADS-B NACp = 7 SE, even with the extrapolation/latency spoiling factor, is essentially equal to the MSSR cross-range SEP at R = 33 NM. The time variation of the ADS-B reported position error, or de-correlation interval for the ADS-B SEP, differs from the MSSR error time variation however. MSSR errors near the antenna bore-sight will generally be de-correlated from update to update, but ADS-B with GPS de-correlation times are on the order of several minutes. This means that this component of an ADS-B separation error value, Es, randomly selected at the indicated probability in the above plot will tend to have little change over this de-correlation time.

If the assumed mean latency value of 0.3 seconds is uncompensated, the results are shown in Figure 3-3; only the orthogonal merge scenario separation error is affected by the mean latency in these two cases since the means cancel in the along-track case.<sup>5</sup> Similarly, any uncompensated along-track error effect associated with a specific aircraft onboard delay uncertainty (24 meters in the TMA case) will basically remain unchanged for that aircraft. However, as shown above, whatever the differences in time and source variation, the magnitude of the effects on the ADS-B separation error, Es, are very small relative to the separation distance.

<sup>&</sup>lt;sup>5</sup> Although the relative separation error is unaffected by the mean error in latency, it is important to emphasize that any correlation of these ADS-B position reports with other sensor position estimates should appropriately account for the mean latency. The other possible operational impact of latency is to subtract from any look-ahead conflict alert time based on the use of old data, but this is not significant for the range of values of interest here.

MSSR ref:  $\sigma Mr \cdot 6076 = 136$  q = 64  $\sigma \phi = 0.068$  R = 33  $\sigma Mx(R) = 0.042$ Op area: v = 320  $\sigma d = 0.15$   $\mu d = 0.3$   $\sigma v = 5$ ADS-B NAC:  $\sigma g8 \cdot 1852 = 38.892$   $\sigma g7 \cdot 1852 = 75.932$ 



Extrapolation time in sec

### Figure 3-3. Uncompensated Latency ADS-B to ADS-B Along-Track SEP, and Orthogonal Track SEP at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR

Comparison of the NACp = 7 orthogonal merge tracks SEP parts of Figures 3-2 and 3-3 for compensated and uncompensated latency is shown in the SEP verses Es format in Figure 3-4 for an assumed extrapolation time, t = 3 sec. Linkage between these two formats is illustrated by noting that in Figure 3-3, based on the cumulative separation error at the 5 percent limit,

 $SOn(\sigma gn, t, v, \sigma v, \sigma d, \mu d) = 0.125$ 

And in Figure 3-4, when Es = 0.125 NM, SEP = POn = 0.05 as shown by:

 $POn(0.125, \sigma gOn, \mu d) = 0.049$ 

While Figure 3-3 references the SSWG MSSR model for comparison, Figure 3-4 also couples the comparison to the RFG MSSR model.



Figure 3-4. NACp = 7 ADS-B to ADS-B SEP for Orthogonal Tracks (With and Without Latency Compensation) Compared With MSSR-MSSR SEP for Above Parameters

### 3.1.2 For Cross-Track Errors

Cross-track errors are of interest in parallel track scenarios. In this case latency (contributing to along-track errors) is of little interest, but velocity track angle errors extrapolate to cross-track errors, and the potential for one of the aircraft pair to turn towards the adjacent aircraft during the extrapolation interval is accounted for by assigning a turn margin to the separation error estimate.

The conservative bound on cross-track position uncertainty distribution due to a cross-track velocity error standard deviation (equal to one half the 95 percent error in m/s) over the extrapolation time, t sec, for the extrapolated aircraft is:

$$\operatorname{stv}(t, \operatorname{sv}) \coloneqq \frac{\operatorname{sv} \cdot t}{1852}$$
 NM

Total cross-trk SEP at 0.05 prob for extrapolation of basic NAC value of one aircraft with velocity The turn margin for a cross-track acceleration and suffermined from examination of John Shaw provided maneuver distribution data in United Kingdom airspace [9]. From his Gaussian fit of this data and the extrapolation time provided in the reference, acceleration probability values of at = 1.7 m/s/s and ae = 1.4 m/s/s are derived for terminal and en route areas on the assumption that the accelerations are primarily due to turns. Cross-track position uncertainty due to a possible turn toward the other aircraft by the aircraft being extrapolated over a time, t sec, is then:<sup>6</sup>

TMA at := 1.7 E-R ae := 1.4 a := at 
$$\sigma tr(t, a) := \frac{0.5 \cdot a \cdot t^2}{1852}$$
 NM

For reference to RFG values, note errors for assumed 6 and 12 sec times agree

TMA  $\sigma tr(6, at) \cdot 1852 = 30.6$  m E-R  $\sigma tr(12, ae) \cdot 1852 = 100.8$  m

The RSS cross-track uncertainty for extrapolation of the basic NACp standard deviation of the earlier received aircraft position,  $\sigma gn$ , with the velocity uncertainty,  $\sigma tv$ , and turn probability uncertainty during the extrapolation interval,  $\sigma tr$  is the term below in brackets. This, RSSed with the reference aircraft position uncertainty times the 1.65 factor is then the 0.05 probability SEP for this scenario, SXn

$$SXn(t, a, \sigma v, \sigma gn) \coloneqq 1.65 \cdot \sqrt{\sigma gn^2 + \sigma gn^2 + \sigma tr(t, a)^2 + \sigma tv(t, \sigma v)^2}$$

TMA examples of the sensitivity of parallel track separation errors to extrapolation velocity 95% errors of 10 m/s/s and 4 m/s/s, and the TMA turn value, at, are shown in Figure 3-5. Two results are of interest here: the SE is relatively insensitive to the assumed velocity errors for either NACp value, and even the turn probability margin effect is unnoticeable if the extrapolation time is limited by a periodic update acceptance window of about three seconds.

<sup>&</sup>lt;sup>6</sup> Computed track angle errors for turning aircraft based on latent state vector velocity data should also be considered in look-ahead applications, but the report delays of several seconds considered here should still provide better estimates than those usually obtained from radar tracker estimates for turning targets.



Extrapolation time in sec

Figure 3-5. NAC 7 and NAC 8 Cross-Track Separation Error Probability at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR at R = 33 NM

## 3.2 En route Area ADS-B to ADS-B No-fault SEP

Comparative requirements for ADS-B to ADS-B separation in E-R airspace use several assumptions that differ from the above terminal area cases: the assumed aircraft speed for position extrapolation is 600 kts, the turn probability factor is a = 1.4 m/s/s, and the MSSR cross-range reference is R = 60 NM with an assumed multi-MSSR environment. The multi-MSSR environment assumption introduces a new parameter in defining the MSSR baseline – the relative bias errors among the multiple MSSRs which influence the assumed coordinate converted accuracy of any selected MSSR as the baseline reference.

### 3.2.1 Multi-MSSR En route Reference SEP

In this case the MSSR range and azimuth measurements relative to the radar site coordinates are converted to the ADS-B WGS-84 coordinate system. Residual bias errors after any correction process are, as is the usual case in product control analyses, assumed to be a Gaussian distribution over the population of radars. The 95 percent bounds on these bias residuals are assumed to +/-1 ACP in azimuth and +/-60 meters in range for the RFG (and +/-133 feet for SSWG).<sup>7</sup> Since the time of the MSSR dwell on a target during an update scan is dependent on the azimuth angle, time registration depends upon target track geometry relative to the MSSR location. The residual time error after registration correction is assumed to be +/- 0.5 seconds (95%). Calculation of the standard deviations of the associated residual bias error components and RSS effects of these errors on the MSSR position estimates follow:

**RFG MSSR Az Error Model**  $\sigma_{10} := 0.054$   $\sigma_{20} := 0.27$   $\alpha := 0.05$ 

Std dev of multiple MSSR residual bias error uncertainties at 95% bound

Az ang  $ACP := 1 \ \phi bo := ACP \cdot \frac{0.088}{1.96}$  Along range  $\delta := 60$   $rbo := \frac{\delta}{1.96}$   $\sigma rbo := \frac{rbo}{1852}$ 

RSS MSSR Az errors and bias error std devs for random aspect angles

$$\sigma_1 := \sqrt{\sigma_1 o^2 + \phi_0 o^2}$$
  $\sigma_1 = 0.07$   $\sigma_2 := \sqrt{\sigma_2 o^2 + \phi_0 o^2}$   $\sigma_2 = 0.274$ 

Redefine  $\sigma$  as cross-range error at: R := 60

$$\sigma_{1x}(R) \coloneqq \sigma_{1} \cdot \frac{\pi}{180} \cdot R \qquad \sigma_{1x}(R) = 0.074 \text{ NM} \qquad \sigma_{2x}(R) \coloneqq \sigma_{2} \cdot \frac{\pi}{180} \cdot R \qquad \sigma_{2x}(R) = 0.287$$

$$pn1(\zeta, R) \coloneqq \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma_{1x}(R)^{2}}} \cdot exp\left(\frac{-\zeta^{2}}{2 \cdot \sigma_{1x}(R)^{2}}\right) \qquad pn2(\zeta, R) \coloneqq \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma_{2x}(R)^{2}}} \cdot exp\left(\frac{-\zeta^{2}}{2 \cdot \sigma_{2x}(R)^{2}}\right)$$

The cross-range RFG error distribution pdf at range, R, is then:

 $pm(\zeta, R) := (1 - \alpha) \cdot pn1(\zeta, R) + \alpha \cdot pn2(\zeta, R)$ 

The RFG along-range multi-MSSR error pdf is:

<sup>&</sup>lt;sup>7</sup> Appendix B considers the sensitivity of the assumed residual bias on the resulting reference value.

**RFG MSSR Range Error Model sd = 70 m**  $\sigma reo := \frac{70}{1852}$   $\sigma reo = 0.038$ 

$$\sigma re := \sqrt{\sigma reo^2 + \sigma rbo^2}$$
  $\sigma re = 0.041$ 

**RFG along range error pdf** 
$$pmR(\zeta, \sigma re) := \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma re^2}} \cdot exp\left(\frac{-\zeta^2}{2 \cdot \sigma re^2}\right)$$
  
One sided 95% along-rng SE is 1.65 factor  $SRr(\sigma re) := 1.65 \cdot \sqrt{2 \cdot \sigma re^2}$   $SRr(\sigma re) = 0.096$ 

The SSWG error models including CD-2 quantization for cross-range and along-range are:

#### AFS 450 MSSR Gaussian Az error model w/ 1 ACP quantization for SSWG:

#### The cross-range SSWG multi-MSSR error distribution pdf at range, R, is then:

$$ps(\zeta, R) := \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma M x(R)^2}} \cdot exp\left(\frac{-\zeta^2}{2 \cdot \sigma M x(R)^2}\right)$$

And,

#### Along-range multi-MSSR errors for SSWG range error model:

Note: SSWG sr = 136 ft w TMA quantization is a basic unquantized 133 ft error  $\sigma rf := 133$   $\sigma ro := \frac{\sigma rf}{6076}$   $\sigma ro = 0.022$ RSS MSSR range errors and bias error std devs for random aspect angles  $\sigma rbo = 0.017$  $\sigma r := \sqrt{\sigma ro^2 + \sigma rbo^2}$   $\sigma r = 0.027$ 

CD-2 range quantization for TMA and E-R: 
$$qt := 64$$
  $qe := 8$   $q := qe$   $q = 8$ 

SSWG quantized range error 
$$\sigma mr(q, \sigma r) := \sqrt{\left[\left(\sigma r\right)^2 + \left(\frac{1}{q \cdot \sqrt{12}}\right)^2\right]}$$
  $\sigma Mr := \sigma mr(q, \sigma r)$   
 $\sigma Mr = 0.045$ 

The SSWG one-sided separation errors at a 0.05 probability are then:

One sided 95% along-rng SE is 1.65 factor 
$$SMr(\sigma Mr) := 1.65 \cdot \sqrt{2 \cdot |\sigma Mr|^2}$$
  $SMr(\sigma Mr) = 0.106$   
One sided 95% x-rng SE is 1.65 factor  $SMx(R) := 1.65 \cdot \sqrt{2 \cdot (\sigma Mx(R))^2}$   $SMx(R) = 0.209$ 

With the multi-MSSR bias effects defined above, and the R = 60 NM reference range, potential ADS-B en route values of NAC = 7 and 6 are compared with the reference in Figure 3-6.

With +/- ACP 95% residual az bias uncertainty, and +/- 60 meter residual range bias uncertainty: R = 60  $\sigma 1 = 0.07$   $\sigma 2 = 0.274$   $\alpha = 0.05$   $\sigma \phi = 0.081$  q = 8  $\sigma g 7 = 0.041$   $\sigma g 6 = 0.123$ 



Figure 3-6. Probability the MSSR-MSSR Cross-Range Surveillance Separation Error With Residual Bias Uncertainty at R = 60 NM is Equal to or Greater Than Es, Compared With NACp = 7 and 6 No-Fault ADS-B to ADS-B Separation Error Probabilities (Without Extrapolation/Latency Effects)

### 3.2.2 En Route Area ADS-B to ADS-B No-fault SEP Examples

At this point, other than for the assumed speed of 600 kts and new turn parameter, en route sensitivity examinations follow the terminal area process. Corresponding en route along-track and orthogonal merge cases are given in Figures 3-7 with latency compensation, and Figure 3-8 without latency compensation.

MSSR ref:  $\sigma Mr \cdot 6076 = 275$  q = 8  $\sigma \phi = 0.081$  R = 60  $\sigma Mx(R) = 0.089$ Op area: v = 600  $\sigma d = 0.15$   $\mu d = 0$   $\sigma v = 5$ ADS-B NAC:  $\sigma g 6 \cdot 1852 = 227.796$   $\sigma g 7 \cdot 1852 = 75.932$ 



Extrapolation time in sec

Figure 3-7. Compensated Latency ADS-B to ADS-B Along-Track SEP, and Orthogonal Track SEP at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR

MSSR ref:  $\sigma Mr \cdot 6076 = 275$  q = 8  $\sigma \phi = 0.081$  R = 60  $\sigma Mx(R) = 0.089$ Op area: v = 600  $\sigma d = 0.15$   $\mu d = 0.3$   $\sigma v = 5$ ADS-B NAC:  $\sigma g 6 \cdot 1852 = 227.796$   $\sigma g 7 \cdot 1852 = 75.932$ 



Extrapolation time in sec

### Figure 3-8. Uncompensated Latency E-R ADS-B to ADS-B Along-track SEP, and Orthogonal Track SEP at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR

The en route cross-range error for parallel track scenarios is given in Figure 3-9. All these en route examples are plotted over the longer extrapolation time since permitted E-R update rates may be lower than those for TMA cases.



Extrapolation time in sec

Cross-track separation error probability at a 0.05 probability level as a function of extrapolation time for above parameters compared with reference MSSR

Figure 3-9. E-R Cross-Track Separation Error Probability at a 0.05 Probability Level as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR

## 3.3 Summary of ADS-B to ADS-B No-fault SEP Results

En route and terminal area no-fault conditions may be compared as follows:

- ADS-B-ADS-B separation with NACp=8 has a lower SEP than MSSR-MSSR separation in the terminal case when bias effects, extrapolation effects, and various configurations of aircraft are considered (see Figure 3-2, 3-3, and 3-5).
- ADS-B-ADS-B separation with a NACp=7 has a lower SEP than MSSR-MSSR separation in the en route case when bias effects, extrapolation effects, and various configurations of aircraft are considered (see Figures 3-7, 3-8 and 3-9).
• The NACv=1 velocity errors have little effect on performance when examining extrapolation of ADS-B data (see Figure 3-9). Similarly, the effect of a possible turn on the extrapolated position uncertainty is small as long as the update acceptance window is limited to 3 seconds in the TMA and 6 seconds E-R.

# 4 ADS-B to MSSR No-Fault Surveillance Separation Error

#### 4.1 ADS-B to MSSR No-Fault SEP Reference Case

Just as in the en route multi-MSSR case above, additional bias error sources and time registration issues must be considered when one aircraft is reporting its position and velocity on ADS-B, and an adjacent aircraft position is determined by MSSR. In this case the MSSR range and azimuth measurements relative to the radar coordinates are converted to the ADS-B WGS-84 coordinate system, and it is expected that any bias in MSSR azimuth and range measurements relative to WGS-84 are minimized in this process by adjusting the MSSR estimates to agree with the time registered ADS-B reported position for dual equipped aircraft.<sup>8</sup> Residual bias errors after this correction process are, as is the usual case in product control analyses, assumed to be a Gaussian distribution over the population of radars. The 95 percent bounds on these bias residuals are taken to be +/-1 ACP in azimuth and +/-60 meters (about +/-200 feet) in range in the following examples.

Since the time of the MSSR dwell on a target during an update scan is dependent on the azimuth angle, time registration of a nearby ADS-B reported position depends upon track geometry relative to the MSSR location. The residual time error after this registration correction is assumed to be +/- 0.5 seconds (95%). Calculation of the standard deviations of the associated residual bias error components, and selection of the greater of the along or cross range components to conservatively represent arbitrary scenario TMA cases, are shown below.

#### For ADS-B to MSSR SEP with residual bias error uncertainties at +/- 95% bounds

The approach assigns bias uncertainty errors to MSSR target, then extrapolates ADSB target.

X-rng: 
$$\phi b := \frac{0.088}{1.96} \quad \sigma x b(R) := \phi b \cdot \frac{\pi}{180} \cdot R \quad \text{Along-rng:} \quad rb := \frac{60}{1.96} \quad \sigma rb := \frac{rb}{1852} \quad \sigma rb = 0.017$$

For general relationship of MSSR residual bias uncertainty to separation direction, assume circular error with Gaussian sd in any direction of interest determined by greater of x-rng or along-rng residual bias component

$$\mathbf{x} = if(\mathbf{x} \mathbf{b}(\mathbf{R}) \ge \mathbf{x} \mathbf{b}(\mathbf{R}), \mathbf{x} \mathbf{b}(\mathbf{R}), \mathbf{x} \mathbf{b}) \qquad \mathbf{x} = 0.026 \qquad \mathbf{R} = 33$$

Since the resultant of these different errors depends upon the particular aircraft latency, the track geometry relative to a particular MSSR location, and the resulting error projection onto the separation direction of the adjacent aircraft is random, the projected standard deviation of the resulting Gaussian distribution is the root sum squared of the components.

<sup>&</sup>lt;sup>8</sup> Using targets of opportunity that are primarily radial tracks for azimuth bias correction, and cross-range tracks for range bias correction minimizes any time difference related errors in this correction process.

For an arbitrary MSSR trgt flt path direction relative to the radar location with along-trk ADSB-MSSR relative time error:  $tb := \frac{0.5}{1.96}$   $\sigma tb(v) := tb \cdot \frac{v}{3600}$ 

Total projected residual bias error related MSSR trgt position uncertainty is then

$$\sigma ab := \sqrt{\sigma b^2 + \sigma t b(v)^2}$$
  $\sigma ab = 0.034$ 

Referring to Section 2.1 for the along and cross range standard deviations of the SSWG MSSR at the range, R = 33 NM, the greater of the two values is now selected to conservatively represent the single MSSR assumed circular error distribution.

$$\sigma M := if(\sigma M x(R) \ge \sigma M r, \sigma M x(R), \sigma M r)$$
  $\sigma M = 0.042$   $R = 33$ 

The convolution of the bias error distribution with the single MSSR basic position error pdf is then used with the ADS-B pdf to assess the capability of ADS-B to MSSR SEP.

ADSB NACP  $\sigma g7 = 0.041$ For SSWG MSSR model R = 33  $\sigma M = 0.042$   $\sigma ab = 0.034$   $\sigma Mb := \sqrt{\sigma M^2 + \sigma ab^2}$  $\sigma Mb = 0.054$   $pMb(\zeta, \sigma Mb) := \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma Mb^2}} \cdot exp\left(\frac{-\zeta^2}{2 \cdot \sigma Mb^2}\right)$ 

SEP of ADSB to SSWG MSSR with relative residual bias, PsaS:

$$PsaS(Es,\sigmaMb,\sigma gn) := \int_{-\infty}^{\infty} pg(y,\sigma gn) \cdot \int_{-\infty}^{y} pMb[(\xi - Es),\sigmaMb] d\xi dy$$

Figure 4-1 compares the mixed sensor result with the ADS-B and MSSR alone cases for an ADS-B NACp = 7.

 $\sigma g7 = 0.041$ 



 $\sigma M = 0.042$ 

Separation error, Es NM

 $\sigma ab = 0.034$ 

# Figure 4-1. ADS-B to MSSR SEP (With Residual Bias Error Uncertainty) Compared With ADS-B to ADS-B SEP for Same NACp = 7 Value. MSSR-MSSR SEPs for SSWG and RFG Models are also Shown for Reference

Part of Figure 4-1 restates the basic result that NACp = 7 SEP is equal to the reference SSWG MSSR SEP result as previously shown in Figure 3-1. However, the NACp = 7 ADS-B to MSSR SEP in Figure 4-1 is greater than either of these results alone due to the relative MSSR bias error that must now be considered in this mixed sensor case. As will be seen in one of the following examples, a more accurate NACp value can sometimes compensate for the bias error, but depending on the relative values, this may not always be the case.

#### 4.2 ADS-B to MSSR No-Fault SEP Examples

For SSWG MSSR model

R = 33

With this non-extrapolated case as background, geometric relationships of ADS-B target tracks extrapolated to MSSR target tracks, and different geometric track relationships to the MSSR location are described. Previously considered scenario SEPs for time synchronized ADS-B targets relative to the MSSR location are then:

#### Parallel ADSB-MSSR tracks:

RSS max MSSR position error with MSSR relative bias error, then RSS turn and velocity errors with ADS-B error of other aircraft (Note latency bias has no effect here)

$$SXs(t, a, \sigma v, \sigma M, \sigma ab, \sigma gn) \coloneqq 1.65 \cdot \sqrt{\left|\sigma M^2 + \sigma ab^2\right| + \left|\sigma tr(t, a)^2 + \sigma tv(t, \sigma v)^2 + \sigma gn^2\right|}$$

#### In-trail ADSB-MSSR trks (or orthogonal ADSB merge w/MSSR trk):

RSS max MSSR position error with MSSR relative bias error, then RSS latency and velocity errors with ADS-B error of other aircraft

 $SA(\sigma gn, t, v, \sigma v, \sigma d, \sigma M, \sigma ab) \coloneqq 1.65 \cdot \sqrt{\sigma M^2 + \sigma ab^2} + \sigma gn^2 + \sigma T(v, \sigma d)^2 + \sigma t v(t, \sigma v)^2$ 

 $SAs(\sigma gn, t, v, \sigma v, \sigma d, \sigma M, \sigma ab, \mu d) \coloneqq SA(\sigma gn, t, v, \sigma v, \sigma d, \sigma M, \sigma ab) + \mu T(v, \mu d)$ 

#### Orthogonal MSSR trk merge w/ADS-B trk:

RSS max MSSR position error with MSSR relative bias error, then RSS with ADS-B error of other aircraft

$$SOs(\sigma gn, \sigma M, \sigma ab) \coloneqq 1.65 \cdot \sqrt{\sigma M^2 + \sigma ab^2 + \sigma gn^2}$$

t := 0, 0.2 6

Sensitivity examples for NACp values of 7 and 8 with compensated latency in TMA along-track and merge scenarios are given in the following two Figures 4-2 and 4-3. Notice the different effect the latency uncertainty (increased to an assumed value of 0.6 sec for these two plots) has on SE depending upon whether the merging aircraft is ADS-B or MSSR.



Figure 4-2. TMA NAC = 7 ADS-B to MSSR SEP at 0.05 Probability as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR (Compensated Latency)



#### Figure 4-3. TMA NAC = 8 ADS-B to MSSR SEP at 0.05 Probability as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR (Compensated Latency)

Similar along-track and merge examples for the en route area assumptions are shown in Figures 4-4 and 4-5 for NACp = 6 and NACp= 7.



Figure 4-4. En Route NAC = 6 ADS-B to MSSR SEP at 0.05 Probability as a Function of Extrapolation Time for Above Parameters Compared With Reference MSSR (Compensated Latency)





#### 4.2.1 Residual MSSR Azimuth Bias Uncertainty

All the ADS-B to MSSR cases so far have assumed residual bias uncertainties of 1 ACP in azimuth, and 60 meters in range, and the plots were for fixed reference MRRS ranges of 33 NM in terminal area cases and 60 NM for en-route examples. Since the cross-range error increases with range for a specific azimuth bias uncertainty, showing the SEP as a function of range for various assumed conditions is a useful way to illustrate the general properties of this effect. We first assume the relative time registration bias,  $\mu d$ , is zero. Figure 4-6 is such a plot with b = 1 ACP representing the conditions shown in Figure 4-1 for a NACp = 7.





MSSR Range, NM

Figure 4-6. In-trail ADSB-MSSR (or Orthogonal ADSB Merge w/MSSR) SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR-MSSR and ADSB-ADSB SEPs for NACp = 7 and μd = 0

Notice that for R = 33 NM as in Figure 4-1, both the ADSB-ADSB and MSSR-MSSR in-trail separation errors are equal at 0.1 NM, and that the ADSB-MSSR (for b =1) is about 0.01 NM greater than this. In this plot format, however, the relationships among the various parameters with range from the MSSR are clear. For example, the flat part of the reference MSSR plot below 20 NM shows the MSSR accuracy is limited by the SSWG assumed along-range accuracy; the effect of increasing residual azimuth bias values on the MSSR-ADSB SEP beyond 20 NM is shown by the increasing values of b in the plot. The effect of higher ADS-B accuracy is shown in Figure 4-7 where all the values are unchanged except for NACp = 8.





MSSR Range, NM

Figure 4-7. In-trail ADSB-MSSR (or Orthogonal ADSB Merge w/MSSR) SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR-MSSR and ADSB-ADSB SEPs for NACp = 8 and µd = 0.

Similar plots for parallel track and a 90 degree MSSR merge with ADS-B scenarios are show in Figures 4-8 and 4-9.



MSSR Range, NM

Figure 4-8. Parallel-track ADSB-MSSR SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR-MSSR and ADSB-ADSB SEPs for NACp = 8

 $\sigma v = 5 \sigma d = 0.15$ 

v = 300

t = 3

 $\mu d := 0$ 



MSSR Range, NM

Figure 4-9. Orthogonal MSSR Merge w/ADSB SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR-MSSR and ADSB-ADSB SEPs for NACp = 8 and µd = 0

We see by comparing Figures 4-7 through 4-9 that the effect of residual azimuth bias is similar in each separation scenario with the MSSR-ADSB SEP with b = 1 ACP about equal to the reference MSSR-MSSR SEP at a range of approximately 30 NM. This cross-over reference range could be reduced slightly if the residual azimuth bias could be reduced to b = 0 ACPs.

#### 4.2.2 Residual Time Bias Uncertainty

ADSB and registration

 $\underline{\sigma}\underline{g}\underline{n} := \sigma g 8$ 

As already discussed, a residual registration time bias introduces an along-track bias error in the reported position. This comparative effect is seen in the next two plots which are similar to Figures 4-7 and 4-9 except the time bias,  $\mu$ d, is now assumed to be 0.3 seconds. Figure 4-10 shows the residual time error can have a noticeable effect on the in-trail or 90 degree ADS-B merge scenarios, but has no effect on the 90 degree MSSR merge as shown in Figure 4-11.

Parallel track scenarios, as shown in Figure 4-8, are insensitive to time bias since the along-track position error is perpendicular to the separation direction.



MSSR Range, NM

Figure 4-10. In-trail ADSB-MSSR (or Orthogonal ADSB Merge w/MSSR) SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR-MSSR and ADSB-ADSB SEPs for NACp = 8 and Indicated µd = 0.3 sec

 $\sigma v = 5 \sigma d = 0.15$ 



 $\mu d := 0.3 t = 3$ 

 $\sigma g n := \sigma g 8$ 

v = 300

ADSB and registration

MSSR Range, NM

Figure 4-11. Orthogonal MSSR Merge w/ADSB SEP for Several Residual Az Bias Values, b ACPs, Compared With MSSR-MSSR and ADSB-ADSB SEPs for NACp = 8 and µd = 0.3 sec

#### 4.3 Summary of ADS-B to MSSR No-Fault SEP Results

Figure 4-1 for basic NACp = 7 shows requirements for ADS-B to TMA MSSR separation are more stringent than for ADS-B to ADS-B separation due to the assumed residual bias error uncertainty when MSSR estimates are converted to WGS-84 coordinates. This effect is illustrated more sharply in the several separation scenarios comparing NACp = 7 with NACp =8 in the TMA Figures 4-2 and 4-3. While NACp = 8 satisfies most TMA scenarios, even this accuracy does not quite compensate for the assumed residual bias error in the in-trail and ADS-B merge cases; the comparative excess error of 0.03 NM should have no operation impact, however, and the risk of lower availability associated with a higher NACp value may not warrant this tighter requirement. Residual azimuth bias uncertainty has a greater effect on MSSR-ADSB SEP with increasing range, but the effect is fairly consistent with different separation scenarios (Figures 4-7 through 4-9). In-trail ADSB-MSSR and 90 degree ADS-B merge SEP scenarios are most sensitive to the residual time bias uncertainty (Figure 4-10).

Comparisons of SEP for NACp = 6 and NACp = 7 for the different E-R scenarios are shown in Figure 4-4 and 4-5. Figure 4-5 shows NACp = 7 meets E-R ADS-B to MSSR separation requirements.

# 5 Fault Condition SEP

Acceptance of an erroneous ADS-B reported position based on an undetected navigation source error is the critical issue in defining ADS-B requirements for safe ATC separation of aircraft. This is especially so if the same navigation source used for aircraft navigation is used as the input for ADS-B based surveillance. The effect of an undetected GPS fault on the surveillance separation error depends upon the satellites and their geometry used in the position determination by each aircraft of a nearby pair, as well as the orientation of the separation between aircraft relative to the satellite set geometries. For example, if both aircraft use the same satellite sets in their GPS position solutions and one the satellites experiences a fault condition, the estimated positions for each aircraft experience the same error, and the relative horizontal separation is unchanged. The following considers the more conservative baseline case of a single aircraft fault condition with the resulting error in the direction of the other aircraft of the pair. The more unusual case of both aircraft experiencing a fault with the resulting errors each in the direction of the other aircraft is examined in Section 5.2.

Although the same RFG MSSR reference ranges of 33 NM and 60 NM are used in the comparative assessment, as long as maximum ADS-B position errors in a fault condition (and any separation loss during the required time to fault detection) are small in terms of the relative minimum separation, and these fault conditions are rare, the comparative results should assure acceptable operational capability over the whole separation service volume of coverage.

#### 5.1 One Aircraft of a Pair in Fault Condition

Assurance (at a 99.9 percent level) that a Receiver Autonomous Integrity Monitoring (RAIM) monitored GPS fault condition does not result in a horizontal position error greater than Rc NM without an alert is conveyed in the ADS-B quantization of Rc as a NIC value. These integrity encoded Rc (or Horizontal Protection Level [HPL]) values are given below with an example for Rc = 0.6 NM, or a NIC = 6.9

 $<sup>^{9}</sup>$  A fault containment value, Rc = 0.6 NM, is used in the following examples to illustrate fault condition behavior better than would a lower value of NIC = 7 which is required for some TMA parallel approach operations not considered here.

# Select a RAIM Horizontal Protection Level, Rc in NM:

Rc := 0.6  $NIC(Rc) := if(Rc \le 0.012, 10, NIC(Rc))$   $MIC(Rc) := if(0.012 < Rc \le 0.04, 9, NIC(Rc))$   $MIC(Rc) := if(0.04 < Rc \le 0.1, 8, NIC(Rc))$   $MIC(Rc) := if(0.1 < Rc \le 0.2, 7, NIC(Rc))$   $MIC(Rc) := if(0.2 < Rc \le 0.6, 6, NIC(Rc))$   $MIC(Rc) := if(0.6 < Rc \le 1, 5, NIC(Rc))$   $MIC(Rc) := if(1 < Rc \le 2, 4, NIC(Rc))$   $MIC(Rc) := if(2 < Rc \le 4, 3, NIC(Rc))$   $MIC(Rc) := if(4 < Rc \le 8, 2, NIC(Rc))$   $MIC(Rc) := if(8 < Rc \le 20, 1, NIC(Rc))$   $MIC(Rc) := if(8 < Rc \le 20, 1, NIC(Rc))$ 

Details on the relationship of the ratio of the GPS error at the time of detection to the containment radius, Rc, are given in reference [10]. The results are summarized below for three snapshots of the fault condition horizontal error with associated probabilities of missed detection:

- Rc = HPL (encoded as NIC(Rc) by ADS-B) with pmd(Rc) = 0.001
- $Rt = 0.59 \times Rc$  (equal to the detection Thd) with pmd(Rt) = 0.5
- Rn = 0.27 x Rc (representative of errors < Th) with pmd(Rn) = 0.99

The RAIM model referenced above also shows that the standard deviation of the fault bias error is given by  $\sigma f = Rc/7.47$ . With these results, the expected ADS-B to ADS-B SEP, PFaa, given a GPS horizontal fault bias condition, Rf, for one aircraft of a pair in proximity becomes:

$$PFaa(Es, \sigma f, \sigma g, Rf) \coloneqq pmd(Rf) \cdot \int_{-\infty}^{\infty} pg(y, \sigma g) \cdot \int_{-\infty}^{y} pg[(\xi - Es + Rf), \sigma f] d\xi dy$$

Latency/extrapolation effects on position uncertainty are ignored in the fault condition case since their magnitudes are small in comparison to the fault bias error. Figure 5-1 compares the resulting fault condition surveillance separation error with the reference MSSR at the above three snapshot values for an assumed Rc = 0.6 NM, or NIC = 6 and a no-fault NACp = 8. The envelope of the fault condition curves represents bounds on the fault bias error at various possible times of fault detection.

These results have important operational implications since they indicate what the position errors on the controller display will be at the time of fault detection, given that an un-excluded GPS fault has occurred. Although a rare occurrence, these results show that in the event of an un-excluded fault, the probability, for example, is 0.001 that the ADS-B SEP is equal to or greater than 0.6 NM, whereas the routine RFG MSSR error at this same probability is only 0.4 NM, and the SSWG assumed MSSR model separation error is only about 0.2 NM. Although these differences are small in terms of a 3 NM separation, how the controller deals with these differential errors at the time of fault detection (in terms of their relationship to the minimum separation standard) must be considered in determining acceptable NIC requirements for any significant future reduction in minimum separation standards with ADS-B.



Separation Error, Es NM

Probability the fault condition ADSB-ADSB separation error is greater than or equal to Es Figure 5-1. Probability the NIC = 6 Fault Condition ADS B to ADS-B Separation Front's Greater Than MS SAMSSR and Great and chosen and chos

RAIM fault detection requirements may also be considered on the basis of the long term likelihood of the controller having to deal with any exceptional surveillance error condition associated with a GPS fault condition. This requirement is somewhat similar to an availability requirement on wide area radar coverage since it indicates the probability that the controller will have to manage the recovery from a non-normal surveillance situation. This long term surveillance separation error probability (termed Separation Reduction Probability [SRP]) here is the SEP, given a no-fault condition, plus the SEP, given a fault occurs, times the probability the fault occurs during the proximity monitoring exposure time. For a GPS fault rate of 10<sup>-4</sup>/hr, and an assumed 30 minute proximity exposure time.

Te := 0.5 PF := Te  $\cdot 10^{-4}$  PF = 5 × 10<sup>-5</sup> SRP(Es,  $\sigma f$ ,  $\sigma g$ , Rx) := (1 - PF)  $\cdot$  Paa(Es,  $\sigma g$ ) + PF  $\cdot$  PAaa(Es,  $\sigma f$ ,  $\sigma g$ , Rx)

This long term SRP likelihood is compared with the reference MSSR at R = 33 NM in Figure 5-2 for NACp = 8 and NIC = 6. Notice that although this long term measure of displayed separation error is greater than that expected with the SSWG MSSR error model, it is less than that expected with the RFG reference model.

These SEP and SRP comparisons at the fixed MSSR reference range of 33 NM are generalized in Appendix C where comparisons are made as a function of the range of applicability (RoA).

 $\sigma f = 0.08$  $\sigma g = 0.021$ Rc = 0.6NIC(Rc) = 6Fhe = 1 $pmd(Rc) = 9.676 \times 10^{-4}$  $Rt := 0.59 \cdot Rc$ pmd(Rt) = 0.49Rt = 0.354 $PF = 5 \times 10^{-5}$  $Rn := 0.27 \cdot Rc$ Rn = 0.162pmd(Rn) = 0.991 $\sigma r = 0.038$ R = 33 $\sigma 1 = 0.054$  $\sigma 2 = 0.27$  $\alpha = 0.05$ 



Separation Error, Es NM

Figure 5-2. Long Term Probability the NIC = 6 SRP ADS-B to ADS-B Separation Error is Greater Than or Equal to Es Compared With the RFG MSSR-MSSR Along-Range and Cross-Range Surveillance Separation Error at R = 33 NM

#### 5.2 Both Aircraft of a Pair in Fault Condition

A rare, but possible, event is for the aircraft to use different sets of satellites in their position solutions except for a fault condition satellite that is common to the position solutions of both aircraft [9]. We first consider the resulting fault condition horizontal errors in each aircraft to be independent in Figure 5-3. This is compared with the case when the two fault condition errors are dependent in Figure 5-4. Based on [11], the reasonable worst case assumption is that due to relative geometry this combined dependent error, Fhe, is 1.5 times the single aircraft fault condition error. Since these common fault conditions are so rare, only the conditional cases,

given the fault occurs, are shown; i.e., the long term separation reduction probability is not included.

As with Figure 5-1, the envelope of the fault condition snapshots in these two cases show how the separation error in a fault condition grows with time when the fault condition produces a ramp bias error. Comparison 0f Figures 5-3 and 5-4 indicate that the dual fault condition separation error is not sensitive to whether the errors in each aircraft are assumed to independent or reasonable worst case dependent. Although these dual fault conditions may be rare, an acceptable minimum NIC value must still assure that the total error, given a fault condition, plus some recovery margin is less than the minimum separation standard.

# 5.3 En route Fault Condition

En route integrity monitoring requirements definition follows the same process as that described above for the TMA except the reference RFG MSSR distributions at R = 60 NM include the residual bias effects assumed in Section 3.2.1 for the multi-MSSR environment. The resulting Figure 5-5 for the SEP, given a fault condition and an assumed NIC = 5, shows characteristics similar to those of the comparative TMA plot in Figure 5-1: ADS-B separation errors at the time of fault detection are greater than those of MSSR at likely probabilities, but rarer large errors are no more likely than those of MSSR. Similarly, the SRP for NIC = 5 shown in Figure 5-6 is no greater than that of the reference MSSR, even for the assumed reasonable worst case fault condition of simultaneous fault conditions in adjacent aircraft with a resulting aggregate loss factor, Fhe = 1.5.

## 5.4 ADS-B to MSSR Fault Condition Differences

Fault condition behavior of SEP and SRP for ADS-B to MSSR is similar to that of ADS-B to ADS-B except only the fault condition of the ADS-B aircraft is considered, i.e., there are no dual fault mode issues. Secondly, as with the multi-MSSR environment, the residual MSSR coordinate conversion bias uncertainty for the aircraft under MSSR surveillance must be included. These differences are negligible and examples are not given here.

# 5.5 Summary of Fault Condition Results

The conditional separation error at SEP = 0.001 for a NIC = 6 is about 0.1 NM greater than the reference MSSR at 33 NM (Figure 5-1), but the long term SRP is lower than the assumed no-fault MSSR reference (Figure 5-2). Assumed reasonable worst case dual fault conditions may exceed the reference MSSR separation error (Figure 5-3 and 5-4) but the occurrence of this condition is rare, and the difference of about 0.5 NM is small compared to the 3 NM minimum separation standard. A NIC = 5 shows similar properties for the E-R reference (Figure 5-5 and 5-6).

# For both aircraft in fault condition with independent bias errors Pf1 and Rf2 towards each other



Separation error, Es NM

Figure 5-3. Conditional Separation Error Probability vs. Separation Error for Different Combinations of NIC = 6 Fault Bias Errors on Each Aircraft at Time of Detection, Given that the Relative Geometry, Measurement Errors, and Satellite Solution Sets in Each Aircraft Produce these Independent Fault Condition Errors in Opposing Directions

# Prob undetected GPS faultHPL separation factorFhe := 1.5Pgf := 1PAaa(Es, $\sigma f, \sigma g, Rf$ ) := Pgf·pmd(Rf) $\int_{-\infty}^{\infty} pg(y, \sigma f) \cdot \int_{-\infty}^{y} pg[(\xi - Es + Fhe \cdot Rf), \sigma f] d\xi dy$

Value of Es with envelope of ADS-B fault conditions below MSSR-MSSR SEP (or some acceptable SEP level) is minimum acceptable separation error for ADS-B in fault condition w/ indicated NIC

Pgf = 1Fhe = 1.5Rc = 0.6NIC(Rc) = 6
$$\sigma f = 0.08$$
 $\sigma g = 0.021$ pmd(Rc) =  $9.676 \times 10^{-4}$ Rt :=  $0.59 \cdot Rc$ Rt =  $0.354$ pmd(Rt) =  $0.49$ Fhe =  $1.5$ Rn :=  $0.27 \cdot Rc$ Rn =  $0.162$ pmd(Rn) =  $0.991$ 

 $R = 33 \qquad \sigma 1 = 0.054 \qquad \sigma 2 = 0.27 \qquad \alpha = 0.05 \qquad \sigma r = 0.038$ 



Separation Error, Es NM

Probability the fault condition ADSB-ADSB separation error is greater than or equal to Es at several probabilities of missed detection of the fault bias error compared with the Assemssive MSSR a long-tange alloc coss range surveillance separation is morater NM. or Equal to Epapenveral Probabilities of Missed Detection of the Fault Bias Error Compared With the RFG MSSR-MSSR Along-Range and Cross-Range Surveillance Separation Error at R = 33 NM (Fhe = 1.5 assumed)



Separation Error, Es NM

Probability the fault condition ADSB-ADSB separation error is greater than or equal to Es Figure 5-5. Probability the NIC = 5 Fault Condition ADS-B to ADS-B Separation Error is Greater at several probabilities of missed detection of the fault bias error compared with the Than or Equal to Es at Several Probabilities of Missed Detection of the Fault Bias Error Compared with the WSSR-MSSR Along-range and Cross-range Surveillance Separation Error at R = 60 NM





Separation Error, Es NM

Figure 5-6. Probability the NIC = 5 and Fhe = 1.5 SRP ADS-B to ADS-B Separation Error is Greater Than or Equal to Es at Several Probabilities of Missed Detection of the Fault Bias Error Compared With the MSSR-MSSR Along-range and Cross-range Surveillance Separation Error at R = 60 NM

# 6 Summary

Terminal area requirements for ADS-B to ADS-B separation, including latency and other time registration issues, and for ADS-B to MSSR separation, including registration and latency issues, have been examined for the RFG reference range of 33 NM. An ADS-B NACp = 7 meets most ADS-B to ADS-B comparative accuracy needs, but NACp = 8 is required for the uncompensated merge scenario and to off-set residual bias uncertainty effects for ADS-B to MSSR separations. A similar process defines requirements for en route separations except the reference MSSR separation error at 60 NM also includes possible residual bias errors. A NACp = 7 meets all E-R scenarios. These requirements at a SEP = 0.05 (for both TMA and E-R) are independent of the SSWG or RFG reference MSSR models used, and are insensitive to reasonable assumptions made in time registration extrapolation of the ADS-B update.

Sensitivity of the 3 NM surveillance separation error to the GPS fault condition behavior on nearby aircraft were then examined for an assumed NIC = 6 and various possible fault condition combinations. The RFG MSSR model is the primary reference assumed in this case since its more accurate representation of MSSR tail error behavior better represents available measurements. Even under the reasonable worst case fault conditions, a NIC = 6 in the TMA, and a NIC = 5 in the E-R environment seem adequate to assure the comparative integrity of the position report.

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# Appendix A Position Estimate Error Distributions and the Separation Error Probability

Cross-range position estimate error distributions, or probability density functions, for the RFG MSSR azimuth angle error model and a distance R = 33 NM from the radar are shown in Figure A-1 for two targets separated by Sa = 0.2 NM.



Separation in NM

#### Figure A-1. Cross-range Error Distributions for MSSR Relative Position Estimates on Two Targets Separated by Sa NM at Range, R NM

These pdfs represent the aggregate error distribution of a large number of measurements. Notice that the difference in these position errors indicate the aircraft may actually be closer or further apart than the mean of the measured separation, Sa, due to uncertainty in the separate position estimates. The sensor characterization questions are then, "What is the distribution of the separation errors based on the difference in these position estimates, and what is the probability the separation error is no greater than a certain value?"

The risk assessment question when the sensor provides surveillance data on adjacent aircraft with unknown separation is then, "What displayed average separation assures safe separation for a certain position measurement error pdf?"

The sensor assessment concept may be developed from the point of view of the separation measurement error distribution for the difference in position estimates on two aircraft separated by Sa. Figure A-2 illustrates that the difference in the pdfs for two position estimates, pm, separated by Sa is the convolution of the two position estimate pdfs.

Convolution of MSSR pdfs, pm, separated by Sa is the MSSR-MSSR separation error pdf, pse:



Figure A-2. Cross-range Separated MSSR pdfs, pm, Compared With Separation Error pdf, pse, Which is the Difference (or Convolution) of these Position Estimation Errors

The cumulative probability, Cse, of the convolution of the position error pdfs, pse, given in Figure A-2 is the probability the MSSR separation estimate is less than or equal to y when the separation is Sa. This is illustrated in Figure A-3.



Figure A-3. Derivation of the Cumulative Probability the Separation Error, Cse ≤ y, When the Separation is Sa

Interchanging the order of integration gives the equivalent cumulative probability result shown as Csp in Figure A-4.



Figure A-4. Order of Integration Changed in Computing the Cumulative Separation Error Probability When the Separation is Sa

From this result, the normalized separation error probability (or alternately, the surveillance separation error) is derived in Figure A-5 in terms of the separation error, Es = Sa - y.<sup>10</sup> The separation error probability, SEP, is then the probability the surveillance measured separation error is Es, or greater.

<sup>&</sup>lt;sup>10</sup> This formulation is due to Bruce DeCleene (FAA AIR-130)



Figure A-5. Normalization of the Separation Error pdf to the Separation, Sa, and Use of the Separation Error, Es, to Yield the Normalized Separation Error Probability, SEP

An alternate path from the cumulative separation error relationship given in Figure A-4 leads to the previously used conditional risk measure, the close approach probability described in Figure A-6. Since the SEP formulation is independent of the separation and aircraft dimensions, it has been found to be a more flexible way to isolate and evaluate surveillance error characteristics.

The ICAO close approach probability (CAP) concept is a conditional risk measure, given that the aircraft are in close proximity. It provides an estimate of the probability that the aircraft are actually within +/- Aw, of each other when the displayed separation is the mean estimate, So. The aircraft width, Aw is 0.033 NM in the following examples.

The CAP is in this case, for values:  $A_W := 0.033$  R = 33  $S_0 = 0.8$ 

 $CAP(So, Aw, R) := \int_{-\infty}^{\infty} pm(y, R) \cdot \int_{y-Aw}^{y+Aw} pm[(\xi - So), R] d\xi dy \qquad CAP(So, Aw, R) = 4.801 \times 10^{-7}$ 

When the slopes of the pdfs are low over the interval, +/- Aw, this is approximated by the ICAO form used previously in the RFG NRA Appendix E:

$$\operatorname{Cap}(\operatorname{So},\operatorname{Aw},\operatorname{R}) \coloneqq 2 \cdot \operatorname{Aw} \cdot \int_{-\infty}^{\infty} \operatorname{pm}(\operatorname{y},\operatorname{R}) \cdot \operatorname{pm}[(\operatorname{y} - \operatorname{So}),\operatorname{R}] \, \mathrm{dy} \qquad \operatorname{Cap}(\operatorname{So},\operatorname{Aw},\operatorname{R}) = 4.581 \times 10^{-7}$$

Figure A-6. Relationship of the Conditional CAP to the Separation Error Distribution, Given a Separation, So

### **Appendix B MSSR Reference Sensitivity Examination**

The terminal area SSWG MSSR cross-range reference 95 percent separation error as a function of range for an azimuth angle standard deviation of 0.072 degrees is shown as the sloping dotted line in Figure B-1. This line assumes the position estimates for nearby aircraft are made in time synchronization. While this assumption is always true for in-trail aircraft on a radial trajectory, how good is this assumption for in-trail orbital trajectories separated by s = 3 NM moving at a velocity, v = 320 kts, a distance R NM from the radar with a scan period, Ts = 5 seconds? Any difference in dwell times on the leading and trailing aircraft in this case either adds or subtracts from the SEP initially determined by the assumed time synchronized position uncertainty estimates also shown in Figure B-1.

#### MSSR reference SEP sensitivity examination for in-trail path and SSWG model:

x-range 95% position SE with time synchronized dwells: SMx(R)

Time difference error in orbital in-trail separation, s, updates for range, R, velocity, v, and scan period, Ts sec:

Total x-rng SEP, TMx:  $TMx(R,s,v) \coloneqq SMx(R) + esx(R,s,v)$   $TMm(R,s,v) \coloneqq SMx(R) - esx(R,s,v)$ Time difference for radial intrail updates is zero, thus, SSWG 95% SE is:  $SMr(\sigma Mr) = 0.052$ For basic ADSB-ADSB SEP:  $SEn(\sigma gn) \coloneqq 1.65 \cdot \sqrt{2 \cdot \sigma gn^2}$   $SEn(\sigma g8) = 0.049$  SMx(17) = 0.05

 $SMx(33) = 0.098 \quad SEn(\sigma g7) = 0.096 \quad TMx(33,s,v) = 0.104 \quad TMm(33,s,v) = 0.091 \quad SMr(\sigma Mr) = 0.052 \quad SMr) = 0.052 \quad SMr(\sigma Mr) = 0.052 \quad SMr) = 0.052 \quad SMr(\sigma Mr) = 0.052 \quad SMr) = 0.052$ 



MSSR range, R NM

Figure B-1. TMA Reference SSWG MSSR Total Cross-range 95% SEP Bounds vs. Range Compared With Cross-range and Along-range Position Only SEPs for s = 3 NM In-trail Separation

As shown here, the dwell time difference is insignificant for our TMA reference range of R =33 NM where the SSWG cross-range SEP is equal to the basic ADS-B to ADS-B SEP = 0.097 NM for NACp = 7. At shorter ranges, however, although the NACp = 8 SEP equals the SSWG along-range SEP = 0.05 NM and the time synchronized cross-range SEP at 17 NM. the actual limits on MSSR cross-range SEP due to dwell time differences can be significant for the assumed conditions.

For en route environments with coverage provided by multiple MSSRs, the relative bias errors among the different sensors must also be considered in describing the reference MSSR SEP. Figure B-2 assumes the relative bias errors are zero. This represents the use of a single radar for separation, or be very close to the low residual bias errors achievable with some multi-sensor tracker implementations.



ACP = 0

 $\delta = 0$ 

1×10<sup>-</sup>

1×10

0

With +/- ACP 95% residual az bias uncertainty, and +/- d meter residual range bias uncertainty:

Separation error, Es NM

Es

à

0.6

0.8

ስ

0.4

Figure B-2. Probability the En Route MSSR-MSSR Cross-range Surveillance Separation Error at R NM With Residual Bias Uncertainty Equal to Zero is Equal to or Greater Than Es, Compared With NACp = 7 and 6 No-fault ADS-B to ADS-B Separation Error Probabilities (Without Extrapolation / Latency Effects)

0.2

Sensitivity of this assumed time synchronized assumption for the position estimates is derived in Figure B-3 for an en route in-trail separation of s = 5 NM, velocity v = 600 kts, distance from radar, R = 60 NM, and MSSR scan period Ts 12 seconds.

#### MSSR reference SEP sensitivity examination for in-trail path and SSWG model:

x-range 95% position SE with time synchronized dwells: SMx(R) ACP = 0  $\delta = 0$ Time difference error in orbital in-trail separation, s, updates for range, R, velocity, v, and scan period, Ts sec:

Total x-rng SEP, TMx:  $TMx(R,s,v) \coloneqq SMx(R) + esx(R,s,v)$   $TMm(R,s,v) \coloneqq SMx(R) - esx(R,s,v)$ 

Time difference for radial intrail separation updates is zero, thus, SSWG 95% SE is: SMr(GMr)



MSSR range, R NM

#### Figure B-3. En Route Reference SSWG MSSR Total Cross-range 95% SEP Bounds vs. Range Compared With Cross-range and Along-range Position Only SEPs for s = 5 NM In-trail Separation and Zero Relative Bias Errors (ACP = 0 and δ = 0 meters)

Similar plots for assumed MSSR relative bias errors of ACP = 1 and  $\delta$  = 60 meters are shown in Figures B-4 and B-5.

With +/- ACP 95% residual az bias uncertainty, and +/- d meter residual range bias uncertainty: ACP = 1  $\delta = 60$ 

 $R = 60 \quad \sigma 1 = 0.07 \quad \sigma 2 = 0.274 \quad \alpha = 0.05 \qquad \sigma \varphi = 0.081 \qquad q = 8 \qquad \sigma g 7 = 0.041 \qquad \sigma g 6 = 0.123$ 



Separation error, Es NM

Figure B-4. Probability the En Route MSSR-MSSR Cross-range Surveillance Separation Error at R NM With Residual Bias Uncertainty ACP = 1 and  $\delta$  = 60 Meters is Equal to or Greater Than Es, Compared With NACp = 7 and 6 No-fault ADS-B to ADS-B Separation Error Probabilities (Without Extrapolation / Latency Effects)
$SMr(\sigma Mr) = 0.106$ 

#### MSSR reference SEP sensitivity examination for in-trail path and SSWG model:

x-range 95% position SE with time synchronized dwells: SMx(R) ACP = 1  $\delta = 60$ Time difference error in orbital in-trail separation, s, updates for range, R, velocity, v, and scan period, Ts sec:

$$Ts := 12 \quad \underset{k \to \infty}{\text{sigma}} = 5 \qquad \underset{k \to \infty}{\text{min}} = 600 \qquad \qquad \underset{k \to \infty}{\text{esx}(R, s, v)} := \frac{Ts}{2 \cdot \pi} \cdot \operatorname{atan}\left(\frac{s}{R}\right) \cdot \frac{v}{3600} \qquad \qquad \underset{k \to \infty}{\text{Ris}} = 5, 6..200$$

Total x-rng SEP, TMx:  $TMx(R,s,v) \coloneqq SMx(R) + esx(R,s,v)$   $TMm(R,s,v) \coloneqq SMx(R) - esx(R,s,v)$ 

Time difference for radial intrail separation updates is zero, thus, SSWG 95% SE is: SMr(GMr)

TMx(60, s, v) = 0.235

TMm(60, s, v) = 0.182

SMx(60) = 0.209



MSSR range, R NM

#### Figure B-5. En Route Reference SSWG MSSR Total Cross-range 95% SEP Bounds vs. Range Compared With Cross-range and Along-range Position Only SEPs for s = 5 NM In-trail Separation and Relative Bias Errors, ACP = $1 \delta = 60$ Meters

Relative bias errors more typical of current FAA mosaic use of radars might be ACP = 2 and  $\delta$  = 120 meters. This impact on the MSSR reference SEP is illustrated in Figures B-6 and B-7.



 $R = 60 \quad \sigma 1 = 0.105 \quad \sigma 2 = 0.285 \quad \alpha = 0.05 \quad \sigma \varphi = 0.113 \quad q = 8 \quad \sigma g 7 = 0.041 \quad \sigma g 6 = 0.123$ 



Separation error, Es NM

Figure B-6. Probability the En Route MSSR-MSSR Cross-range Surveillance Separation Error at R NM With Residual Bias Uncertainty ACP = 2 and δ = 120 Meters is Equal to or Greater Than Es, Compared With NACp = 7 and 6 No-fault ADS-B to ADS-B Separation Error Probabilities (Without Extrapolation / Latency Effects)

#### MSSR reference SEP sensitivity examination for in-trail path and SSWG model:

x-range 95% position SE with time synchronized dwells: SMx(R) ACP = 2  $\delta$  = 120 Time difference error in orbital in-trail separation, s, updates for range, R, velocity, v, and scan period, Ts sec:

 $\mathsf{Total x-rng SEP, TMx: } \mathsf{TMx}(R,s,v) \coloneqq \mathsf{SMx}(R) + \mathsf{esx}(R,s,v) \qquad \mathsf{TMm}(R,s,v) \coloneqq \mathsf{SMx}(R) - \mathsf{esx}(R,s,v)$ 

Time difference for radial intrail separation updates is zero, thus, SSWG 95% SE is: SMr(GMr)



MSSR range, R NM

#### Figure B-7. En Route Reference SSWG MSSR Total Cross-range 95% SEP Bounds vs. Range Compared With Cross-range and Along-range Position Only SEPs for s = 5 NM In-trail Separation and Relative Bias Errors, ACP = $2 \delta$ = 120 Meters

These results are summarized in Table B-1 below. The TMA single sensor mode MSSR has no relative bias error, and the effect of the slight difference in dwell times for the 3 NM separation and 320 kt velocity have no effect on the choice of the SSWG reference. A value of SMx = 0.10 NM is used as the TMA reference in Figure B-1 and subsequent plots. The assumed residual bias errors for the E-R MSSR have a progressively greater effect, however. In recognition of the fact that bias errors must be considered in multi-senor environments, but without assuming they are completely removed, we will assume values of ACP = 1 and  $\delta$  = 60 meters for our examinations. This results in SMx = 0.21 NM as shown in Figure B-6 and subsequent plots. As shown in Table B-1, dwell time differences produce latency related errors of only +/- 0.03 NM around this value.

Op Area/and Ref Range R NM	Bias Errors ACP/ð meter	Position Only 95% SE SMx NM	Max Latency 95% SE TMx NM	Min Latency 95% SE TMm NM	Basic NACp Equivalent Value
TMA/33	0/0	0.10	0.10	0.10	7
E-R/60	0/0	0.18	0.20	0.15	7
E-R/60	1/60	0.21	0.24	0.18	7
E-R/60	2/120	0.28	0.31	0.26	6

 

 Table B-1. Summary of Sensitivity Results for Reference MSSR 95% Separation Error With Residual Bias Error and Dwell Time Latency

Similar comparative results are summarized in Figure B-8 with the position RFG MSSR model (i.e., no differential dwell time latency considerations) used as a reference rather than the SSWG model used in the previous examples. Notice here, as in Table B-1 for the SSWG MSSR reference, that a NACp = 6 value only meets the R = 60 NM range of applicability requirement when an assumed residual azimuth bias uncertainty is 2 ACP. Again, as in the above table, a NACp = 7 meets ROA requirements down to less than 40 NM even with zero bias.



Figure B-8. Comparison of ADS-B NAC Values With RFG MSSR Reference Model Combined With Several Assumed Residual Azimuth Bias Uncertainty Values

# Appendix CADS-B Fault Condition Separation Errors<br/>Compared with Two MSSR Error Models<br/>for Various References Range Assumptions

#### C.1 Introduction and MSSR Reference Models

Initial requirements for use of ADS-B by air traffic control are being determined on the basis of a comparative assessment with currently accepted MSSR surveillance characteristics. Separate models for this reference have been used by the FAA/SSWG and the RFG since previous measurements on MSSR error distributions, without the availability of GPS as a truth reference, left some doubt about the most accurate representation of this distribution. These models are summarized in Figure C-1. Since the MSSR cross-range error increases with range, and the ADS-B fault condition error limit is independent of range, selection of the most appropriate range of applicability (RoA) for this comparison is also of interest. The RFG and SSWG MSSR references are represented by how their no-fault errors determine the cross-range surveillance separation errors (SSE) vs. range for different values of separation error probability (SEP).





Position Uncertainty Error (NM)

#### MSSR reference models for along-range errors and cross-range errors at R = 33 NM.

#### Figure C-1. Comparison of RFG and SSWG Error Components at R = 33 NM

The alternate reference SEPs are shown in Figure C-2 at a range of 33 NM, the range at which the RFG along-range and cross-range errors are equal.



#### **Comparison of along-range SSEs** $\sigma re = 0.038$ $\sigma Mr = 0.022$



Comparison of cross-range SSEs at range, R R := 33



Separation Error, Es NM

Figure C-2. Comparison of RFG and SSWG Along-Range and Cross-Range SSEs at R = 33 NM

The cross-range SSEs in Figure C-2 can be plotted as a function of range by representing the curves with sample values of the separation error probability at SEP = 0.05,  $10^{-3}$ , and  $10^{-8}$ . Figure C-3 shows this range dependence for the RFG and SSWG models.



RoA NM RFG cross-range SEPs at 0.05, 10^-3, 10^-8 as a function of range



RoA NM

SSWG cross-range SEPs at 0.05, 10^-3, and 10^-8 as a function of range

Figure C-3. MSSR Cross-Range SSEs vs. RoA at SEP =  $10^{-8}$ ,  $10^{-3}$ , and 0.05

#### C.2 ADS-B Separation Errors in GPS Fault Condition

Separation errors for ADS-B, given a GPS fault condition, are now described at different probabilities of fault detection and then comparisons are made with the above no-fault MSSRs separation errors as a function of range of applicability (RoA). Figure C-4 plots the fault condition SEP for an ADS-B integrity containment radius of 0.6 NM (NIC = 6). The three snapshots represent the separation error at probabilities when: 1) the fault bias is Rt NM at the expected time of detection (pmd = 0.5), 2) the fault bias is Rn NM when the probability of missed detection is pmd = 0.99, and 3) when the fault error is Rc NM when pmd = 0.001 (the NIC containment radius).



Rc = 0.6 NIC(Rc) = 6 Rt = 0.354 Rn = 0.162  $\sigma f = 0.08$   $\sigma gn = 0.021$ 

Separation Error, Es NM

Probability the ADSB-ADSB separation error, given a fault condition, is greater than or equal to Es at several probabilities of missed detection of the fault bias error

Figure C-4. ADS-B SEP, Given a Fault With Rc = 0.6 NM, at pmd = 0.99, 0.5, and 0.001

Notice in the above for Rc = 0.6 NM that the separation error is Es = 0.48 NM when SEP = 0.05. A similar plot for a NIC = 7 containment radius of 0.2 NM has a value of Es = 0.16 NM. These fault condition SSEs are compared with the MSSR SSEs as a function of range

for the same value of SEP = 0.05 in Figure C-5. These comparisons have the following features:

- RFG and SSWG models have basically the same 95 percent SSE bounds
- ADSB fault condition SSE bounds for both NIC values exceed the reference MSSR errors at this SEP



MSSR RoA, NM

RFG and SSWG X-RNG SSEs compared with NIC= 6 and 7 SSEs at SEP = 0.05

#### Figure C-5. ADS-B Fault Condition SSEs for NIC = 6 and 7 Compared With RFG and SSWG MSSR SSEs at SEP = 0.05

Fault condition SSEs are compared with the MSSR SSEs as a function of range for the separation error probability,  $SEP = 10^{-3}$  in Figure C-6. These comparisons have the following features:

- RFG reference SSE exceeds SSWG bound at 0.999
- ADS-B fault condition SSE with NIC = 6 exceeds SSWG reference at this SEP, but is less than RFG x-range SSE at R = 60 NM
- NIC = 7 equals the SSWG x-range SSE at 40 NM



MSSR RoA,NM

#### RFG and SSWG X-RNG SSEs compared with NIC= 6 and 7 SSEs at SEP = 10^-3

## Figure C-6. ADS-B Fault Condition SSEs for NIC = 6 and 7 Compared With RFG and SSWG MSSR SSEs at SEP = 10<sup>-3</sup>

Fault condition SSEs are compared with the MSSR SSEs as a function of range for the separation error probability,  $SEP = 10^{-8}$  in Figure C-7. These comparisons have the following features:

- RFG reference exceeds SSWG x-range SSE bound for rare events (SEP = 10-8) due to higher tails.
- NIC = 6 fault condition SSE at this SEP equals the no-fault RFG SSE at half the maximum coverage range
- NIC = 7 fault condition SSE equals the SSWG reference SSE at about 35 NM at SEP = 10-8





Figure C-7. ADS-B Fault Condition SSEs for NIC = 6 and 7 Compared With RFG and SSWG MSSR SSEs at SEP = 10<sup>-8</sup>

#### C.3 Long Term Separation Reduction Probability

The long term likelihood of loss of ADS-B separation (including the weighted probability of a GPS fault), or the SRP, is shown in Figure C-8 with the SRP given at different probabilities of fault detection. Assumed values for this illustration are NACp = 8 and NIC = 6; a close proximity exposure time of 30 minutes; and an MSSR reference range of 33 NM.

## Long term probability the ADSB separation is reduced by Es, or the Separation Reduction Probability (SRP), given close proximity for Te hrs and GPS fault rate of 10E-4 /hr/user (compared with reference MSSR)

Te := 0.5 PF := Te  $\cdot 10^{-4}$  PF = 5  $\times 10^{-5}$ 

 $SRP(Es, \sigma f, \sigma gn, Rf) \coloneqq (1 - PF) \cdot Paa(Es, \sigma gn) + PF \cdot PFaa(Es, \sigma f, \sigma gn, Rf)$ 

$$R := 33$$
 Rc = 0.6



Separation Error, Es NM



Figure C-8. Long Term SRP for NIC = 6, NAC = 8 Compared With Reference MSSRs

Comparative plots of the separation reduction probability for NACp = 8 combined with NIC values of 6 or 7 at the three SRP sample points are given in the following three figures. Figure C-9 for the ADS-B SRP = 0.05 (including the time weighted probability of a GPS fault) shows the separation error is determined by the NACp= 8 condition at this SRP and is better than either MSSR reference SRP beyond about 15 NM.





RFG and SSWG X-RNG SEPs compared with NIC= 6 and 7 SRPs at Prob = 0.05

Figure C-9. SRP = 0.05 for NAC = 8, and NIC = 6 or 7 Compared With MSSRs

Similar comparative behavior is shown in Figure C-10 for  $SRP = 10^{-3}$  where the long term ADS-B behavior is still primarily determined by the NACp = 8 value.







#### Figure C-10. SRP = 0.001 for NIC = 6 and 7 Compared With Reference MSSRs

The long term fault containment limit differences in the NIC = 6 and 7 cases do show a difference at the SRP =  $10^{-8}$  sample point as shown in Figure C-11. The long term rare event SRP =  $10^{-8}$  for NIC = 6 is better than the RFG MSSR over most of its coverage in this case, and about equal to the SSWG SEP at a range of 60 NM.



MSSR RoA, NM

RFG and SSWG X-RNG SEPs compared with NIC= 6 and 7 SRPs at Prob = 10^-8

Figure C-11. SRP =  $10^{-8}$  for NIC = 6 and 7 Compared With Reference MSSRs

#### C.4 Summary of Results

- The RFG and SSWG MSSR cross-range errors are similar at SEPs above 0.02, but RFG tail errors increase SSEs at lower values.
- No-fault NACp = 8 SSE is better than both MSSRs.
- The most critical consideration in the use of ADS-B for ATC separation is the relative level of the reported position error at the time of GPS fault detection.
- Fault condition SSEs for NIC = 6 exceed the no-fault MSSR references at higher SEPs, but equal the RFG at half the coverage range of 60 NM. The NIC = 7 case is generally equal to the mid-range value of the SSWG reference at the lower probabilities of SEP.
- Long term SRP for ADS-B with NIC = 6 or 7 is generally better than the RFG MSSR, and as good as the SSWG at the maximum range for rare events. NIC = 7 is as good as the SSWG reference even in the very rare probability case.

## Appendix DADS-B Latency and Effects on DisplayedAircraft Separation Errors

#### **D.1** Introduction

Previous comparisons of ADS-B reported position accuracy assumed the asynchronously received reports would be time registered using the velocity in the state vector report. If both time registered ADS-B and radar position estimates incur similar delays before displayed for ATC separation, the relative comparison of the two surveillance sources can be made at the sensor output level as previously treated. The sensor comparison, however, must be made at the display level if time registration cannot be assumed. The issue addressed here then is: how bad can the uncompensated latency error be without degrading the quality of displayed separation of adjacent aircraft?

We consider two cases: 1) the reported positions from adjacent aircraft are each current (low latency), but arrive with an asynchronous time delay of td seconds, and 2) both position estimates are delayed for td seconds either before broadcast by each aircraft, or by the ground receive site. Recent data collection results verify that the error associated with report latency is an along-track bias, or translational error, in the estimated position of a moving aircraft – no cross-track error is introduced by latency for non-turning aircraft [2].

### **D.2** Analyses

#### **D.2.1** Aircraft Movement During the Latency Interval

Translational movement, d NM, over a period, td seconds, for a non-maneuvering aircraft at a speed, v knots, is plotted in Figure D-1 for latency times of td = 0.5 sec, td = 1.0 sec, and td = 2.0 sec. Notice that a speed of 270 kts results in a bias error of 0.15 NM for a delay, td = 2 sec. This same delay results in a bias of 0.3 NM with an en route speed of 540 kts.

As noted above, latency produces no cross-track position error unless the aircraft starts a turn just after broadcast of the delayed report. What effect would a possible turn have on the accuracy of this latency related position adjustment? An aircraft in a 3 degree/second turn produces a lateral acceleration of 1 g (about the maximum for ATC maneuvers) at a speed of about 360 kts. Cross track displacements (in feet) for this turn rate are shown in Figure D-2 as a function of speed for latency intervals of td = 1 sec and td = 2 sec. Here, the 360 kt 1 g turn has a cross-track displacement of only 64 feet, while higher en route speeds at this same value of td = 2 sec produce off-sets less than 90 feet. Reasonable turn rates do not then seem to preclude use of velocity to correct reported positions for latency values of several seconds.



Figure D-1. Along-Track Bias Error, d, for Uncompensated Delay, td at Speed, v



Figure D-2. Cross-track Displacement in Feet vs. Speed for 3 deg/sec Turn and Two Latency Values: td = 1 sec, and td = 2 sec

#### **D.2.2 Effect of Asynchronous Latency on Separation**

#### **D.2.2.1 Relative Track Separation Considerations**

Although time correction of the reported position is planned for future use of ADS-B, initial automation applications display the asynchronously received positions. The effect of asynchronous ADS-B update time, td, on displayed separation is most easily demonstrated by considering two in trail aircraft at the same speed separated by a distance, S. Assume the aircraft move a distance, d = v td, during the interval, td. When the trailing aircraft reports its position first, and the leading aircraft moves a distance, d, before reporting its position, the separation computed from these uncompensated asynchronous ADS-B reports is S + d and the normalized separation error is, et = (S + d) / S. Conversely, when the leading aircraft reports first, and the trailing aircraft reduces the separation by a distance, d, before reporting, the normalized separation error is then el = (S - d) / S. For example if d = 0.15 NM in Figure D-1 and S = 3 NM, et = 1.05 and el = 0.95. Since either event is equally likely, these normalized errors cluster about the actual separation, S.

Asynchronous delay effects are dependent on the relative track angle and separation configuration when the aircraft are not in trail. Geometry for two off-set aircraft with a subtended track angle,  $\alpha$ , separated by S, moving at speeds, v, is shown in Figure D-3 (a). Each aircraft moves a distance, d, along its flight path in time, td. If the report from aircraft 2 flying along the reference baseline is received first, and it then moves a distance, d, before the report from aircraft 1 is received when the actual separation is then S, the computed separation from these reports is  $(d^2 + S^2)^{1/2}$ . When S= 3 NM and d = 0.15 NM, this normalized error is about 1.001.

Now assume a report from aircraft 1 at position, P1, is received at time, t = to. If the asynchronous report from aircraft 2 is received at later time, to + td, when it is at P2, then the separation calculated from these uncompensated reports is D in the figure. However, aircraft 1 has moved to P3 during this interval, so the actual separation at the time of the uncompensated separation estimate is S in the figure. The normalized separation error based on uncompensated reports is then, D/S.

This normalized uncompensated separation error, D/S, is shown in Figure D-4 as a function of the subtended track angles (the relative flight directions) for an actual separation of S = 3 NM. The figure shows the error variation with  $\alpha$  for latency bias errors of d = 0.15 NM (along track bias for a speed of 270 kts and td = 2 sec), and for d = 0.3 NM (along track bias for en route speeds of 540 kts and td = 2 sec). No appreciable error occurs with parallel flight paths ( $\alpha$  = 0 degrees). As would be expected, the maximum effect is for  $\alpha$  = +/- 90 degrees (aircraft 1 is flying directly towards or away from the flight direction of aircraft 2). In the d = 0.15 NM case, the uncompensated separation estimate is 1.05 times the actual separation when one aircraft is flying towards the other. The same relative flight paths, if S = 3 NM were extended to the en route environment with higher speeds (d = 0.3 NM), produce a normalized error of D/S = 1.10.



a) Aircraft 2 reports t<sub>d</sub> sec after aircraft 1 reports.



#### b) Both aircraft reports delayed t<sub>d</sub> sec.

Figure D-3. Effects of Uncompensated Latency on Reported Separation, D, Compared With Actual Separation, S



## Figure D-4. Reported/Actual Separation for Actual Separation, S = 3 NM, as a Function of Subtended Track Angle for Two Latency Bias Values

Asynchronous times greater than two seconds produce roughly proportionally larger separation errors. Latency corrected positions using the state vector instantaneous velocity are required if these relative errors are unacceptable to ATC. As shown in [2], variations in latency, combined with the basic navigation position error distribution, produce corresponding variations in the along-track position errors.

#### **D.3** Effect of Common Latency on Separation

Common delay of both reports from adjacent aircraft can occur in the ADS-B ground receiver and communication infrastructure as well as by delayed broadcast of the navigation source data by the aircraft. It is the differential delay of ADS-B data relative the delay of baseline radar data that is of chief interest here, however. Assuming common delay of both radar and ADS-B reports by the ground communication and display systems, sources of common delay of interest then are those unique to ADS-B: delays in delivery of the navigation source data before broadcast by the aircraft, and any excessive processing delays by the ground receiver before state vector report output. These sources of uncompensated delay are of interest relative to the nominal 0.5 second ASR output delay (0.8 second for Mode-S), or 1.5 second output delay for ARSRs when ADS-B data are combined with radar estimates.

Since the effects of common delay on separation are sensitive to the separation geometry as well as the relative flight paths, we examine these effects for the three relative separation configurations shown in Figure D-3 (b). Error characteristics are bounded by assuming both aircraft are at the same speed, i.e., the displacements during the delay, d, are equal.

Case 1 for parallel flight paths ( $\alpha = 0$  deg) is easy. Separation at the time of measurement, D, is the same as actual separation after delay, S, when both aircraft have moved a distance, d. Note that if the displacements are unequal, the differential movement is represented by  $\alpha = 0$  deg in Figure D-4.

Case 2 for orthogonal flight paths ( $\alpha = 90 \text{ deg}$ ) might represent a condition experienced in the merge of two streams of traffic into a single stream. Aircraft 1 moves toward the initial position of aircraft 2 during the delay interval, while aircraft 2 moves along its flight direction so that the initial separation, D, becomes the actual separation, S, after common delay of both initial position reports. Figure D-5 shows the normalized separation error, D/S, as a function of the along-track bias for initial separations of D = 3 NM and 5 NM. See Figure D-1 to relate this bias, d, to delay, td, and speed, v. Notice that although both aircraft are moving in this case, the normalized errors for D = 3 NM with d = 0.15 NM and d = 0.3 NM do not differ greatly from the asynchronous case shown in Figure D-4 for the same values when  $\alpha = 90$  deg. Relative errors for the larger D = 5 NM initial separation are of course smaller as shown in Figure D-5.



Normalized separation error when both reports delayed

Figure D-5. Reported Separation / Actual Separation When Both Reports Are Delayed for Orthogonal Tracks ( $\alpha = 90$  deg)

Case 3 for closing parallel off-set flight paths ( $\alpha = 180 \text{ deg}$ ) represents a worst case example for the effect of common report latency on separation since the actual separation is reduced by the displacement of both aircraft during the delay interval. Initial separation is defined by the off-set, So, and the lateral separation, Do, so the initial separation is determined by the root sum squared of these values. Setting both values to 2.12 NM yields an initial separation of 3 NM; setting both values to 3.54 NM yields an initial separation of 5 NM. These separations for closing traffic shown in Figure D-6 are probably unrealistic in controlled airspace, but they do afford a comparative way to examine common mode latency under extreme conditions. The normalized error in this case for 3 NM initial separation is 1.07 when d = 0.15 NM, and 1.2 when d = 0.3 NM (our terminal area and en route bias error displacements for td = 2 sec). Normalized results for an initial separation of 5 NM are less.





Figure D-6. Reported Separation / Actual Separation When Both Reports are Delayed for Closing Off-set Tracks ( $\alpha = 180 \text{ deg}$ )

#### **D.4** Conclusions

Accurate compensation for report latency in the along-track direction is available by correcting the delayed position report using the reported state vector instantaneous velocity. Any cross-track errors due to aircraft turns experienced during this delay interval are small for the intervals of interest. Uncompensated ADS-B latency errors are of primary concern in relationship to similar latency effects with radar when data from these two surveillance sources are combined.

Uncompensated errors due to asynchronous delay of ADS-B reports, and errors due to common delay of both adjacent aircraft reports have been examined. Although resulting effects on displayed separation are somewhat sensitive to the relative flight path configuration, normalized separation errors less than 5% are expected for 3 NM separation in the terminal area for uncompensated delays; similar normalized errors for 3 NM separation at en route speeds are less than about 10%.

Common delays of adjacent aircraft ADS-B position data is of interest relative to similar sensor output delays with radar separation. Terminal area radar delays are 0.5 - 0.8 sec, en route radar delays are 1.5 sec.

#### **D.5** References

- [1] S. R. Jones, August 2005, Automatic Dependent Surveillance Broadcast Surveillance Requirements to Support Air Traffic Control Separation Standards, MTR 05W000008, The MITRE Corporation, McLean, VA.
- [2] S. R. Jones, March 3, 2006, "ADS-B Report Latency Effect on Track Errors," Report to Future Surveillance Separation Standards Working Group, The MITRE Corporation, McLean, VA.

### Appendix E Glossary

Acronym	Definition		
1090 ES	1090 MHz Extended Squitter		
ACP	Azimuth Change Point		
ADS-B	Automatic Dependent Surveillance-Broadcast		
ATC	Air Traffic Control		
CAP	Close Approach Probability		
<b>CD-2</b>	Common Digitizer-2		
E-R	En Route		
FAA	Federal Aviation Administration		
GPS	Global Positioning System		
HPL	Horizontal Protection Level		
ICAO	International Civil Aviation Organization		
MSSR	Monopulse Secondary Surveillance Radar		
NACp	Navigation Accuracy Category for Position		
NIC	Navigation Integrity Category		
NM	Nautical Miles		
pdf	Probability Density Function		
RAD	Radar-Controlled Airspace		
RAIM	Receiver Autonomous Integrity Monitoring		
RFG	Requirements Focus Group		
RoA	Range of Applicability		
RSS	Root Sum Squared		
SEP	Separation Error Probability		
SRP	Separation Reduction Probability		
SSE	Surveillance Separation Error		
SSWG	Separation Standards Working Group		
TMA	Terminal Area		
WAAS	Wide Area Augmentation System		