Modeling Departure Rate Controls for Strategic Flow Contingency Management

Shin-Lai (Alex) Tien[1], Christine Taylor[2], Craig Wanke[3]
The MITRE Corporation, McLean, VA, 22102

Ongoing research is currently focused on the need to improve the strategic traffic flow management decision making processes. The research effort in this paper is part of a greater research initiative aimed at developing quantitative analysis and design capabilities for flow contingency management. In our prior study, a flow-based queuing network model for air traffic management was proposed and various traffic management initiatives were modeled and tested with a realistic traffic and weather scenario. Under a flow-based environment, when there are multiple initiatives proposed that impact more than one departure flow, their interactions become convoluted. Therefore, this paper presents an enhancement to the model for departure controls in order to properly account for operational realities and provide better inputs for the queuing simulation. The goal is to approximate today's execution of departure delay programs in a flow-based model and focuses on correctly capturing the interaction effects when multiple initiatives are present. A mathematical model is formulated to determine time-varying departure rates for individual departure flows. Numerical experiments are conducted on a test network and a nation-wide case. When the multiple initiatives are solved in a coordinated fashion, there exists a tradeoff relationship between system-wide delay savings and balancing delays across multiple flows. The performance of the proposed model in addressing such a tradeoff relationship as well as operational realities is discussed and compared with a native apportioning algorithm.

I. Introduction

STRATEGIC traffic flow management (TFM) addresses predictions of significant capacity/demand imbalances two or more hours in the future. To address the needs of strategic TFM for the Next Generation Air Transportation System (NextGen), decision support tools are needed to quantify the predictions and the outcomes to decision makers. A component of the envisioned system is Flow Contingency Management (FCM), which aims to quantify the impact of predicted large-scale congestion, especially resulting from weather or other off-nominal events[1], and enable mitigation strategies to be evaluated prior to implementation. In Ref. 2 and 3, a concept of operations for FCM was proposed that integrates probabilistic weather-impact forecasts with a National Airspace System (NAS) queuing network model to aid decision makers in the development of contingency plans for multiple potential outcomes. The resulting contingency plans provide coordinated strategic control actions, including Traffic Management Initiatives (TMIs) that can mitigate the predicted weather-impact. Figure 1 depicts the associated framework for the proposed FCM concept.

In Figure 1, a critical component in the framework is the feedback loop for developing and evaluating contingency plans. This process requires that decision makers or automation specify a set of control actions and evaluate the impact of the proposed plan. Given that strategic controls must be enacted hours in advance, while significant weather and traffic uncertainty exist, it is essential that FCM captures the impact of these actions accurately. As such, a dynamic queuing network model for traffic simulation is constructed in Ref. 4 to simulate aggregated demand predictions and evaluate the impact of both the weather-impacted constraints and the controls imposed on the system in the proposed contingency plan.

[1] Senior Engineer, TFM Evolution and System Engineering, 7515 Colshire Drive, M/S N450.
[2] Lead Engineer, TFM Evolution and System Engineering, 7515 Colshire Drive, M/S N450, and AIAA member.
[3] Sr. Principal Engineer, TFM Evolution and System Engineering, 7515 Colshire Drive, M/S N450, and AIAA member.

In the queuing simulation, Traffic Management Initiatives (TMIs) that comprise a large set of the strategic controls enacted today and envisioned in the NextGen environment can be modeled as rate constraints that influence demand propagation through a route-based network. TMIs, such as a Ground Delay Program and an Airspace Flow Program, need to be translated into specific departure rates imposed at the origin airport to mitigate downstream congestion resulting in the accumulation of ground delay as opposed to air delay. In order to accurately evaluate the impact of the proposed TMIs and their effectiveness at mitigating downstream congestion, it is essential that the departure rates be computed correctly.

This paper proposes a mathematical model that effectively translates the execution of departure delay programs into a flow-based context and addresses the situation where multiple flow programs are planned either in a sequential or coordinated fashion. The proposed model shall be able to observe operational realities in current practice and determine the appropriate time-varying departure rates for individual departure flows. In the rest of the paper, we begin by discussing how each of the various controls can be captured within the FCM framework in Section II, as these controls are flight specific when used in an operational context. In Section III, we will review the queuing network model in Ref 4 and discuss the implementation issues and proposed strategies for defining these rates. In Section IV, a mathematical model will be formally introduced, and its properties will be discussed and illustrated via a numerical example in Sections V and VI. Conclusions and suggestions for future study are summarized in Section VII.

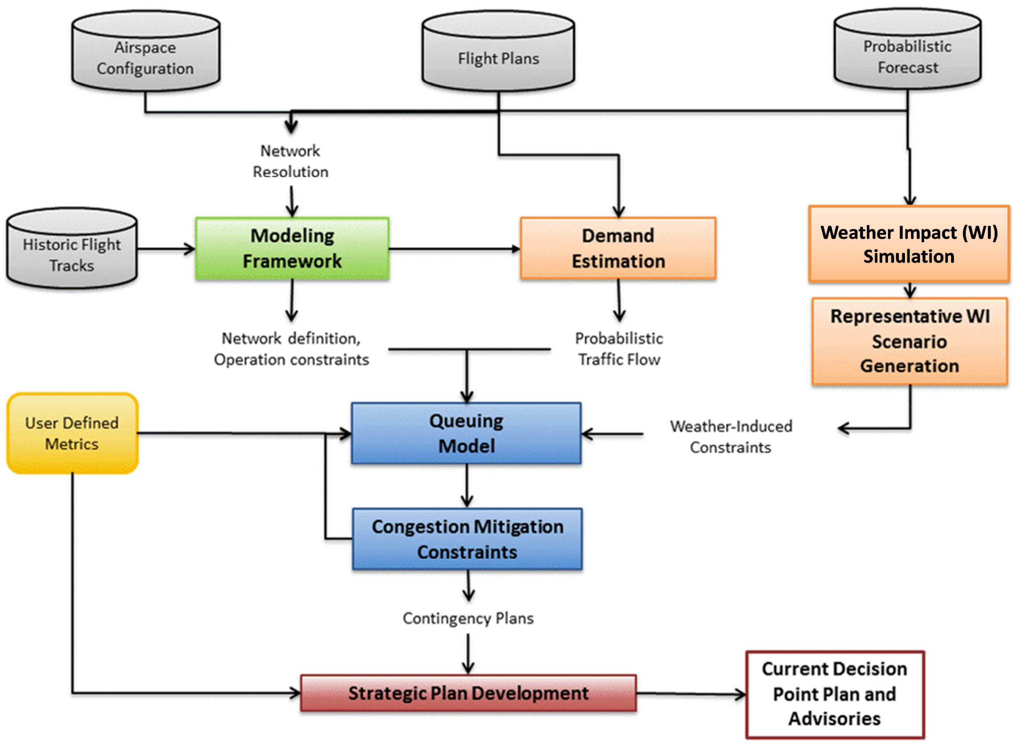

Figure 1. Flow Diagram of the FCM Framework

II. Background

In the current strategic planning process, NAS operations are periodically assessed, and potential events that create demand-capacity imbalances are addressed through the implementation of congestion-mitigating control actions. Two commonly utilized control actions, Ground Delay Programs (GDPs) and Airspace Flow Programs (AFPs), are taken a few hours ahead of the predicted event and impose delays on the ground before flights depart.

A GDP is a traffic management procedure that delays aircraft at their departure airport so as to resolve the future congestion at the arrival airport. The delay assigned to an aircraft at the departure airport is determined by the allowable arrival rate at the destination airport at a future time. Flights scheduled to arrive at a GDP airport are managed through the assignment of estimated departure clearance times (EDCTs) based on their arrival times using a slot allocation rule called Ration-By-Schedule (RBS). A flight may be exempt from a GDP if its departure airport is outside of the GDP scope, which may be defined by distance or other criteria. An Airspace Flow Program (AFP) is similar to a GDP, but only applies to flows intersecting a flow constrained area (FCA). A flight-specific EDCT for an AFP is assigned based upon the estimated transit time to the AFP constrained area. All the flights that will pass through a FCA are subject to an AFP, and thus no exempt flights are considered with an AFP. When a flight is subject to both an AFP and a GDP, under current practice a prioritization rule may be used to determine its EDCT, which will be summarized in Section II.A.

When a program (GDP or AFP) is initiated, decision makers need to specify several program parameters, i.e. the program location, start time, duration, and the scope that defines which flights are included in the program (only for GDP). Since such a program needs to be planned hours in advance before the predicted depart time, the initial decision on the program's parameters may be revised or the program itself may be revoked as the weather or traffic predictions evolve. In addition, as there might be multiple programs planned sequentially throughout the day, the implementation of a later program could interact with or build upon previous ones. Capturing the operational realities associated with program implementation, which will be reviewed in Section II.A, is essential in the NAS strategic planning process and should be addressed in the model development of the FCM.

We will review the existing studies and current practice of TMI modeling and implementations and their implication to the FCM modeling framework.

A. TMI Modeling and Implementation in a Flight-Based Model

Various aspects of designing TMIs (mainly GDPs) have been studied in recent literature. To examine program planning, Cook and Wood[5] employed a Monte-Carlo simulation approach that generates weather clearing times and scope in order to evaluate the impact on traffic if the GDP terminates before the weather clears. Mukherjee et al.[6] considered the uncertainty in airport capacity forecasts and proposed a framework that sequentially determines GDPs with updated weather information and then assigns arrival slots to individual flights. In order to improve the slot assignment mechanism, Ball et al.[7] have shown that a distance-based slot rationing method utilizes capacity more efficiently under uncertain weather. For coordinating multiple TMIs, Churchill et al.[8] proposed a deterministic integer programming model that solves the slot assignment for individual flights under multiple TMIs, and a Monte-Carlo experiment on capacity parameters was conducted in Churchill et al.[9] in order to assess the impact of stochastic capacity variation.

Recently, researchers have focused on the mutual exclusivity assumed between multiple TMIs. Barnhart et. al.[16] discussed the potential inefficiency in current practice resulting from the assignment of conflicting EDCTs when multiple TMIs are present and proposed a mathematical model that balances the defined fairness and efficiency metrics while assigning departure slots to individual flights. In this paper, we will utilize the term *controlling element* to specify the TMI that is predominant in determining the departure rate when multiple TMIs are in place.

According to Ref. 10 and discussions with Subject Matter Experts (SMEs), if a flight already has an EDCT from an earlier GDP, no EDCT will be issued for an AFP, and that flight will still appear as known demand to the AFP, even though it is controlled by a GDP. In practice, a flight that is subject to both an AFP and a GDP, the GDP will be the controlling element as delivery rates for airports are more precisely defined by the Airport Acceptance Rate (AAR) than the less-well-understood rates associated with constrained airspace regions.

Hence, depending on which TMI was implemented (or canceled) first, the EDCTs may be managed differently. We have summarized the following possibilities:

- If the GDP is implemented first, the AFP will not change the flight's GDP-assigned EDCT and the controlling element will remain the GDP.
- If the AFP is implemented first, the GDP will attempt to use the AFP-assigned EDCT and will only add additional delay if it cannot make this flight fit into the GDP program rate within system-defined parameters. If a new EDCT is assigned, the controlling element for this flight will change to the GDP and the flight will participate in all GDP revisions and schedule adjustment processes. Attempts to maintain the AFP-assigned EDCT promote consistency for the flight operator.
- If the GDP is canceled and the AFP remains in place, the EDCT will remain unchanged, but the controlling element will change from the GDP to the AFP and the flight will participate in all AFP revisions, compressions, etc.

Moreover, since the scope of a GDP defines whether a flight is under EDCT control, operational treatment between exempt and non-exempt flights needs to be distinguished. In practice, all flights destined to a GDP airport, whether originating from an exempt or a non-exempt airport, are assigned controlled times of arrival (CTA) and EDCTs. The reason is to attempt to increase the accuracy of the GDP delivery rate and to allow flight operators to substitute their flights from any airport with the any other of their flights. For an exempt flight, its CTA and an EDCT are assigned equal to its ETA and its scheduled wheels-off time.

B. Modeling TMIs in a Flow-Based Model

The above operational rules must be captured in the FCM queuing simulation in order to increase the accuracy of the NAS performance estimates and better assist the decision making process. As a strategic planning tool, FCM models demand and traffic propagation in a flow-based environment. As such, it is necessary that the flight-based TMIs be translated into flow-based controls in order to be captured in the FCM queuing model, described in detail in Section III.

For GDPs, the proposed approach requires that decision makers specify program parameters, i.e. GDP airport, program rate, start time, duration, and the scope. Given the specified parameters, the controlled departure rates at each airport are determined by a model that properly apportions the available GDP rate by demand. These controlled departure rates are provided as inputs to the FCM queuing model in order to simulate the impact of the GDP on temporal and spatial traffic distributions along with the impact of other en route constraints (e.g. mile-in-trail restrictions, sector congestion, etc.). After the simulation, decision makers would have the opportunity to analyze the arrival rate (where actual transits and en-route delays are included) to determine if an adjustment in the GDP rate is desirable.

For AFPs, the decision maker will determine the AFP rate, flow constrained area (FCA), times, and directionality. Essentially, the departure rates at each airport to the flow-constrained regions would be determined in a similar manner as specified for the GDP.

In the following sections, the FCM queuing model will be reviewed, and the challenges of modeling the real-world operations will be further discussed. A mathematical model will then be proposed to determine the controlled departure rates while addressing the interactions among multiple TMIs.

III. A Flow-Based Air Traffic Management Model

To motivate the enhanced TMI modeling capability proposed in this paper, the network structure and queuing model utilized in FCM are briefly introduced in this section. More details can be found in Ref. 4 and 11.

A. FCM Network Structure

The FCM framework employs the concept of heterogeneous network resolution[11]. Specifically, nodes in the FCM network represent NAS resources, albeit at different levels of aggregation, where the selection of the appropriate level of aggregation is determined by the modeling fidelity necessary to capture the control actions.

Within the area of control, the origin and destination nodes are defined to represent individual airports, where the flow enters and leaves the network, respectively. The origin and destination nodes are connected by a series of sector boundary nodes that represent directional crossings (i.e. between a pair of sectors two nodes are defined, one for each direction of crossing flow). The arcs connecting the nodes are origin-destination (O-D) specific and are derived from an analysis of historic sector crossings, or more specifically historic sector triplets (sequence of upstream, current, and downstream sectors), as illustrated in Figure 2. Using sector triplet data, network size can be reasonably limited by only representing realistic flow patterns across sectors, and the transit time through a sector can be captured more accurately.

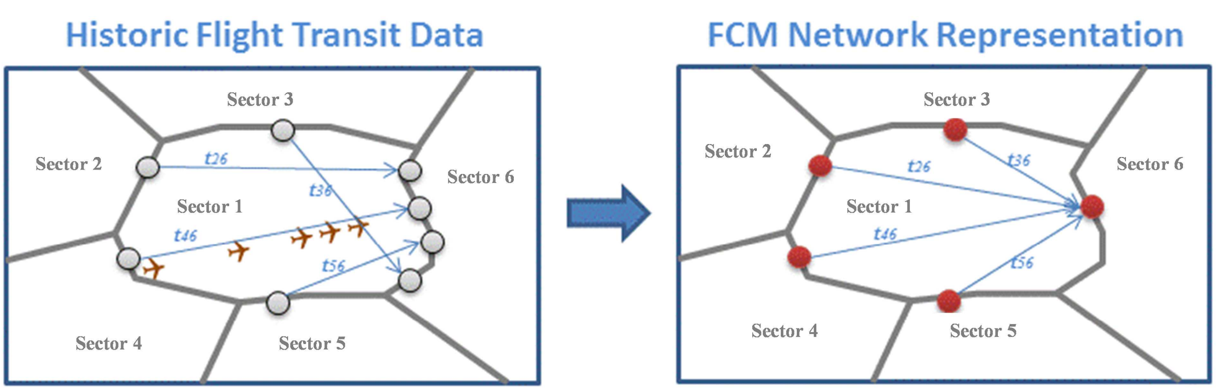

Figure 2. Network Representation of a Sector: Each node is associated with two sectors, representing a boundary crossing. Each arc is associated with a sector triplet (previous-current-next).

Outside of the area of control, the network model represents NAS resources as aggregated clusters of individual resources, and the origin and destination nodes represent multiple airports clustered together using heuristic clustering criteria[12]. The origin and destination nodes are connected by a series of Air Route Traffic Control Center (ARTCC) boundary nodes and ARTCC triplets. The associated demand between the O-D pair corresponds to the total demand between the airports represented in the clusters. As the multiple aggregation levels are simply represented as nodes and arcs within the network, an integrated modeling framework is developed that is computationally tractable yet provides the detail necessary to simulate and evaluate the desired flow impact.

Given that FCM is a strategic planning tool, a "route" in the FCM network would describe how an aggregate demand proceeds from an origin to a destination in a more aggregate sense, as opposed to defining a specific jet route, filed flight plan, or trajectory for an aircraft.

B. A Queuing Network Model for Air Traffic Management

Let us define a network $G = (\mathbf{N}, \mathbf{A})$, where $\mathbf{N}$ is the node set and $\mathbf{A}$ is the arc set. Without loss of generality, it is assumed that there is a super source node s that feeds the traffic into the system and a super sink node e where the traffic terminates. For each O-D pair, a set of candidate routes $\mathbf{R}_{od}$ are identified. A route here is defined by an origin node, a sequence of intermediate nodes, and a destination node. The rest of the notations of the queuing model are defined as follows:

f_{rji}^{t} = Inflow to node i from node j at time t on route $r \in \mathbf{R}_{od}$

g_{rij}^{t} = Outflow from node i to node j at time t on route $r \in \mathbf{R}_{od}$

N_{rij}^{t} = Service rate provided (allocated) on the arc from node i to j at time t for the flow with origin o and destination d on route $r \in \mathbf{R}_{od}$. In addition, $\bar{N}_{ij}^{t} = \sum_{r \in \mathbf{R}_{od}, o \in O, d \in D} N_{rij}^{t}$ is a shorthand notation of the service rate provided on the arc from node i to node j at time t

p_{rso}^{t} = Proportion of demand of a particular O-D pair assigned to route r at time t, carried by the arc between the super source s and an origin o

b_{rij}^{t} = Backlog at node j from node i at time t for the flow on route $r \in \mathbf{R}_{od}$

The queuing effect is modeled by dynamic stochastic service rates provided at each node. This formulation differentiates the service rate and backlogs at each queue by Origin-Destination-Route (O-D-R) triplets. Under a discrete-time approximation, the fundamental queuing phenomena are governed by the following functional relations:

$$g_{rij}^{t} = f_{rij}^{t+\bar{t}_{rij}} \qquad \text{for all } r \in \mathbf{R}_{od}, o \in \mathbf{O}, d \in \mathbf{D}, t \in \mathbf{T} \tag{3.1}$$

$$g_{rjl}^{t+1} = min\{b_{rij}^{t} + f_{rij}^{t+1}, M_{rij}^{t+1}\} \qquad \text{for all } r \in \mathbf{R}_{od}, i, j, l \text{ in succession on } r \tag{3.2}$$

$$b_{rij}^{t+1} = max\{0, b_{rij}^{t} + f_{rij}^{t+1} - M_{rij}^{t+1}\} \qquad \text{for all } r \in \mathbf{R}_{od}, i, j \text{ in succession on } r \tag{3.3}$$

$$f_{rso}^{t} = p_{rso}^{t} \times Demand(o, d, t) \qquad \text{for all } r \in \mathbf{R}_{od}, o \in \mathbf{O} \tag{3.4}$$

Eq. (3.1) is the flow conservation constraint which states that the flow entering arc *(i,j)* arrives at j after a nominal travel time. In particular, $\bar{t}_{rso} = 0$ is always true, and thus $f_{rso}^{t} = g_{rso}^{t}$. Eq. (3.2) states the outflow from j after being served at the queue of *j*, where M_{rji}^{t} is Poisson-distributed with a given service rate N_{rji}^{t} and could follow any type of distribution for stochastic queues. Eq. (3.3) describes the backlog formation at *j*. Both Eq. (3.2) and (3.3) describe the discrete-time approximation of a continuous stochastic queue. The service rate could also be deterministic, depending on the modeling needs for various TMI controls (as modeled in Ref. 4).

This discretization is tractable and operates as follows: For each queue at every unit time interval, the updated backlog is determined by the existing backlog (i.e. number of aircraft waiting in the queue), the inflow (number of aircraft approaching a boundary intersection point in a unit time interval), and the service rate provided (maximum number of aircraft that can be served in a unit time). If the sum of the existing backlog and inflow is larger than the service rate, the outflow will equal the service rate, and the updated backlog will be the difference; otherwise, the existing backlog and inflow will pass to the downstream, and the updated backlog is zero.

Eq. (3.4) describes the demand of an O-D pair split by route at an origin node. Each route has a proportion rate, which can be either specified by manual input or determined through optimization. The demand generation function itself should address the uncertainty at a strategic timeframe, and it can be estimated from historic data or modeled as a stochastic process, e.g. Poisson process.

C. Challenges for Modeling TMIs and Their Interactions

For the purpose of FCM, the queuing model must also simulate the implementation of congestion mitigation plans. A set of TMIs, such as ground delays, sector or flow controlled area rate restrictions, rerouting, or other necessary initiatives, may be proposed in a congestion mitigation plan to alleviate the congestion due to an imbalance between predicted capacity and predicted demand. The design of management actions or TMIs requires controlling the passage rate at either origin or sector boundary nodes, depending on TMI types. The set of mitigation controls proposed to manage the flow will result in changes to flow propagation.

Our prior study has demonstrated how individual TMIs would be implemented in the FCM queuing model. In Ref 4, the overall rate of a departure-controlled TMI, i.e. GDP or AFP, is allocated to affected flows via an apportioning mechanism prior to running the queuing simulation. This approach ensures the main role of the queuing simulation as an evaluation tool for traffic propagation under uncertainty. As such, the procedure to determine a controlled rate to individual flows is crucial to accurately simulate TMI's impact and should represent operational reality as much as possible.

A rate apportioning algorithm for GDP or AFP that observes the first-scheduled-first-serve principle in a flow-based sense is described as follows:

(FSFS Apportioning Algorithm)

- Step 0 – (Initialize) Define the program location, program start and end times, target program rate, and scope. Identify exempt and non-exempt flows (or routes), their initial demands, and their estimated transit times to the constrained resource. Set time t = program start time.
- Step 1 – At time t, do the following:
    - Determine the departure rates of the exempt flows by allocating the target program rate *<u>proportionally</u>* to the current demand plus any backlog remaining from the previous time step,, where the time of departure = (t – the estimated transit time to the constrained resource).
    - If the target rate is not met, apply the same process for the non-exempt flows by using the available rate.
    - Evaluate backlogs incurred at time t for all flows.
- Step 2 – If t = program end time, then stop; otherwise, set t = t +1 and go to Step 1.

This apportioning algorithm, however, is designed for a single constrained resource only, and therefore does not account for the situation when there are multiple flow control programs that affect the same departure flow. Although it can be applied sequentially to multiple programs, there may be cases where a later program has excess capacity to accommodate a flow that is subject to an earlier one. As such, the sequential application may result in an inefficient allocation of resources, and thus unnecessary backlogs.

In addition, from the literature and discussions with SMEs, it is understood that when both a GDP and an AFP are enacted at different timeframes for the same departure airport, their interactions become convoluted, especially when represented in a flow-based model. Specifically, there are cases where the interaction between GDPs and AFPs could affect the rate apportionment procedure:

1. *GDP before AFP*

If a GDP begins controlling departures at an origin node prior to the AFP departure rate constraint, the GDP constraint is the controlling element and the AFP can be effectively ignored in the FCM simulation. As such, the model will treat this situation as if only a GDP rate constraint was placed.

2. *AFP before GDP*

If an AFP departure rate constraint is in place prior to the GDP departure constraint, the goal is to maintain the AFP departure rate, if feasible. However, if that isn't possible, the flow will be subject to the GDP. As such, we propose the following process for capturing the interaction of AFPs and GDPs.

- *If the GDP rate is greater than the AFP rate* – For the case when an O-D pair is subjected to both controls, but the GDP control rate is greater than the AFP rate, we first process the AFP flow up until the AFP rate limit, and include additional demand as AFP-generated backlog. The remaining GDP rate (i.e. the GDP rate minus the AFP rate) will be used to process non-AFP flow and excess flow will be included as GDP-generated backlog.
- *If the GDP rate is less than the AFP rate* – For the case when an O-D pair is subjected to a GDP control rate that is less than the AFP rate, we process the AFP-destined demand first, up until the GDP rate. If additional AFP-destined demand exists it is included as backlog with the non-AFP-destined demand as GDP-generated

backlog. Thus, essentially, AFP-destined demand is processed in each time bin as requested, up to the allowable departure rate for that origin node (i.e. the GDP rate), but if the GDP rate modifies the departure time (i.e. limits the flow greater than the AFP rate), the flow is processed as delayed departure demand and operates solely under the GDP rate control.

- *GDP ends, AFP remains in place* – Once a GDP ends, we effectively revert to an AFP-only situation. If the AFP was put in place after the GDP, we need to begin controlling via an AFP (as we only controlled via a GDP prior to this event). If however, the AFP was put in place prior to the GDP, we need to include AFP-destined demand that is in excess of the AFP rate limit in the AFP-generated backlog.

Thus, to sufficiently capture real-world TMI impacts within FCM, a rate apportionment model is needed that captures the interactions between GDPs and AFPs appropriately.

IV. Modeling Departure Rate Controls

A. Problem Definition

As this paper aims to approximate the execution of GDP and AFP in the FCM context, the model proposed must address the operational realities when multiple flow programs are enacted, either in a sequential or coordinated fashion. In a flow-based modeling environment such as FCM, a departure flow may be subject to more than one traffic management initiative at different timeframes. The goal here is to convert overall program rates from various TMIs in a coordinated fashion into controlled rates for individual departure flows while mimicking operational realities to the extent possible. For the remainder of this paper, the term "TMI" refers to a GDP or an AFP, and the term "flow" represents a route defined in the FCM network that carries known, time-varying demand (obtained from a schedule or a demand forecast model). The departure rate of a flow is limited by a cap value that is determined after considering the effect of all the TMIs across the multiple departure flows.

To solve this rate control problem in a flow-based model, a time-state network representation is proposed. Each "state" represents a controlling TMI, and the sequence of "state" is set in the ascending order of the time a TMI is in effect at the departure airport. Figure 3 illustrates the network for a single departure flow, where each node represents a time-state incidence, and the horizontal and vertical links are directional and represent the transition between consecutive times and consecutive states, respectively.

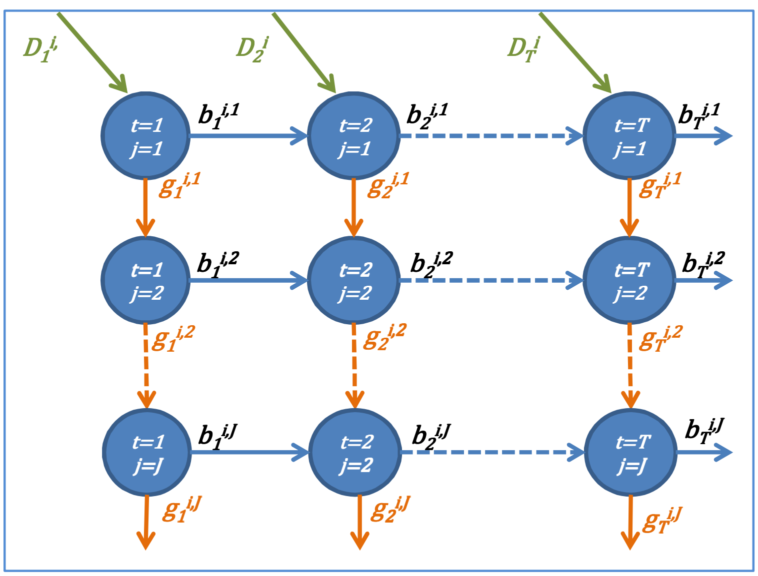

Figure 3. Time-State Network for Modeling Departure Rate Controls

With this network representation, departure demand can be seen as a flow commodity. Specifically, at each time period, the demand of the flow, D, enters the network via the first controlling TMI and travels along the network so that the first-scheduled-first-served policy is maintained. The backlog variable b associated with a horizontal link is the amount of demand passing from one period to the next and is considered delay accumulation caused by a controlling TMI. The outflow variable g associated with a vertical link is the amount of demand passing from one

TMI to the next at the same period and is considered the transfer of the controlling role between TMIs. The amount of demand exiting the last TMI would be the controlled flow rate to be set at that time period for the FCM queuing simulation. This representation would allow us to describe how demand is either delayed by a TMI or transferred to another one with the controlling role and to track backlogs and outflows varying across the planning horizon.

There are several inherited limitations from the network representation to be mentioned:

1. This network problem considers only the TMI impact on departure rates prior to simulation, so it does not take into account weather uncertainty or any enroute in-trail controls.
2. The demand of departure flows is treated as input. Any reroute, schedule uncertainty, or cancellation should be evaluated before generating the demand input for this model.
3. The transit time to a TMI is estimated under nominal conditions, and therefore does not capture en-route congestion or delay. For planning purposes in the strategic timeframe, the best estimate of transit time, for example, historically flown time adjusted for the wind forecast, is sufficient. The actual transit time may vary in the FCM simulation since weather impact and sector congestion are also simulated.
4. When a flow is no longer subject to any TMI, the model assumes that all the accumulated backlogs will turn into outflows in the next time period, and that the FCM queuing simulation should comply with the overall departure capacity at an airport and impose backlog as needed. In practice, decision makers would modify the program duration if significant demand is predicted near the end of a TMI.

B. Linear Programming Formulation

We formulate a linear programming (LP) model to solve this time-state network problem while jointly considering multiple rate restrictions and exemption incurred by TMIs. Without loss of generality, it is assumed that a TMI, either GDP or AFP, may have exempt and non-exempt flows, and all the flows that are involved in any of the TMIs must be included in the formulation.

Notations:

- Set of time period: $\{1, \ldots, T\}$.
- Set of departure flows that are involved in any of the TMIs: $\{1, \ldots, I\}$.
- Set of TMIs: $\{1, \ldots, J\}$.
- $\mathbf{J}_i$: Set of TMIs that controls flow *i*, where $\mathbf{J}_i \subseteq \{1, \ldots, J\}$.
- $\mathbf{F}_j$: Set of flows that is subject to TMI *j*, where $\mathbf{F}_j \subseteq \{1, \ldots, I\}$.
- $\mathbf{E}_j$: Set of flows that is subject to TMI *j* but is exempt from rate control, where $\mathbf{E}_j \subset \mathbf{F}_j$.
- d_t^i: Deterministic demand (given) of flow i at time t.
- $b_t^{i,j}$: Decision variable that represents backlog of flow i attributed to control j at time t.
- $g_t^{i,j}$: Decision variable that represents outflow of flow i from controlling TMI j to the next at time t.
- M_t^j: The overall program rate at time t at the location that TMI *j* takes place. This is specified by decision makers or via weather impact forecast.
- m_t^j: Decision variable that represents capacity insufficiency of control j at time t.
- $\bar{S}_j$: Control start time at the location of TMI *j*.
- $\bar{E}_j$: Control end time at the location of TMI *j*.
- $\bar{t}_{i,j}$: Constant flight time from flow *i* to the geographical location that TMI *j* takes place.
- $\bar{s}_{i,j} = \bar{S}_j - \bar{t}_{i,j}$: Local control start time of TMI *j* on flow *i*.
- $\bar{e}_{i,j} = \bar{E}_j - \bar{t}_{i,j}$: Local control end time of TMI *j* on flow *i*.
- $c_t^{i,j}$: cost coefficient for backlogs.
- ρ: cost coefficient for capacity insufficiency.
- ε : cost coefficient for proportional equity.

Minimize:

$$Objective\ Function\ 1 := \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{t=1}^{T} c_t^{i,j} b_t^{i,j} + \sum_{j=1}^{J} \sum_{t=1}^{T} \rho m_t^j \tag{4.1}$$

Subject to:

$$d_t^i + b_{t-1}^{i,j} = g_t^{i,j} + b_t^{i,j} \quad , for\ j \in \{1\},\ i \in \{1,\dots,I\},\ t \in \{1,\dots,T\}, \text{ and } b_0^{i,j} = 0 \tag{4.2a}$$

$$b_{t-1}^{i,j} + g_t^{i,j-1} = g_t^{i,j} + b_t^{i,j} \quad , for\ j \in \{2,\dots,J\},\ i \in \{1,\dots,I\},\ t \in \{1,\dots,T\}, \text{ and } b_0^{i,j} = 0,\ g_t^{i,0} = 0 \tag{4.2b}$$

$$\sum_{i \in \mathbf{F}_j \setminus \mathbf{E}_j} g_{t-\bar{t}_{i,j}}^{i,j} + \sum_{i \in \mathbf{E}_j} g_{t-\bar{t}_{i,j}}^{i,j} + m_{t-1}^j \le M_t^j + m_t^j \quad , for\ j \in \{1,\dots,J\}, t \in \{\bar{S}_j\ ,\dots,\bar{E}_j\ \} \tag{4.3}$$

$$g_t^{i,j}, b_t^{i,j}, m_t^j \ge 0 \tag{4.4}$$

The first term in the objective function (4.1) is the weighted sum of the backlog variables. Since demand and TMI rates are deterministic, the backlogs only represent ground delay. The cost coefficient $c_t^{i,j}$ may be defined to prioritize delays by flow, TMIs or time periods. It is set as follows:

$$c_t^{i,j} = \begin{cases} f(t,j), \text{for } i \in \mathbf{F}_j \setminus \mathbf{E}_j, t \in \{\bar{s}_{i,j} \dots \bar{e}_{i,j}\}, \text{and } j \in \{1,\dots,J\} \\ \qquad \delta \quad , \text{otherwise.} \end{cases} \tag{4.5}$$

For flow i subject to and not exempt from TMI j and for the time periods inside the local control time, i.e. $\bar{s}_{i,j} \le t \le \ \bar{e}_{i,j}$, the backlog cost coefficient could be defined by a function of time and TMI sequence $f(t,j)$. For the time periods outside of the local control time or for exempt flow and the flows not subject to a TMI, the backlog cost coefficient is set to a sufficiently large constant $\delta \gg f(t,j) \ge 0$ so that backlog will be much less favorably accumulated.

The second term in the objective function penalizes the situation when capacity is insufficient to handle the demand of exempt flows of a TMI. The cost coefficient ρ is chosen by the rule $f(t,j) \ll \rho \ll \delta$ so as to distinguish from the backlog cost and to show the favoritism among two cost items. The discussion of Constraint type (4.3) will provide more details.

Constraints (4.2a) and (4.2b) describe flow conservation on this time-state network. Demand enters the network and is processed by the first TMI and then the subsequent ones. Each node in Figure 3 represents an incidence of a TMI and a time period. At each node, the total inbound flow should equal the total outbound flow. The inbound flow consists of the backlog from the last period of the current TMI and the outflow from the previous TMI (or demand if there is no previous TMI) while the outbound flow consists of the backlog and the outflow of the current period. The optimization will determine the distribution of backlog and outflow based on the capacity limitation specified by constraint (4.3).

Constraint (4.3) is augmented from the inequality $\sum_{i \in \mathbf{F}_j} g_{t-\bar{t}_{i,j}}^{i,j} \le M_t^j$, which sets a limit on the total outflow across all OD pairs that are subject to the TMI and expected to arrive at in a fixed time period. This is the main coupling constraint that relates the networks of all the impacted flows under a TMI. The program rate M_t^j is seen as the capacity to be allocated by all the outflow variables. Ideally, when the sum of total outflows is over capacity, backlog accumulates, and the inequality before augmentation would be feasible at all times; however, there exists situations where the exempt outflows of a GDP exceeds the program rate for some time periods if the program rate and scope are not properly chosen. To ensure that all the exempt flows always receive priority and have no backlog accumulation, i.e. $b_t^{i,j} = 0$ for $i \in \mathbf{E}_j$ and that the inequality still holds for such a situation, an auxiliary variable m_t^j is employed to capture capacity insufficiency (the excessive outflows over the capacity) and to penalize the objective function at the unit cost of ρ if $m_t^j > 0$. The capacity insufficiency from the last period m_{t-1}^j is carried over and participates in allocating capacity at current time.

To prove that at optimality, the capacity insufficiency variable m_t^j would be positive only if the sum of exempt outflows exceeds the current program rate, let us assume that the optimal objective function value is C^* and that at a particular time $\tilde{t}$ of TMI j, the optimal solution says that ${m_{\tilde{t}}^j}^* > 0$, and $\sum_{i \in \mathbf{F}_j \setminus \mathbf{E}_j} {g_{\tilde{t}-\bar{t}_{i,j}}^{i,j}}^* + \sum_{i \in \mathbf{E}_j} {g_{\tilde{t}-\bar{t}_{i,j}}^{i,j}}^* + {m_{\tilde{t}-1}^j}^* =$

$M_{\tilde{t}}^{j} + m_{\tilde{t}}^{j^*}$. Note that the constraint must be binding at optimality when $m_t^{j^*} > 0$, or otherwise the objective function value can always be reduced by decreasing m_t^j until the equality does not hold. Assume $\sum_{i\in\mathbf{E}_j} g_{\tilde{t}-\bar{t}_{i,j}}^{i,j}{}^* + m_{\tilde{t}-1}^{j}{}^* \leq M_{\tilde{t}}^{j}$, that is, the sum of exempt outflows plus capacity insufficiency from the last period is less than the current rate. Thus, the sum of non-exempt outflows can be reduced by $m_{\tilde{t}}^{j^*}$ units, and governed by constraint (4.2) the reduced amount would become backlog. Without loss of generality, assume that such backlog does not dissipate until the end of the TMI. The new objective function value is $C' = C^* - \left(\rho - \sum_{i\in\mathbf{F}_j\setminus\mathbf{E}_j} \sum_{t=\tilde{t}-\bar{t}_{i,j}}^{\bar{e}_{i,j}} f(t,j)\right) < C^*$, and since $f(t,j) \ll \rho$ as mentioned earlier, this contradicts the optimality assumption.

Constraint (4.4) is the non-negativity constraint of the decision variables.

C. Addressing TMI Interactions

The proposed linear programming model specifically addresses the interaction of the sequence of TMIs. Through cost settings and constraint formulations in an optimization problem, the favoritism on particular flows or TMIs observed in the following prioritization scenarios may be modeled:

- *Prioritization Policy 1: A departure flow is exempt from a GDP.*
  When a flow is destined to a GDP airport but exempt from the program, it will still utilize the program rate and thus reduce the rate made available to the non-exempt flows. If total exempt demand exceeds the program rate, which implies that the GDP scope needs adjustment, the non-exempt flows will not be served before the backlogs of exempt flows dissipate. Equation (4.5) specifies that the backlog cost coefficient of the exempt flows is higher than that of the non-exempt ones, thereby favoring the exempt flows in the optimization. In addition, constraint (4.3) makes the model robust enough to handle the situation when the capacity is lower than the exempt demand. As such, a flow exempt from a TMI will not have backlog accumulation.
- *Prioritization Policy 2: A GDP is set prior to an AFP.*
  As noted in Section II, given that the GDP program rate is more precisely defined than the AFP rate, the GDP becomes the controlling element when a flow is subject to a GDP prior to an AFP. To account for this operational reality, the flows subject to a GDP prior to an AFP will be treated as the "exempt" flows of the AFP, and similar to Scenario 1, the objective function and constraint (4.3) will guarantee the outflow from the GDP equals the outflow of the AFP for such flows.
- *Prioritization Policy 3: An AFP is set prior to a GDP.*
  In a flight-based model, the EDCT set by an AFP needs to be maintained, if feasible. For flows in this scenario, since the outflow of an AFP is not guaranteed to be exempt from a GDP restriction, the backlog cost coefficient, $c_t^{i,GDP}$, can be set to a higher penalty than other cost coefficients on flows not subject to the AFP, in order to discourage backlog accumulation.

D. Delay Balance Considerations

The proposed linear programming model solves multiple resource allocation problems on a network flow representation. The objective function (4.1) can be considered as the efficiency objective since its solutions would define system-optimum cost of backlogs. It is commonly known that due the linear nature of the objective function, the optimization results might not produce equitable allocations due to solution symmetry, i.e. a unit of backlog of a flow costs the same in the objective function as that of another flow. Consequently, there is no cost incentive for balancing backlogs between flows, which does not represent the first-scheduled-first-served principle like RBS in today's GDP execution.

A more balanced solution could be achieved by including an equity objective in the optimization. Here the term "equity" represents the balance of backlog distribution among individual flows in the context of a flow-based model, which is different from the equity concept in air traffic management. As such, the rate apportioning problem should also be optimized over "*proportional equity*", which can be measured by how the available program rate and backlogs are assigned to individual flows proportionally to their demands[18]. Let us first define the shorthand notes for cumulative demands, outflows and backlogs, respectively: $D_t^{i,j} = \sum_{v=\bar{s}_{i,j}}^{t} d_v^i$, $B_t^{i,j} = \sum_{v=\bar{s}_{i,j}}^{t} b_v^i$, and $G_t^{i,j} =$

$\sum_{v=\bar{s}_{i,j}}^{t} g_v^i$. For each TMI $j \in \{1, \ldots, J\}$, we use four metrics to measure the equity of allocating outflows and backlogs:

- The highest proportion of cumulative outflow to demand: $OH_j = \max_{i \in \mathbf{F}_j \setminus \mathbf{E}_j, t \in \{\bar{s}_{i,j} \ldots \bar{e}_{i,j}\}} \left\{ \frac{G_t^{i,j}}{D_t^{i,j}} \right\}$
- The lowest proportion of cumulative outflow to demand: $OL_j = \min_{i \in \mathbf{F}_j \setminus \mathbf{E}_j, t \in \{\bar{s}_{i,j} \ldots \bar{e}_{i,j}\}} \left\{ \frac{G_t^{i,j}}{D_t^{i,j}} \right\}$
- The highest proportion of cumulative backlog to demand: $BH_j = \max_{i \in \mathbf{F}_j \setminus \mathbf{E}_j, t \in \{\bar{s}_{i,j} \ldots \bar{e}_{i,j}\}} \left\{ \frac{B_t^{i,j}}{D_t^{i,j}} \right\}$
- The lowest proportion of cumulative backlog to demand: $BL_j = \min_{i \in \mathbf{F}_j \setminus \mathbf{E}_j, t \in \{\bar{s}_{i,j} \ldots \bar{e}_{i,j}\}} \left\{ \frac{B_t^{i,j}}{D_t^{i,j}} \right\}$

An equity objective function (4.6) is formulated for a minimization problem, which can be linearized and thus solved with the proposed linear program. Problems with such an objective are called *balanced linear programming problems*[17]. It is intended to simultaneously maximize the lowest proportion and minimize the highest proportion, and thus achieve a more equitable distribution.

$$\mathit{Objective\ Function}\ 2 \coloneqq \sum_{j=1}^{J} \varepsilon\left(OH_j - OL_j + BH_j - BL_j\right) \tag{4.6}$$

Ideally, for each TMI, the goal is to distribute the outflows or backlogs proportionally to the outflow from a previous controlling TMI, for each flow. However, if the outflow variable is used as the denominator in the proportional equity metric, the linearity of the formulation cannot be preserved. Therefore, the original demand is used to compute the equity metrics in order to approximate such a goal.

V. Numerical Experiment

We use a numerical example to show how the proposed LP approximates today's execution of GDP and AFP in a flow-based environment. The proposed LP is first solved with only the efficiency objective to show the difference between the LP and the FSFS algorithm. Then, by including the equity objective, we show how the solutions evolve to more closely approximate the FSFS solution as the equity/efficiency tradeoff ratio increases.

A. Problem Setup

To demonstrate the proposed LP, a hypothetical example is designed to illustrate the potential benefit of TMI coordination and the interaction between two competing objectives, namely efficiency and equity, in allocating available rates. Figure 4 depicts a geographical network of three origin-destination pairs, where four departure flows are modeled. Hourly demand is assumed constant throughout the day and the estimate of transit time to two destination airports and an AFP are summarized in Table 1.

In this example, we assume the following TMIs are proposed and implemented at different timeframes:

- At 0515, a GDP at Airport is planned from 1100 to 1700 at an hourly rate of 10 due to low ceiling forecast.
- At 0715 an AFP at FCA1 is planned from 1200 to 2000 at an hourly rate of 10 due to thunderstorm in en-route airspace.

Each TMI has three flows under consideration, and local start/end times of a TMI are computed from the TMI duration and the estimated transit time. Since the GDP is planned at an earlier time, Prioritization Policy 2 discussed in Section IV.B is applied, which gives priority to the flows that are subject to a GDP prior to an AFP. There is no exempt demand for both TMIs.

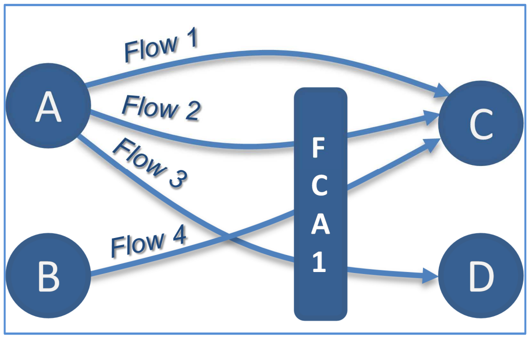

Figure 4. An Example Network

Table 1. Input Parameters for the Rate Control Models

Departure Airport	Flow ID	Hourly Demand	Transit Time (Hour)	
			To Airport C	To FCA1
A	1	4	2	-
	2	4	2	1
	3	4	-	1
B	4	4	2	1

B. Numerical Results with the Efficiency Objective

This rate control problem is formulated and first solved as an efficiency-only problem with the backlog cost coefficient $f(t, j) = 1$. The efficiency-only setting illustrates the potential impact if no equity is considered in departure rate allocation. For comparison purposes, the FSFS apportioning algorithm explained in Section III.C is applied *sequentially* to the GDP and then the AFP, where due to Priority Policy 2 the flows out of GDP would not be delayed by the AFP until the end of the GDP. The results from both apportioning models are summarized in Table 2.

Table 2. Optimization Results – Efficiency Only Model vs. FSFS Algorithm

Apportioning Model		First-Scheduled-First-Served Algorithm (FSFS)		LP with Efficiency Only Objective	
TMI Type		GDP	AFP	GDP	AFP
Duration		11:00 – 17:00	12:00 – 20:00	11:00 – 17:00	12:00 – 20:00
Program Rate		10	10	10	10
Backlogs	Flow 1	18.66	-	24	-
	Flow 2	18.67	24.16	8	58
	Flow 3	-	32.32	-	0
	Flow 4	18.67	24.16	24	2
	Total	56.00	80.65	56	60

For the FSFS model, the GDP produces the same backlog for Flows 1, 2, and 4, since the rate and backlog are proportionally allocated. In addition, as Flows 2 and 4 have the same demand and transit time, the AFP produces the same backlog for each flow as well. The AFP produces more backlogs on Flow 3 because of the prioritization policy imposed. In general, the FSFS model maintains the proportional equity for both TMIs in allocating the rates.

For the proposed model with the efficiency only objective, the sum of the backlog is less than that from the FSFS model, but the backlog distribution is rather unbalanced. Although the total GDP backlog from the LP model is the same as that of the FSFS model, the GDP backlog for Flow 1 is greater. However, this solution produces less total AFP backlog because Flow 1 is subject to GDP only, and the GDP duration is shorter than thee AFP duration. Thus, the backlog accumulated on Flow 1 would not have a longer lasting impact than that of the other flows.

Figure 5 illustrates the cumulative backlogs for each flow on individual TMIs under the two models as a function of time. For the GDP, the FSFS model demonstrates the most equitable allocation, where the backlog at each time period for each impacted flow is the same. For AFP, Flows 2 and 4 have no backlog before and at the end of the GDP because of the prioritization policy. With the efficiency-only objective, the LP arbitrarily assigns more backlogs to Flow 2 than to Flow 4. In fact, the total backlog would not change by any backlog redistribution between Flow 2 and 4 since they have the same set of model parameters.

C. Tradeoff between Efficiency and Equity

In the last example, the proposed model with the efficiency-only objective solves two rate control programs in a coordinated fashion. Using the LP model, a more “efficient” but less balanced rate allocation is produced. However, this efficiency gain is mainly a result of more delays assigned to flows that have restrictions that end earlier, i.e. Flow 1. In today’s execution of GDP, the Ration-By-Schedule rule implies an equitable allocation of arrival slots. To approximate the equity consideration, the effectiveness of the equity objective will be examined.

There exists a tradeoff between the objectives of efficiency (4.1) and equity (4.6). With the backlog cost set to 1, the coefficient ε reflects the tradeoff ratio of equity to efficiency. The efficiency-only model discussed above is a special case of ε=0. Figure 6 summarizes the total backlog of each TMI under various values of ε. Figure 7 illustrates the cumulative backlog distribution over time.

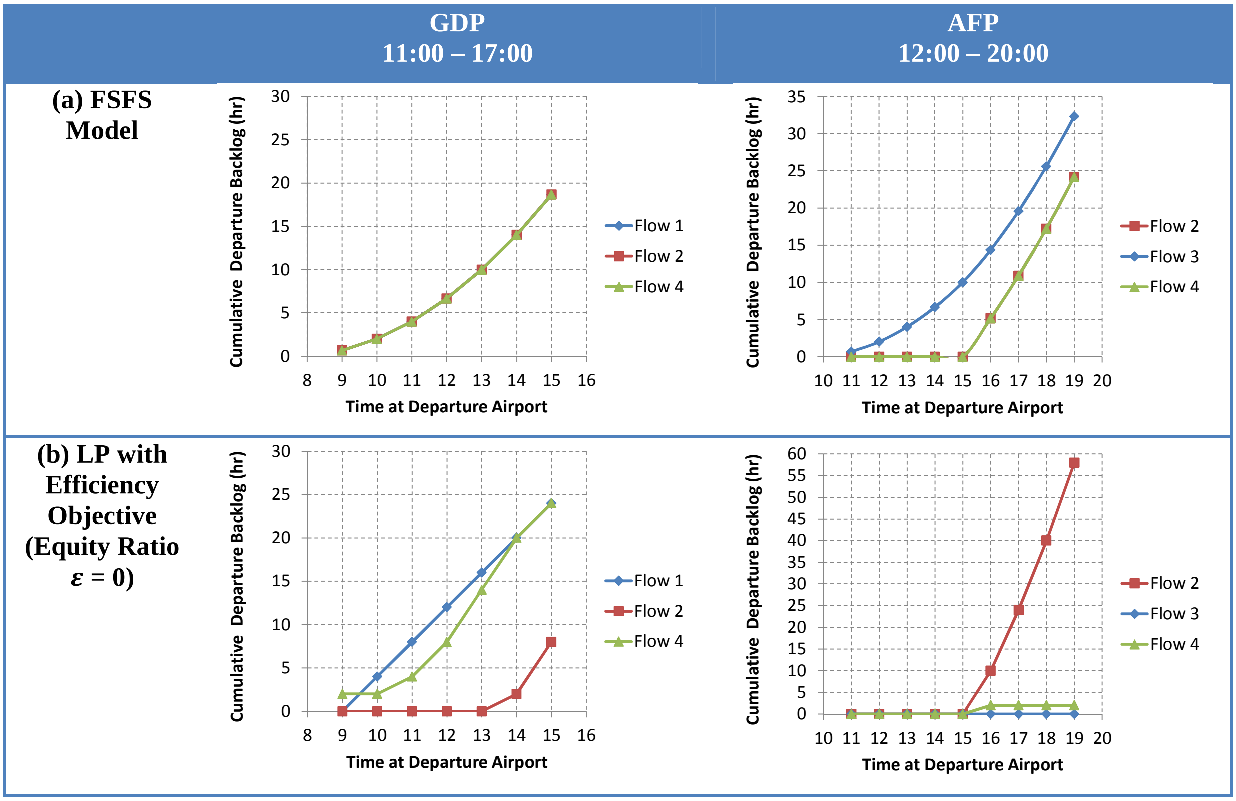

Figure 5. Temporal Cumulative Backlog Distribution – FSFS Model vs. LP with Efficiency Only Objective

In Figure 6, when ε increases from 0 to 1, the total backlog remains constant; however the distribution among flows under both TMIs in Figure 7(a) is more equitable. Flows 2 and 4 have more comparable backlogs and similar distributions. This demonstrates the effectiveness of the equity formulation in reducing solution symmetry and producing a more balanced solution..

As the equity ratio increases, the cumulative backlog distributions become more and more proportional to demand, even though the total backlog cost increases. In Figure 7(d), the cumulative backlog distributions at ε=50 look similar to those of the FSFS model and the total backlog is just slightly less than the FSFS model.

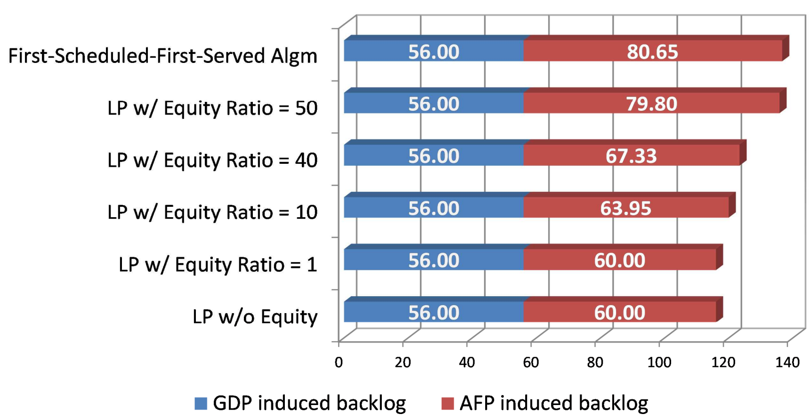

Figure 6. Total System Backlog under Various Apportioning Models

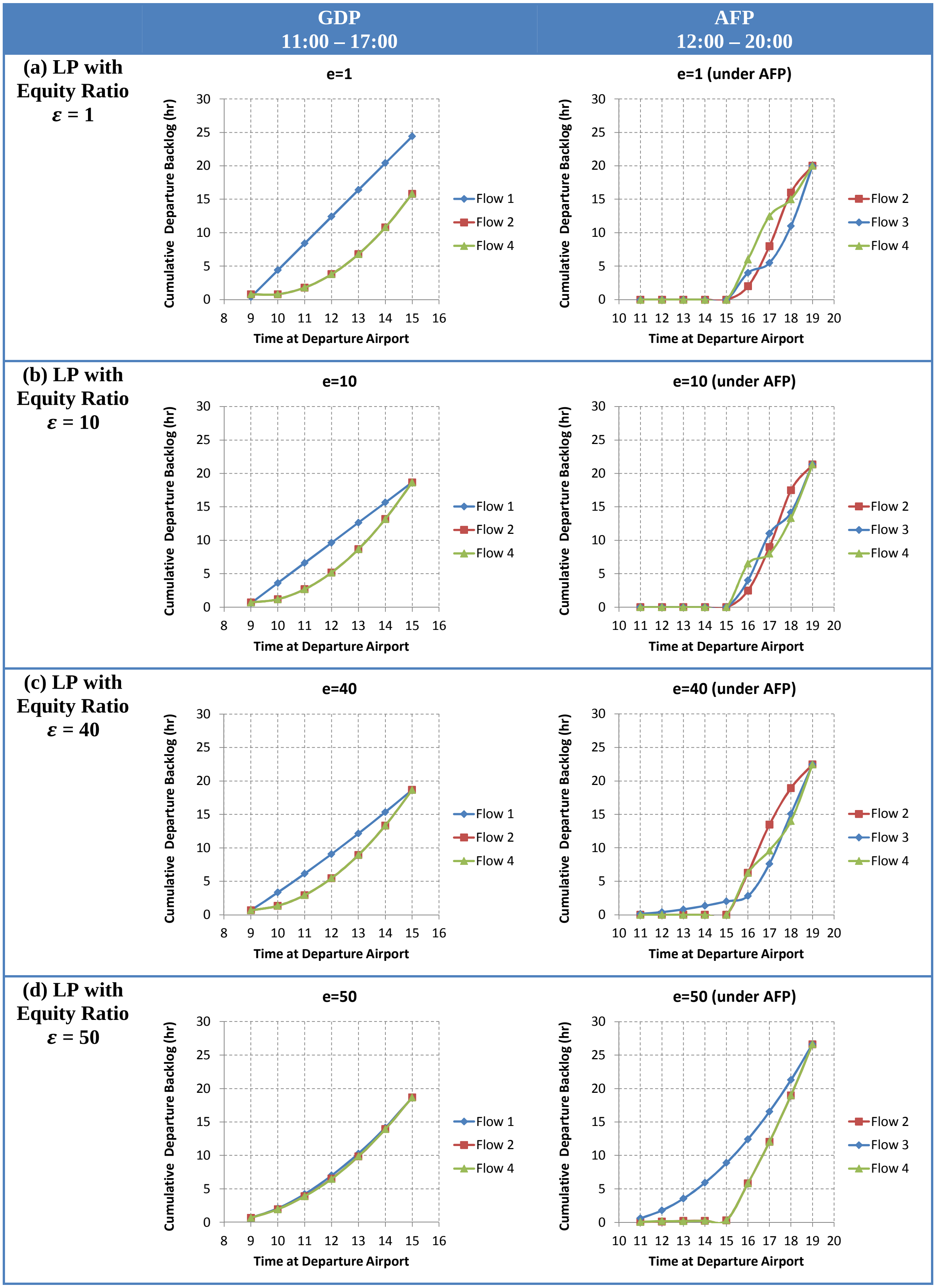

Figure 7. Temporal Cumulative Backlog Distribution under Various Apportioning Models

The sensitivity of backlog cost defines a range for selecting the proper value of ε. In this example, any ε between 0 and 50 represents the tradeoff between the two objectives. More importantly, the specified program rate at GDP and AFP are met at all times, which means that the capacities at the constrained resources are fully utilized and that the proposed LP serves its purpose.

For a general approach for determining the tradeoff ratio in this multi-objective optimization problem, let us assume that $C^q$and C^b are the minimum values for the equity and efficiency objective functions, respectively. One of the transformation methods that combine two objectives with different units could be used to normalize the objective function value of equity by C^q and that of efficiency by C^b. This transformation implies that C^b/C^q would be an adequate value for the tradeoff coefficient. Some complicated forms of transformation may be considered to articulate a priori the reference on one of the objectives based on modeler's judgment. More details on choosing weighting coefficients for a multi-objective optimization can be found in Marler and Arora[14] and Kosil and Silvennoinen[15].

D. Summary

In this specially created example, we put emphasis on verifying the features of the proposed LP and the tradeoff between efficiency and equity objectives. The assumed uniform demand profile intensifies the tradeoff relationship between efficiency and equity.

The proposed LP is able to model the prioritization policy observed in the real-world operations and is intended to provide input values to the FCM queuing model that are more operationally consistent. As such, using the proposed LP model provides decision makers with an accurate prediction of the impact of the controls, which can aid in the development of effective strategic plans.

Our findings are summarized as follows:

1. The reduction in total backlog achieved when only the efficiency metric is used is a due to an inequitable allocation of backlog to flows that have restrictions ending earlier than other flows. However, this is a result of an assumption inherent to the LP model, namely that backlogs are only modeled during the TMI implementation period and are ignored after this time. In reality, those backlogs will be handled by local departure capacity constraints, addressed by other tactical actions, or they will simply dissipate soon. (Note that the local impact at the departure airports after the end of a GDP or an AFP is not model by the slot assignment tool currently used by ATCSCC since it is generally considered outside of the study scope of such a strategic planning problem.)
2. The potential benefit from coordinating multiple TMIs can be exemplified by the efficiency-only model, which illustrates the most optimistic saving at a system-level in *delay caused by TMIs* when jointly solving resource allocation problems at the same time, despite equity concerns.
3. Increasing the weight on the equity objective brings the LP solutions closer to the FSFS results, which is an approximation of the flight-based execution of GDP or AFP in the flow-based context.
4. When multiple NAS resource allocation problems are planned in a coordinated fashion, the objective of minimizing system-wide delay and that of allocating individual delays equitably are in competition. The equity ratio in the proposed model could reflect decision makers' preference on balancing two objectives, but further analysis is required on defining the conditions when the tradeoff relationship should be considered.

VI. NAS-wide Case Study

A. NAS Scenario Setup

To demonstrate the FCM decision framework, a NAS-wide congestion problem is derived from historic traffic and weather. The weather forecasts are taken from September 26, 2010, where only weather contained within the Atlanta ARTCC (ZTL) is considered in this example. The weather-impact model in Ref 13 provides a probabilistic trajectory of weather-impact for the given probabilistic weather forecast shown in Figure 8. Weather-impact is then translated into reduced capacity of ZTL sectors, which is a necessary input of the proposed queuing model.

Figure 8. Weather Forecast on 9/26/10 at 0500 EST for 4, 8, 12, and 16 hours Look-Ahead Time

As the corresponding traffic on that date is obviously impacted by the weather event, we instead utilize traffic predictions from a date with relatively little weather impact. Traffic from August 30, 2010 provides the demand flow in this example since it has extremely low weather coverage and few TMIs and thus little impact on traffic in ZTL. The actual departure rate for each O-D pair from ETMS data is analyzed for every 15-minute time bin.

The network representation is derived from historic filed flight plans for August 30, 2010. Note that this date was purposely chosen to coincide with the traffic day selected for the analysis in order to best represent the actually flown traffic options. To define the level of aggregation for the different NAS resources, we selected ZTL as the area of control. Using the approach described Ref 11, we construct the relevant connections for each O-D layer of the network in order to describe the entire network shown in Figure 9. All origin and destination pairs outside of ZTL are aggregated into *clusters* by using the clustering method in Ref 12, and the individual airports within ZTL are represented as individual nodes. Outside of ZTL, the enroute nodes are represented as connecting ARTCC boundaries and within ZTL sectors, enroute nodes are defined as connections between sector boundaries. The resulting network defines 3,773 routes (or departure flows) for 1,722 cluster pairs. Table 3 provides the details on the network size.

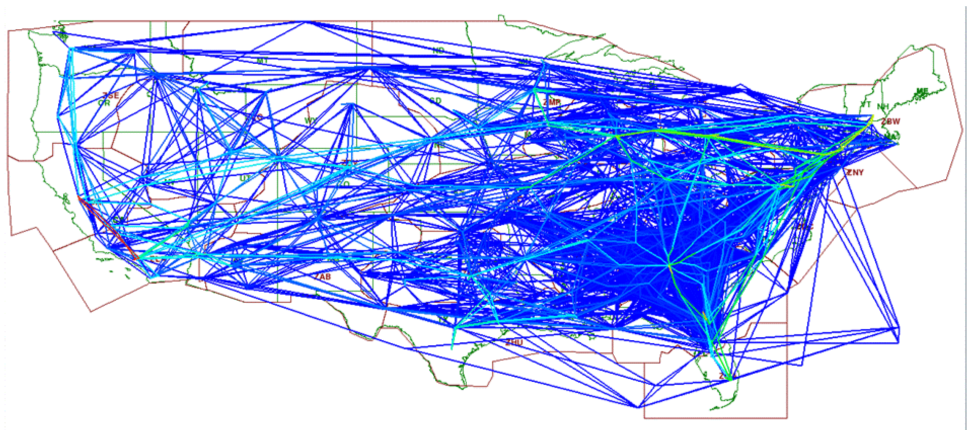

Figure 9. FCM Network of the ZTL Example

Table 3. FCM Network Statistics

Network Property	Size
Number of Airport Clusters	68
Number of Cluster Pairs with Demand	1,722
Number of FCM Network Nodes	1,006
Number of Routes	3,773
Number of FCM Network Arcs	23,768

B. Evaluation of Control Plans

Suppose that given the weather forecast at 1000Z, the southwest traffic of ZTL will be impacted from 1045Z to 2145Z, where the highest impact is expected between 1530Z and 2030Z. The following plans are then discussed on the strategic teleconference and planned to mitigate the weather impact:

- Airspace Flow Program from 1500Z to 2000Z on flows using south and west of ZTL at a rate of 60 flights per hour.
- Ground Delay Program to ATL from 1600Z and 2200Z at a rate of 72 flights per hour.

We solve the rate apportioning problem with FSFS and the proposed LP, respectively:

- To apply the FSFS algorithm, two delay programs need to be solved sequentially, which means the demand inputs for the GDP shouldn't come from the scheduled departures but depend on the rate results of AFP.
- To apply the proposed LP, two delay programs are formulated as an optimization problem with both the efficiency and equity objectives, which has 84,004 decision variables and 87,614 linear constraints.

The departure rates apportioned from the FSFS algorithm approximate today's slot assignment logic in a flow-based sense and represent the result of two uncoordinated programs under an equitable first-scheduled-first-served principle. Meanwhile, the proposed LP solves two rate apportioning problems in a coordinated fashion while considering the tradeoff between efficiency and equity objectives.

To compare the results, we examine the arrival profiles at ATL estimated directly from both models without running the simulation in order to see how an earlier program could affect the later one. Figure 10 summarizes the scheduled and expected arrivals in a 15-minute interval during the GDP periods. It shows that the LP-apportioned arrivals meet the target GDP rate well. On the other hand, the FSFS-apportioned arrivals are influenced by AFP's apportioning results and do not fully utilize the maximum rate allowed because GDP is implemented later than an AFP that has a more restrictive rate.

The LP results seem to be more efficient than the FSFS from the perspective of GDP design but are actually accompanied with a relatively unbalanced distribution of backlogs at the AFP's rate apportionment. Figure 11 compares total backlogs of top 10 origin-destination pairs that are only impacted by the AFP and shows that the LP optimization disproportionally allocates more backlogs to the flows only impacted by AFP. Such an unbalance distribution among various departure flows is not surprising because the coordination through optimization finds a system-optimum solution that might not be optimal to individual flows.

For the proposed LP, the equity objective balances the backlog distribution within each of individual flows during the program duration. Figure 12 shows the cumulative demands and backlogs attributed to AFP of the flows from the Orlando airport cluster (MCO) to the Atlanta airport (ATL). These flows account for 12% of the ATL arrivals on the selected traffic day. It is observed that the LP-apportioned backlogs grow proportionally with the demands as desired by employing the equity objective, and its growth trend is comparable with but faster than that of the FSFS-apportioned backlogs.

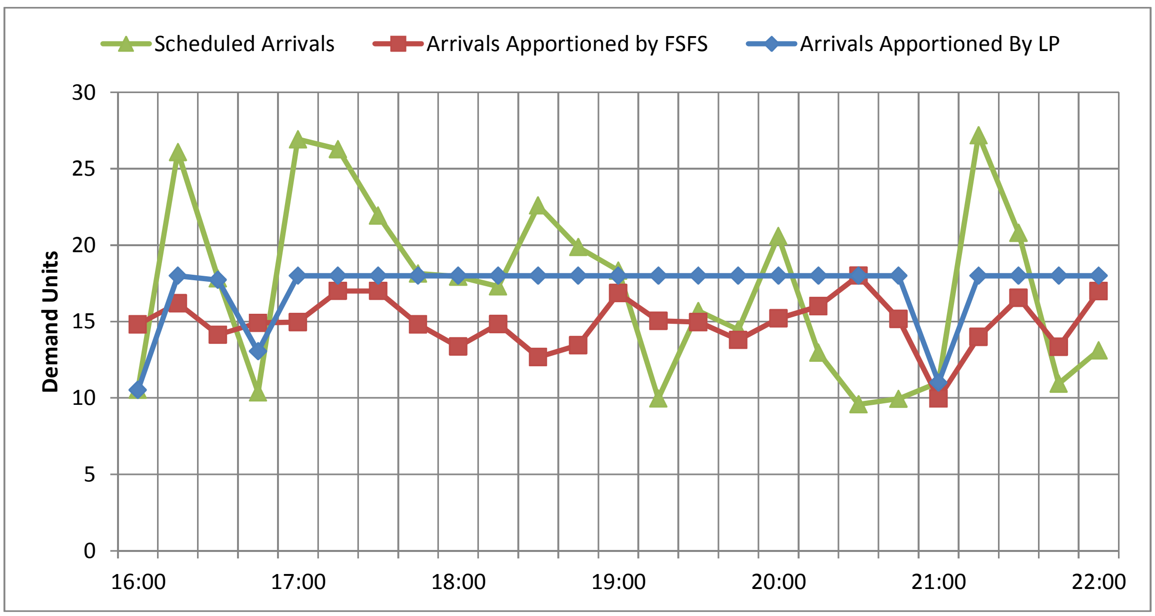

Figure 10. ATL Arrival Profiles during the Periods of the Ground Delay Program

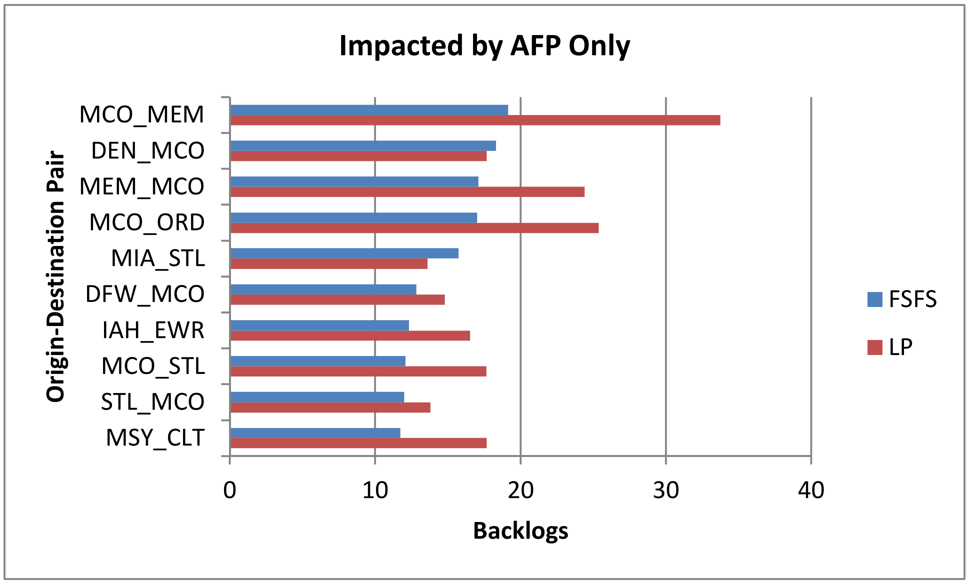

Figure 11. Total Backlogs of Top 10 Origin-Destination Pairs Only Impacted by the AFP

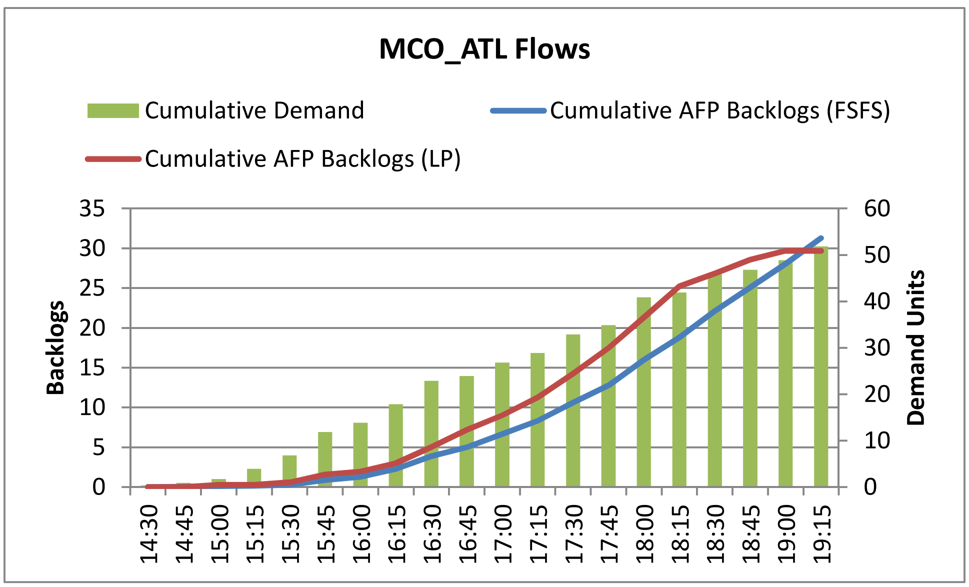

Figure 12. Cumulative Backlog Distribution of MCO-ATL Departure Flows

VII. Concluding Remarks and Future Works

In today's operations, traffic flow management utilizes many resources to obtain an accurate forecast of the NAS in an attempt to identify where and when it is necessary to exert controls to balance aircraft demand when airspace becomes constrained. The FCM framework offers the possibility that both demand and available airspace be viewed and controlled within a single source, where a crucial modeling effort is to simulate traffic propagation over the NAS under weather impact and enacted TMIs.

The proposed rate control model approximates today's execution of departure delay programs in a flow-based sense and is capable of solving multiple departure restrictions in a coordinated fashion in order to provide better inputs to the FCM queuing simulation. It can also address the TMI prioritization rules observed in the real-world operations. Numerical experiments are conducted on a test network as well as on a NAS-wide scenario to illustrate the rate allocation results and the tradeoff between efficiency and equity objectives.

Future work shall focus on analyzing the correlation among weather uncertainty, the design of departure delay programs, and the apportioning results via the FCM queuing simulation.

In addition, when multiple NAS resource allocation problems are planned in a coordinated fashion, the objective of minimizing system-wide delay and that of allocating individual delays equitably are in competition. Decision makers' preference on balancing two perspectives must be involved in the tradeoff analysis. More studies on defining the conditions when the tradeoff relationship should be considered and on associated policy implications are recommended.

Lastly, for the execution of departure delay programs in a flight-based sense, it is recognized that airlines might substitute arrival slots based on their own preferences and operational concerns, which could change the apportioned rates for the FCM simulation. The slot substitution process has the subtleties that cannot be foreseen at a strategic planning timeframe and would not be captured in a flow-based environment. Further study on interpreting the flow-based results to the flight-based operations is suggested.

Acknowledgements

The authors would like to thank Mary Hokit and Tom St. Clair of The MITRE Corporation for their valuable operational insights, Tudor Masek, Rachel Ethier, Steve Zobell, John Huhn, Dr. Lixia Song, Mike Klinker, and Dr. Glenn Roberts of The MITRE Corporation for their valuable suggestions and supports, and Kevin Workman and Ali Rakei of QinetiQ North America for supporting software development and integration.

NOTICE

References

[1] FAA ATO NextGen and Operations Planning, June 30, 2009, NextGen Mid-Term Concept of Operations for the National Airspace System, Washington, DC.

[2] Taylor, C., Wanke, C., Zobell, S., Holland, R., Wang, L., Wan, Y., Roy S., and Xue, M., "Flow Contingency Management Decision Framework," The MITRE Corporation MP100309, Sept. 2010.

[3] Taylor, C., Wanke, C., Wan, Y., and Roy, S., "An Operational Concept for Flow Contingency Management," AIAA Aviation Technology, Integration, and Operations Conference, Virginia Beach, VA, Sept. 2011.

[4] Tien, S., Taylor, C., Yi, Z., Wan, Y., and Wanke, C. "A Route-Based Queuing Network Model for Air Traffic Flow Contingency Management," AIAA Aviation Technology, Integration, and Operations Conference, Virginia Beach, VA, Sept. 2011.

[5] Cook, L., and Wood, B., "A Model for Determining Ground Delay Program Parameters Using a Probabilistic Forecast of Stratus Clearing," The 8th USA/Europe Air Traffic Management Research and Development Seminar, Napa, California, June, 2009.

[6] Mukherjee, A., Hansen, M., and Grabbe, S., "Ground Delay Program Planning Under Uncertainty in Airport Capacity," AIAA Guidance, Navigation and Control Conference, Chicago, Illinois, August 2009.

[7] Ball, M., Hoffman, R., and Mukherjee, A., "Ground Delay Program Planning Under Uncertainty Based on the Ration-by-Distance Principle," Transportation Science. Vol. 44, No. 1, February 2010.

[8] Churchill, A., Lovell, D., and Ball, M., "Coordinating Multiple Traffic Management Initiatives with Integer Optimization," The 4th International Conference on Research in Air Transportation, Budapest, Hungary. June, 2010.

[9] Churchill, A., and Lovell, D., "Assessing the Impact of Stochastic Capacity Variation on Coordinated Air Traffic Flow Management," In the Proceedings of TRB 2011 Annual Meeting, Washington, D.C. January, 2011.

[10] FAA, Airspace Flow Program Best Practice, retrieved from http://tfmlearning.faa.gov/AFP_Best_Practices_v5.pdf on November 1, 2011.

[11] Taylor, C., Masek, T., Wanke, C., Wan, Y., and Roy, S., "Defining Multi-Resolution Networks for Flow Contingency Management," AIAA Aviation Technology, Integration, and Operations Conference, Virginia Beach, VA, Sept. 201.

[12] Wang, L., Taylor, C., and Wanke, C., "An Airport Clustering Method for Air Traffic Flow Contingency Management," AIAA Aviation Technology, Integration, and Operations Conference, Virginia Beach, VA, Sept. 2011.

[13] Xue, M., Zobell, S., Roy, S., Taylor, C., Wan, Y., and Wanke, C., "Using Stochastic, Dynamic Weather-Impacted Models in Strategic Traffic Flow Management", in Proceedings of 91st American Meteorological Society Annual Meeting, Seattle, WA, January 2011.
[14] Marler, R., and Arora, J. "Survey of Multi-Objective Optimization Methods for Engineering," Structural and Multidisciplinary Optimization, Vol. 26, No. 6, 2004, pp. 369-395.
[15] Koski, J. and Silvennoinen, R., "Norm Methods and Partial Weighting in Multi-Criterion Optimization of Structures," International Journal for Numerical Methods in Engineering, Vol. 24, June 1987, pp. 1101-1121.
[16] Barnhart, C., Bertsimas, D., Caramanis, C., and Fearing, D., "Equitable and Efficient Coordination in Traffic Flow Management," Transportation Science, Vol. 46, No. 2, May 2012, pp. 262-280.
[17] Ahuja, R., "The Balanced Linear Programming Problem," European Journal of Operational Research, Vol. 101, 1997, pp. 29-38.
[18] Brown, R., "The Sharing Problem," Operations Research, Vol. 27, No. 2, 1979, pp. 324-340.

