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MITRE TECHNICAL REPORT

Conversion Between Sine Wave and Square Wave Spatial Frequency Response of an Imaging System

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Abstract

The spatial frequency response of an imaging system, known as the Modulation Transfer Function (MTF), is a primary image quality metric that is commonly measured with a sine wave target. The FBI certification program for commercial fingerprint capture devices, which MITRE actively supports, has an MTF requirement. In some cases, however, a square wave ("bar target") must be used in testing, which results in a similar quantity called the Contrast Transfer Function (CTF). This document reports on an investigation of the mathematical relationship between the MTF and CTF, methods for converting between the two, and derives an equivalent CTF from the given spec MTF, for use in the FBI certification program. The methodology presented is applicable to the general case, i.e., whenever conversion between the MTF and CTF of an imaging system is needed.

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Section 1

Introduction

1.1 Background

The Federal Bureau of Investigation (FBI) recently fielded an Integrated Automated Fingerprint Identification System (IAFIS) that greatly enhances the FBI's identification services to other federal agencies, and to local, state, and regional law enforcement agencies. IAFIS is a large, complex information system that utilizes off-the-shelf and advanced technologies. Agencies interacting with IAFIS require some components of these technologies to maintain interoperability.

Among other components, IAFIS incorporates fingerprint scanners to convert inked fingerprint cards, hardcopy latent fingerprints, and live subjects' finger scans (livescan) into digital data, at either 500 pixels per inch (ppi) or 1000 ppi, with output at 8 bits per pixel (256 gray levels). MITRE is supporting the FBI's certification process for vendor fingerprint products, whose foundation is the IAFIS Image Quality Specifications (IQS) (FBI, 1999). The IQS defines the quantitative image quality requirements for fingerprint scanners and printers. Products are certified by a process whereby the vendor generates test data by following a set of established test procedures (FBI, 1995) and MITRE analyzes the test data for conformance and compliance with the specification requirements. This successful certification process has helped to ensure a standard level of image quality for fingerprint comparisons, classification, feature detection, and automated search reliability, and supports systems interoperability between national, state, and local law enforcement agencies.

One of the critical components of the IQS test suite is the testing performed to ascertain a fingerprint scanner's spatial frequency response, known as the Modulation Transfer Function (MTF). Fundamentally, the MTF is a measure of the contrast transmission capabilities of an imaging system, as a function of spatial frequency. The MTF has proven to be an excellent measure of the sharpness and detail rendition of an imaging system (Tannas, 1985). For fingerprint scanners, it indicates the devices' capability to capture fingerprints at a quality level that is needed for successful latent image comparison, as well as for inked or livescan fingerprint search/matching and examination. MTF assessment has successfully quantified the elusive term 'image quality' across fingerprint capture devices of many different designs.

A sine wave target is specified for use in determining the MTF in IQS testing. A sine wave is the fundamental pattern for directly determining the spatial frequency response of a linear system; a pure sine wave input to a linear, spatially invariant system is output as a pure sine wave of the

same frequency, generally with reduced amplitude¹. It has a high signal-to-noise ratio, and the fact that each spatial frequency is individually imaged has been found to be very useful in fault detection/isolation of scanner problems. In addition, the *maximum value* MTF is the IQS-specified quantity of interest and a sine wave target readily lends itself to such a measurement, because the single sine period in each frequency pattern that exhibits the highest computed modulation can be readily identified. Finally, the target is available as a commercial off-the-shelf product²; a typical sine wave target is shown in Figure 1.

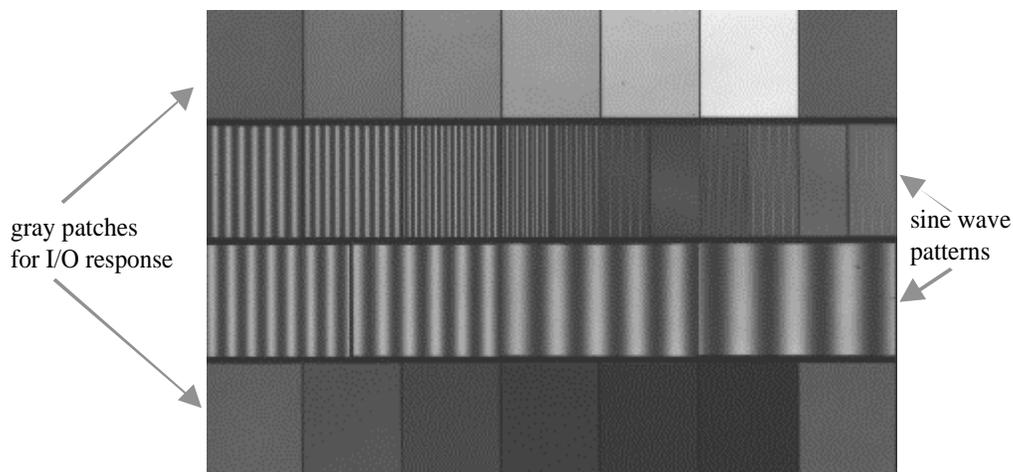


Figure 1. Typical Sine Wave Target

¹ Other targets can be used to measure the MTF, e.g., a narrow line target, point target, random pattern, or sharp edge. In these cases the target is broken up into its sine wave components, via Fourier analysis, to compute the sine wave MTF. Each target type has its own set of advantages and disadvantages.

² Three vendors (first is manufacturer): <http://www.sinepatterns.com>, <http://www.edmundoptics.com>, <http://www.jmloptical.com>

For IQS testing, the sine wave target is scanned and the resulting digital image is processed through a MITRE-developed computer algorithm which computes the scanner's MTF (MITRE, 2001). The essence of the technique is illustrated in Figure 2 for a single sine wave frequency: the gray levels corresponding to the peak and valley in the sine wave image are identified, these gray levels are converted to target space reflectance values by traversing the scanner's input/output response curve, the image modulation is computed, and divided by the target modulation to obtain a single point on the MTF curve. The process is then repeated for a series of sine wave frequencies until the complete MTF curve is generated.

A complication in applying the MTF concept to image scanners results from the fact that scanners perform discrete sampling over finite areas. A scanner samples a continuous, analog image input at discrete locations over discrete time intervals, integrating all incident light energy over a given time interval and within a small area around each sampling point, where the small area is the individual detector element ("pixel") area. One of the consequences of this discrete sampling by finite-sized detector elements is that the scanner acts as a space variant system, which implies that the scanner MTF can be multivalued (Park, et.al, 1984; Feltz, et.al., 1990). In the context of the sine wave target, this space variance means that the phase between this target and the scanner's detector array plays a role in the actual MTF achieved. Here, phase refers to the location of the detector pixel within a sine wave period, at the time the pixel is collecting light energy. As the target-detector phase changes, the scanner MTF can change, within limits. In practice, one measures the minimum, maximum, or average MTF. It has been established procedure in IQS testing to use the maximum measured MTF (after noise filtering) for comparing to the spec MTF for compliance verification purposes.

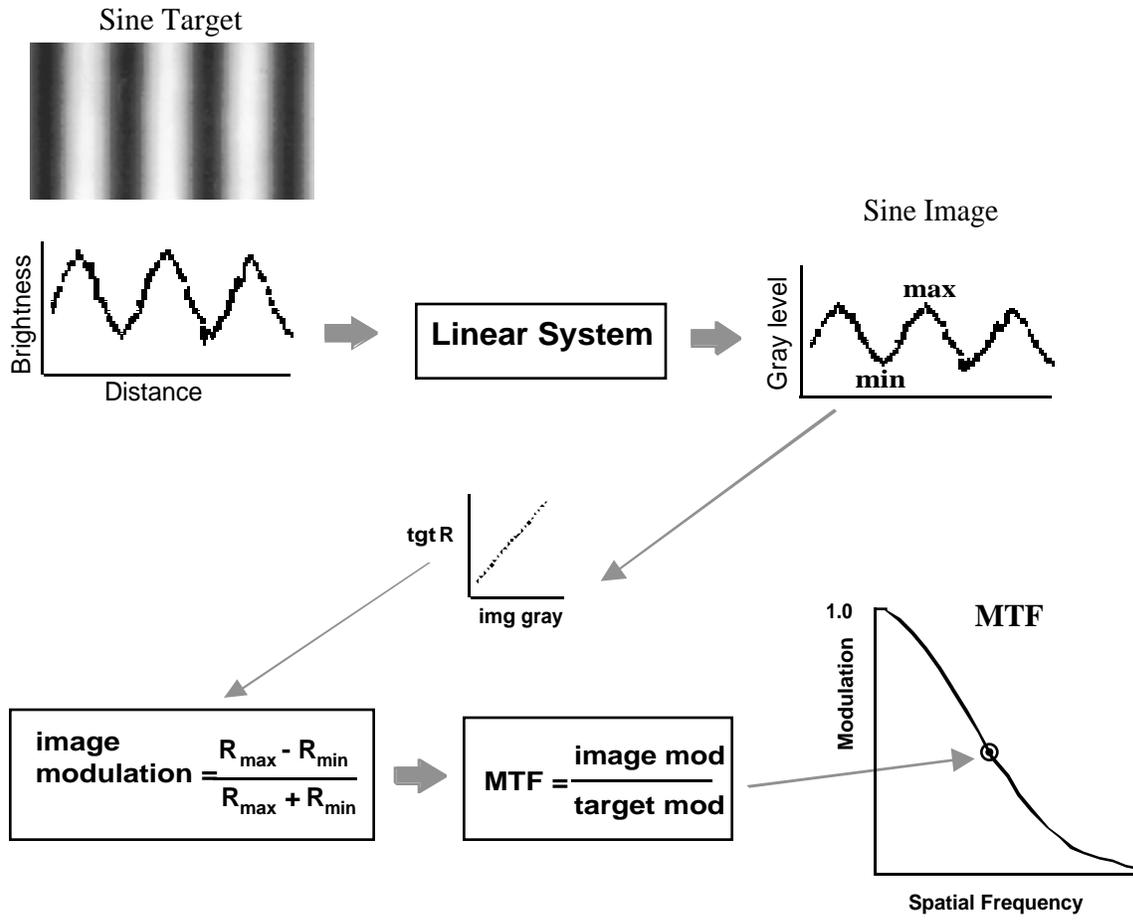


Figure 2. Path of Single Sine Wave to Generate MTF Data Point

1.2 Problem Statement

There are two fundamentally different types of fingerprint scanners certified for use in the FBI IAFIS program. In one type, inked fingerprints on card stock ("ten-print card") or latent prints on paper are scanned. Typically, this scanner is a general purpose, Commercial-Off-The-Shelf (COTS) or modified COTS flatbed, desktop scanner operating in reflection mode, with a linear Charge-Coupled Device (CCD) scanning array detector and associated signal processing to produce the digital output image. The common imaging mode utilized in these scanners is always compatible with scanning a reflective sine wave target.

The other type of scanner, here called a livescanner, directly scans a person's finger and is designed solely for that purpose. In the most common livescanner design a subject's finger is pressed against ("plain impression") or rolled across ("rolled print") the surface of a glass prism. The image is created by the difference in index of refraction (n) between skin and air, utilizing the optical imaging principle of "total internal reflection" (TIR), also called "frustrated internal reflection" (FIR) (Smith, 1966). Within certain defined angles of incident illumination, light traveling from the prism surface to finger ridges in contact with the glass will be absorbed by the skin ($n_{\text{glass}} \cong n_{\text{skin}}$), while light traveling from the prism surface to finger valleys, where a glass-air gap exists ($n_{\text{glass}} > n_{\text{air}}$), will be reflected back into the glass. The reflected light is detected by a linear or two-dimensional CCD array to produce the digital output image.

FIR imaging in livescanners depends on the three-dimensionality of skin, which is either in contact or not in contact with a glass surface, and the relative indices of refraction between glass, air, and skin. Many livescanners cannot adequately image a two-dimensional, continuous gray tone sine wave target; the output image suffers from low signal-to-noise ratio, low contrast, and very few gray levels, making it impractical to compute a valid MTF. It has been found through experimentation, however, that livescanners can adequately image reflective or transmissive bar targets, mainly because a bar target is extremely high contrast with only two gray levels, black and white, and it can be easily fabricated on a glass substrate.

The fundamental problem addressed in this document can be stated as follows:

The FBI image quality specification specifies the minimally acceptable values for a scanner MTF when it is measured using a sine wave target. Since a bar target must be used in some product certification cases, the problem is to answer the questions: (1) is there a relationship between the spatial frequency response derived from a bar target and the MTF derived from a sine wave target?, (2) what is this relationship?, and (3) what is the best approach for handling a bar target derived spatial frequency response, given that the specification is for a sine wave target derived MTF?

Section 2

Problem Solution - Synopsis of Results

The analyses and theory presented over the years in technical literature confirms that, for the same imaging system, the bar target derived spatial frequency response and sine target derived MTF are not equal to each other, but, as will be seen, there is a definite mathematical relationship between them. The difference between the two quantities is often highlighted by denoting the spatial frequency response obtained from a bar target as the Contrast Transfer Function (CTF) or square wave response, to distinguish it from the term 'MTF', which is reserved for the sine wave transfer function. This terminology distinction between bar target CTF and sine target MTF is used in the remainder of this document.

In solving the problem at hand, where a sine wave target MTF is specified but the imaging system under test may require use of a bar target, one can either:

- (1) convert the spec sine wave MTF to the equivalent bar CTF, which then becomes the spec CTF when bar targets are used,
- or,
- (2) convert each CTF measured with a bar target to its equivalent sine wave MTF, for comparison to the original spec sine wave MTF.

The first approach is more straightforward to apply in the testing environment, with less chance of computational error, because the modulations of a sample are directly computed from the bar target image and compared to the 'spec' bar CTF, with no further manipulation or conversion of sample data. Therefore, the first approach is the solution approach utilized.

The method of construction of the spec CTFs are discussed in detail in Section 3; here we present the final results in terms of the proposed spec CTFs for IQS certification compliance verification. Table 1 gives the proposed spec CTFs for a 500 ppi scanner and for a 1000 ppi scanner, which are equivalent to the corresponding IQS Appendix F spec sine wave MTFs, also given in Table 1 (see also Figure 6). The following points should be noted with respect to Table 1:

- The CTF values in Table 1 are unnormalized and assume a target modulation of 1.0 at all frequencies.

- In IQS testing, the bar target frequencies cannot be predefined because the exact target, either COTS or special design, is a vendor choice. The actual CTF is moderately dependent on the number of bars at a given frequency; Table 1 values are the average of 5-bar, 15-bar, and 50-bar CTFs. A bar target with less than 5 bars per frequency should never be used for CTF measurement³.
- For a bar target containing frequencies not given in Table 1, the following equations can be used to obtain the CTF modulation values at those other frequencies.

$$\begin{aligned}
 &500 \text{ ppi, } f = 1.0 \text{ to } 10.0 \text{ cy/mm,} \\
 &\text{CTF} = 3.04105\text{E} - 04 * f^2 - 7.99095\text{E} - 02 * f + 1.02774 \qquad (1)
 \end{aligned}$$

$$\begin{aligned}
 &1000 \text{ ppi, } f = 1.0 \text{ to } 20.0 \text{ cy/mm,} \\
 &\text{CTF} = -1.85487\text{E} - 05 * f^3 + 1.41666\text{E} - 03 * f^2 - 5.73701\text{E} - 02 * f + 1.01341 \qquad (2)
 \end{aligned}$$

³ A 3-bar target, such as the USAF 1951 tribar target, is not appropriate for CTF measurement because the peak and valley image amplitudes are erratic, only the center bar and two adjacent spaces are available for modulation calculation, and because obtaining good phasing between the target and scanner detector pixels is difficult. A minimum of a 5-bar target is required for stability, such as the NBS 1010A target (a.k.a., NBS 1963A target, ANSI/ISO Test Chart #2); better is a 15-bar target, such as the T90 target.

Table 1. IQS Spec MTFs and Corresponding, Proposed Spec CTFs

Frequency (cy/mm)	MTF 500 ppi	CTF 500 ppi	MTF 1000 ppi	CTF 1000 ppi
0.5		0.978		0.979
1.0	0.905	0.948	0.925	0.957
1.5		0.909		0.930
2.0	0.797	0.869	0.856	0.904
2.5		0.830		0.879
3.0	0.694	0.791	0.791	0.854
3.5		0.752		0.829
4.0	0.598	0.713	0.732	0.805
4.5		0.674		0.782
5.0	0.513	0.636	0.677	0.760
5.5		0.597		0.738
6.0	0.437	0.559	0.626	0.716
6.5		0.521		0.695
7.0		0.483		0.675
7.5		0.446		0.655
8.0	0.312	0.408	0.536	0.636
8.5		0.370		0.617
9.0		0.333		0.598
9.5		0.296		0.580
10.0	0.200	0.259	0.458	0.563
10.5				0.546
11.0				0.529
11.5				0.513
12.0			0.392	0.497
12.5				0.481
13.0				0.466
13.5				0.451
14.0			0.336	0.437
14.5				0.423
15.0				0.409
15.5				0.395
16.0			0.287	0.382
16.5				0.369
17.0				0.356
17.5				0.344
18.0			0.246	0.332
18.5				0.319
19.0				0.308
19.5				0.296
20.0			0.210	0.284

Section 3

Technical Details of Solution

A straightforward series expansion formula derived nearly half a century ago (Coltman, 1954) will quite accurately convert a linear imaging system's sine wave target MTF to its equivalent bar target CTF, or vice versa, under certain conditions. These conditions include the assumption that the imaging system is analog, e.g., a photographic film camera, and the assumption that the number of target bars is infinite, i.e., the bar target is a true square wave of infinite extent. The Coltman formula and its application is described in more detail in Section 3.2, but note that the implicit assumptions in this formula do not hold for the case of interest here. Specifically, a digital scanner is a discrete sampling system, not a continuous analog system, and a real bar target used to measure the CTF contains a finite number of bars, sometimes as few as 5 bars per frequency. Under these conditions, and using a curve extrapolation strategy, the Coltman formula can still give a reasonably accurate CTF, as discussed in Section 3.2.

However, we seek a method for converting the MTF to the CTF which directly takes into account the discrete sampling of a digital scanner, the finite number of bars in a bar target, and results in a high accuracy CTF, which is a true-equivalent of the FBI's IQS specified sine wave MTF. The method developed in Section 3.1 meets these conditions.

3.1 Solution Approach

3.1.1 Methodology

The following procedure was developed for converting a discrete sampling imaging system's MTF to its equivalent CTF for a given number of target bars:

- (a) the given system MTF is divided by the Fourier transform of the sampling pixel,
- (b) the result of (a) is multiplied by the Fourier transform of the bar target,
- (c) the result of (b) is Fourier transformed,
- (d) the result of (c) is convolved with the sampling pixel, and
- (e) the maximum and minimum values from the result of (d) are located, from which the CTF modulation values are computed.

Steps a-e are illustrated in the following math-graphic, where F denotes Fourier transformation and \otimes denotes convolution. Steps a-e are individually discussed in detail in the following.

monotonically decreases to zero frequency⁵. Experiments run demonstrate that CTF results up to Nyquist frequency are not greatly affected by any reasonable choice of M(f) values above Nyquist frequency.

For computational efficiency, a curve fit to the entire M(f) was performed, with the following results for 500 ppi and 1000 ppi MTFs; also plotted in Figure 3.

500 ppi, $f \leq 10.0$ cy/mm:

$$M(f) = 4.8597E-07 * f^7 - 1.856125E-05 * f^6 + 2.846349E-04 * f^5 - 2.300554E-03 * f^4 + 1.060994E-2 * f^3 - 2.276723E-2 * f^2 - 7.703449E-2 * f + 1.0000090 \quad (4)$$

500 ppi, $f > 10.0$ cy/mm:

$$M(f) = \exp(-9.991357E-02 * f^{1.06}) \quad (5)$$

1000 ppi, $f \leq 20.0$ cy/mm:

$$M(f) = -2.99661568E-8 * f^5 + 3.04795483E-6 * f^4 - 1.35961019E-4 * f^3 + 3.9614024E-3 * f^2 - 7.76516183E-2 * f + 1.00001941 \quad (6)$$

1000 ppi, $20.0 < f \leq 80.0$ cy/mm:

$$M(f) = -9.90555556E-7 * f^3 + 2.1147619E-4 * f^2 - .0184142778E0 * f + 0.627922857 \quad (7)$$

Steps (b), (c) - These steps are conveniently performed by utilizing a closed form equation for computing the image profile of an N-bar target, given a system MTF, that has been reported in the literature (Barakat and Lerman, 1967, Equation (10)). With a change of notation, Barakat's Equation (10) becomes:

$$j(x) = \frac{1}{3\pi} \int_0^{f_c} \frac{\cos(2\pi xf) \sin(4\pi N L f) M(f)}{f \cos(2\pi L f)} df \quad (8)$$

where,

$j(x)$ = image intensity profile as a function of distance x , prior to pixel sampling

N = number of bars in bar target, at specific frequency

L = half-bar width

f_c = cut-off frequency of M(f): 40 cy/mm for 500 ppi, 80 cy/mm for 1000 ppi

⁵ For the special cases, not considered here, of optics with large aberrations, large defocus, or a central obscuration (apodized aperture), the optics MTF still goes to zero, but not necessarily monotonically. Also note that, at first glance, it might seem that the spec MTF could be extrapolated first, and then divided by sinc(.); but this approach would not give a smooth result because sinc(.) oscillates and goes through zero modulation, at frequencies above $2x$ Nyquist.

Step (d) - Equation (8) is convolved with a rectangle function representing the pixel sampling performed in a scanner, to obtain the final output image profile:

$$i(x) = \int r(t)j(x-t)dt \quad (9)$$

where,

$r(t)$ = rectangle function, width = 0.0508 mm for 500 ppi,
width = 0.254 mm for 1000 ppi

Step (e) - Finally, the maximum and minimum values of the total imaging system's bar image profile are identified from $i(x)$. This is equivalent to the integral of $r(x)j(x)$ with $r(x)$ centered at a valley in $j(x)$ and then centered at a peak in $j(x)$. An alternative method is to constrain all $r(x)$ locations to be integer multiples of the pixel width, and then determine which two $r(x)$ locations result in maximum modulation. In this case the maximum and minimum may not be centered on the peak and/or valley of $j(x)$, respectively. In modeling an actual scanner's performance this second method may be more realistic, but for identifying the CTF that most closely corresponds to a spec MTF, the first method makes more sense.

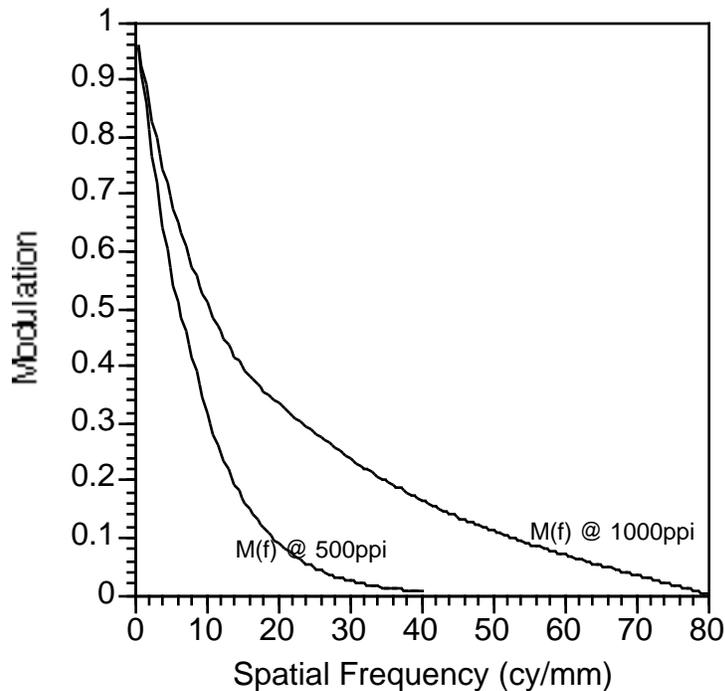


Figure 3. Curve Fits to: $M(f) = \text{Spec MTF} / \text{Sinc}(\pi fw)$

3.1.2 Computer Program

A computer program was developed in Fortran-90 to perform steps b–e, which incorporates the separately computed curve fits given in Equations (4-7). Some features of this computer program are as follows:

- Calculations are performed in double precision mode.
- Integrations are performed via Simpson's rule with a large number of small integration increments to account for the high frequency oscillations of the trig functions.
- Evaluation is only performed over 1.25 periods of the bar target, centered on the middle bar, because the highest peak and lowest valley always occur at and adjacent to the middle bar in this symmetrical model.
- The input bar target is assumed to have a modulation of 1.0.

A number of code verification tests were run; the major tests were as follows:

- Identified required integration increments by running series of increment sizes and observing where stable output occurred.
- Processed a 50-cycle sine wave target with 0.0508 mm and 0.0254 mm sampling pixel widths and with the 500 ppi and 1000 ppi spec MTFs, respectively. The output MTF should equal the original spec MTF in each case, which they did to within less than 0.4% for 500 ppi case and to within less than 0.7% for the 1000 ppi case; these results are given in the printouts in the Appendix. The errors that did occur are due to a combination of the inevitable computational errors and small errors in the curve fits to the input MTFs. Note that for this case of a sine wave target input replacing a bar target input, Equation (2) has a different denominator, given by:
 $f [1-(4Lf)^2]$.
- Processed bar targets, with various numbers of bars, with a diffraction-limited optics MTF; the latter is a known, closed form equation out to the optics cut-off frequency. The output CTFs were accurate to within the accuracy that could be extracted from published data (Bass and VanStryland, 1994).
- Verified that the polynomial curve fits to the MTFs in step (a) were smooth, monotonically decreasing curves with no 'hidden' dips or spikes.

3.1.3 Results

The CTF results for bar targets containing 5, 15, and 50 bars are plotted in Figures 4 and 5 for the 500 ppi case and 1000 ppi case, respectively. Note that the CTF approaches a 'steady state' as the number of target bars increases, e.g., there is very little difference between the 15 bar and 50 bar CTFs. At the opposite extreme, however, for a low number of 5 target bars, there is a distinct rise in the CTF.

The most relevant spec CTF, for comparison to test data results from a real scanner, would be to use the spec CTF derived from the same number-of-bars target as is actually used in the testing. This is impractical, however, because of the wide variety of bar target designs, both COTS and specially fabricated, which, for example, could have a different number of bars in each frequency pattern in the same target. Because nearly all practical bar targets would in fact have between 5 and 50 bars at any given frequency, it is convenient to define a single spec CTF as the average of the 5, 15, and 50 bar CTFs. Taking this one step further, the final spec CTFs at 500 ppi and 1000 ppi are defined as the curve fits to the average bar CTFs, which are shown in Figure 6 (see also Table 1). Note that, as fully expected by the underlying theory, the spec CTFs are everywhere higher than their respective spec MTFs.

One processing anomaly did occur. The CTF values calculated in the neighborhood of $2/3$ of the Nyquist frequency, i.e., close to 6.56 cy/mm for 500 ppi and 13.12 cy/mm for 1000 ppi, showed an unnatural dip in modulation; the calculated values are only about 92% of what would be expected from a smooth curve. Some effort was expended in trying to track down the cause of this but it remains an unknown. The values near $2/3$ Nyquist are reported in the Appendix but were not used in generating the spec CTFs because they are believed to be anomalous data points, i.e., 6.5 cy/mm for 500 ppi; 12.5 and 13.0 cy/mm for 1000 ppi computed modulation values were not used.

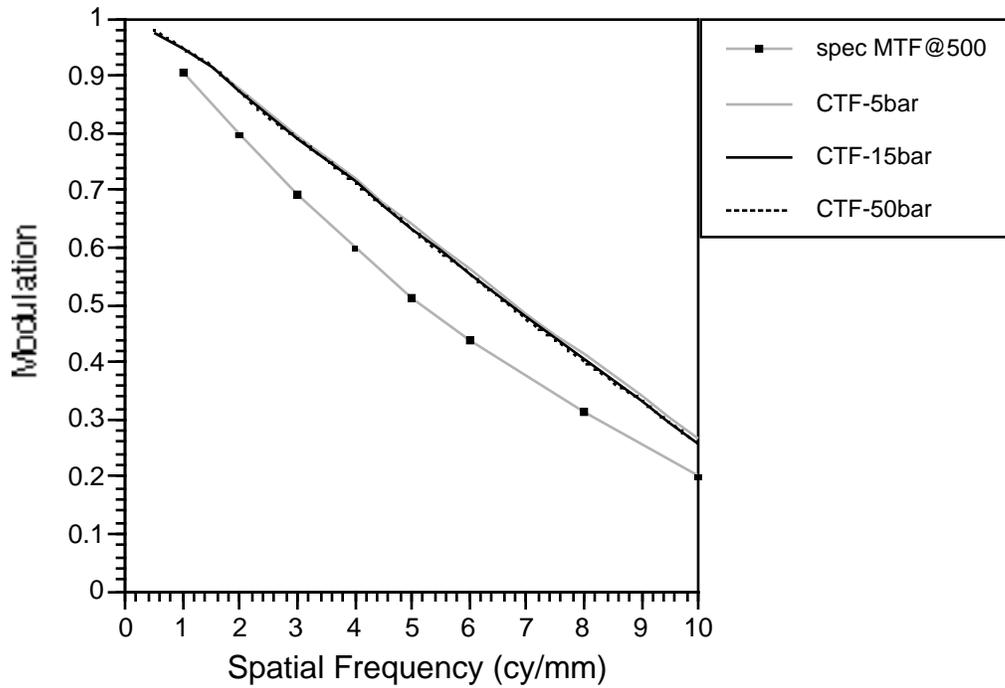


Figure 4. Computed 500 ppi CTFs for 5, 15, 50-Bar Targets and Spec MTF

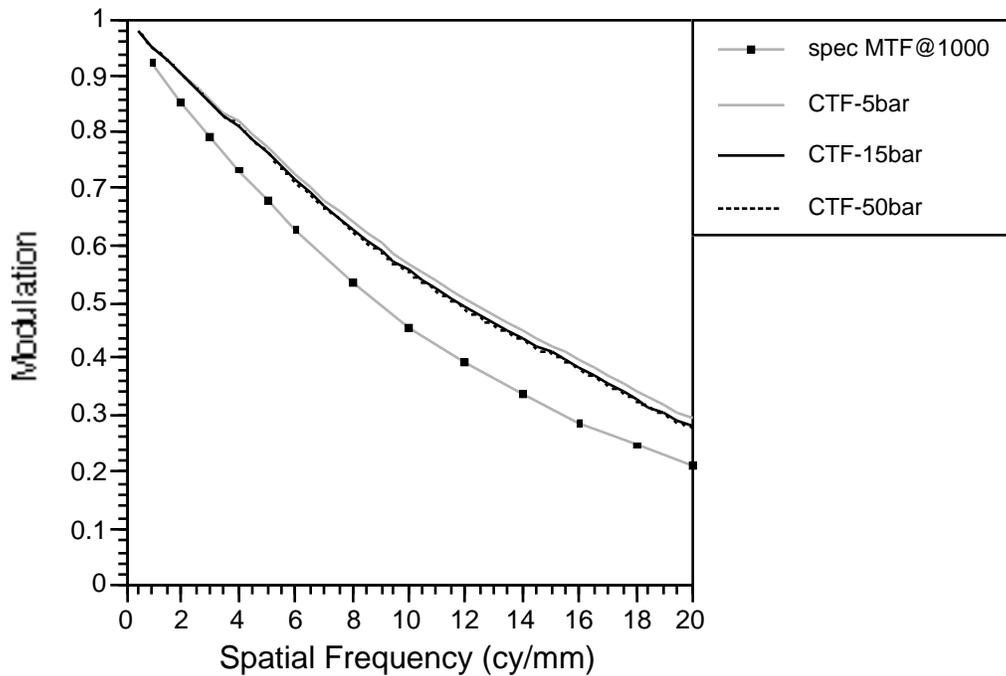


Figure 5. Computed 1000 ppi CTFs for 5, 15, 50-Bar Targets and Spec MTF

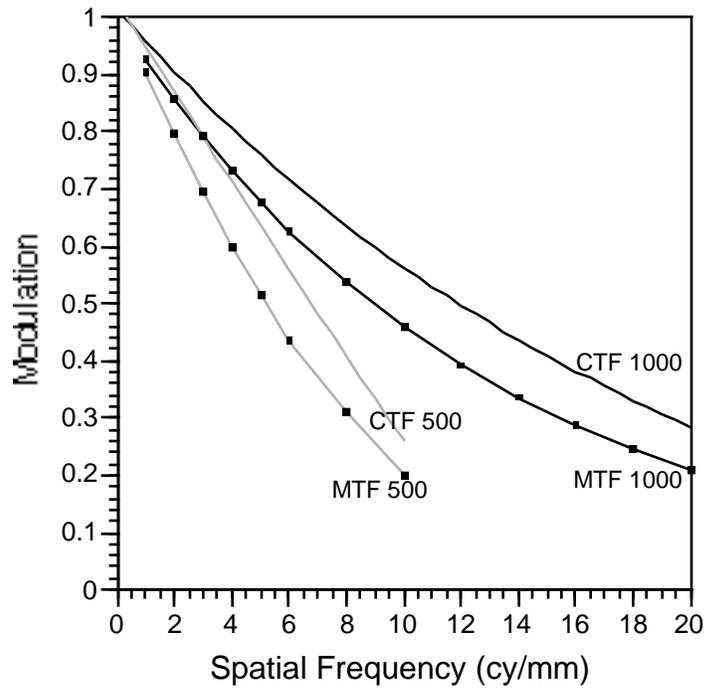


Figure 6. Final Spec CTFs Compared to Original Spec MTFs

3.2 Alternative Approach - Coltman Formula

For analog imaging systems, such as a photographic film camera, it is possible to measure the sine wave MTF up to the frequency at which the MTF goes to zero, which is called the 'cut-off' spatial frequency in the optical field, equivalent to the 'band-limit' frequency in electrical engineering parlance. For this case, a simple series expansion formula derived nearly half a century ago (Coltman, 1954) will quite accurately convert the square wave CTF to its equivalent sine wave MTF, or vice versa.

Given the CTF, the Coltman formula to determine the MTF, is

$$M(f) = \frac{\pi}{4} \left[C(f) + \frac{C(3f)}{3} - \frac{C(5f)}{5} + \frac{C(7f)}{7} + \frac{C(11f)}{11} - \frac{C(13f)}{13} - \frac{C(15f)}{15} - \frac{C(17f)}{17} + \frac{C(19f)}{19} \dots \right] \quad (10)$$

and given the MTF, the Coltman formula to determine the CTF, is

$$C(f) = \frac{4}{\pi} \left[M(f) - \frac{M(3f)}{3} + \frac{M(5f)}{5} - \frac{M(7f)}{7} + \frac{M(9f)}{9} - \frac{M(11f)}{11} + \frac{M(13f)}{13} - \frac{M(15f)}{15} + \frac{M(17f)}{17} - \frac{M(19f)}{19} \dots \right] \quad (11)$$

where, $M(f)$ = sine wave MTF
 $C(f)$ = bar target CTF
 f = spatial frequency

Note that Equation (10) is an irregular sequence in terms of sign and term constants, whereas Equation (11) is a strictly alternating, sequential series. The practical application of the formula can be illustrated by the following simple example. Suppose a CTF has been measured with a bar target out to an imaging system's cut-off frequency of 30 cy/mm, then to compute the sine wave MTF at a single frequency, $f = 2$ cy/mm, requires use of the CTF modulation values at $f = 2, 6, 10, 14, 22, 26,$ and 30 cy/mm, i.e.,

$$M(2) = \frac{\pi}{4} \left[C(2) + \frac{C(3 * 2)}{3} - \frac{C(5 * 2)}{5} + \frac{C(7 * 2)}{7} + \frac{C(11 * 2)}{11} - \frac{C(13 * 2)}{13} - \frac{C(15 * 2)}{15} \right] \quad (12)$$

The fact that the frequency of evaluation increases with each succeeding term in the expansions given in Equations 10 and 11 indicates that for evaluation at higher and higher frequencies, fewer and fewer terms are used, because no modulation values are available above the cut-off frequency. In fact, for evaluation frequencies greater than 1/3 of the cut-off frequency, only the first term exists, resulting in the simple relationships,

$$M(f) = \frac{\pi}{4} C(f) \quad \text{and} \quad C(f) = \frac{4}{\pi} M(f) \quad , \text{ for } f > \frac{\text{cut - off frequency}}{3}$$

There are several possible approaches for applying the Coltman formula to derive the CTF from the MTF of a sampled imaging system, these are:

- (1) Simply multiply the MTF by the $4/\pi$ leading constant in Equation (11); this is a crude application, but is sometimes seen in the literature.
- (2) Apply the full Equation (11) to the MTF, using as many terms as is possible, given that all terms higher than the Nyquist frequency are zero-valued.
- (3) Extrapolate the MTF curve beyond Nyquist frequency, until it smoothly reaches zero modulation; then apply the full Equation 11 with as many terms as is possible.

Figure 7 gives the CTF results of these three approaches when using the spec MTF at 500 ppi as the input to the Coltman formula (Equation (11)), and compares the resulting Coltman CTFs to the spec CTF derived via the technique described in Section 3.1. Clearly, the $4/\pi$ approach and spec-MTF-truncated-at-Nyquist approach lead to large errors at a significant number of frequencies. On the other hand, the spec-MTF-extrapolated-beyond-Nyquist approach gives results very close to the spec CTF derived in Section 3.1.

In fact, the integration kernel in Equation (8) has similarities to the Coltman formula in Equation 11, if one expands Equation (8) into a series. To illustrate this similarity, Figure 8 shows the CTF of a diffraction-limited lens (difflim) having a cutoff frequency of 10.0 cy/mm, as computed by the Coltman formula and by the method in Section 3.1. For the latter, as a further test of the computer program, the CTF for a 50-bar target was computed two ways, which should give equivalent results: (1) point sampling was assumed, with a pixel width = 0.0001 mm, pixel pitch = 0.050 mm (Nyquist=10.0 cy/mm) and using difflimMTF as input, and (2) a finite pixel sampling width = 0.050 mm was assumed (Nyquist=10.0 cy/mm, pixel pitch=0.050 mm), using [difflimMTF / sinc(.05 π f)] as input. As is seen in Figure 8, all three computed CTF curves overlap, and they also agree with published data for the CTF of a diffraction-limited lens (Bass and VanStryland, 1994).

In summary, the technique described in Section 3.1, which admittedly does involve more complex computations than the Coltman approach, is believed to be the more accurate approach for the application at hand. The Section 3.1 approach can account for a small number of target bars, whereas the Coltman formula inherently assumes an infinite number of bars. Also, the section 3.1 approach is more conducive to modeling the discrete-area sampling of a sampled imaging system, including pixel widths that may be less than pixel pitch, whereas the Coltman formula is fundamentally for an analog, continuous imaging system.

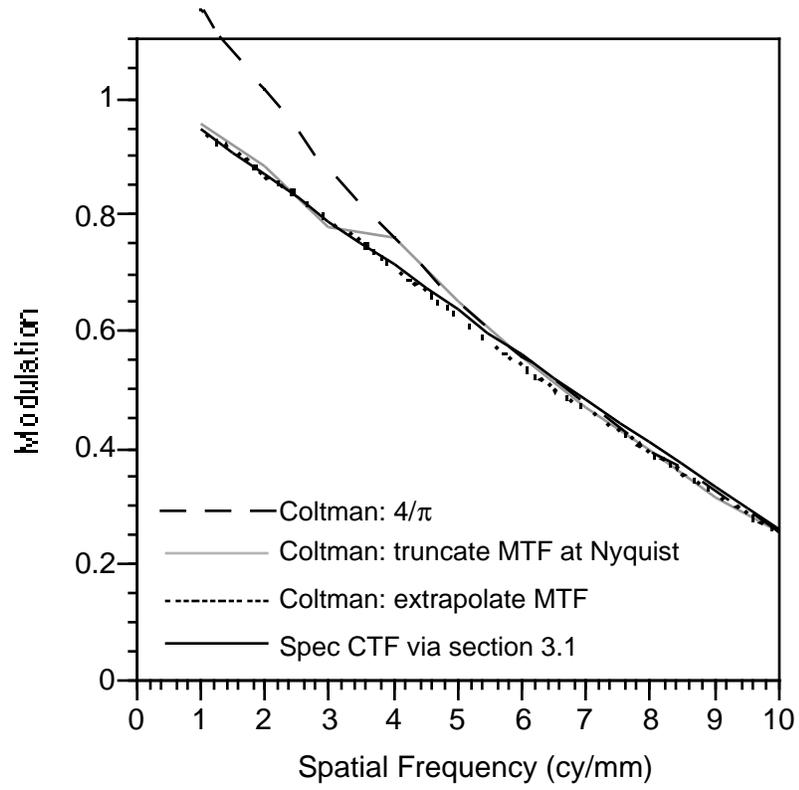
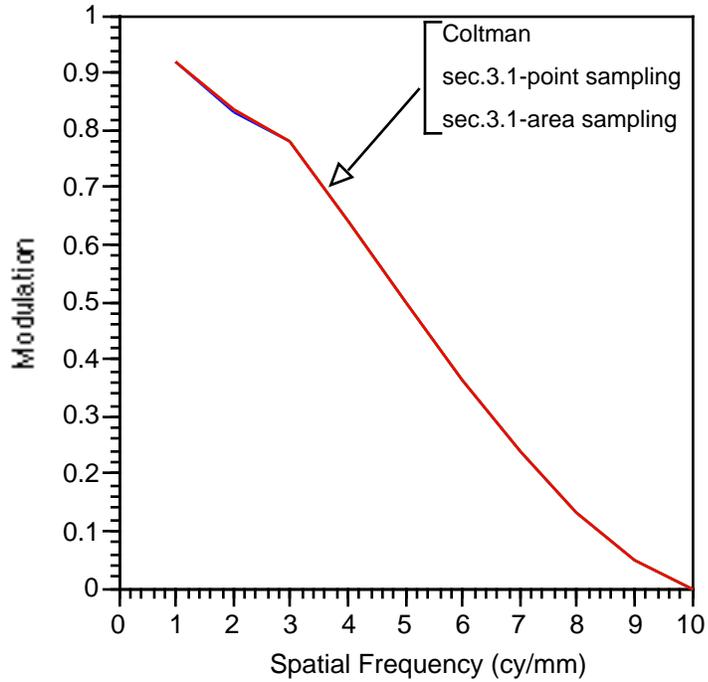


Figure 7. Variations of Coltman Formula Application



cy/mm	difflimMTF (input)	Coltman CTF	sec 3.1 CTF (point sampling)	sec 3.1 CTF (area sampling)
1	0.872889	0.917271	0.91875183	0.91875183
2	0.74706	0.830332	0.8348011	0.8348012
3	0.623838	0.778428	0.7798474	0.77984791
4	0.504632	0.642517	0.64397621	0.64397734
5	0.391002	0.497839	0.49920773	0.4992092
6	0.284757	0.362564	0.3637381	0.363738
7	0.18812	0.239522	0.239309	0.2404192
8	0.104088	0.132529	0.133088	0.13308893
9	0.037386	0.04760133	0.04783677	0.04783712
10	0.000000	0.000000	0.00015346	0.00015338

Figure 8. Lens CTF Calculated via Coltman and Section 3.1 Method

3.3 CTF Normalization

In actual computation of a scanner's CTF from a scanned bar target, the maximum image modulation is first determined from the highest peak and lowest valley combination found within one period. It would often lead to a more accurate result if the peak and valley image gray levels were first converted to their equivalent values in target space before computing image modulation, because this normalizes out gray level measurement differences between the target and scanner-produced image. Bar targets, however, are rarely fabricated with the surrounding uniform gray patches (see Figure 1) necessary to construct the conversion curve. This leaves the choice of either denoting the curve generated from the image modulations as the scanner CTF, or normalizing this curve by a very low frequency modulation value and denoting the result as the scanner CTF. On the upside, the latter approach has the advantage of normalizing-out the effect of a bar target whose effective modulation, as 'seen' by the particular scanner, may be less than 1.0. On the downside, the latter approach will normalize out the degrading effect of any existing light flare or veiling glare that is a part of the scanner performance/design. Analysis of test data from several IQS certified scanners that used bar targets, indicate that CTF normalization by the modulation at a very low frequency (less than 5% of Nyquist) is usually possible and straightforward, and results in a good measure of the CTF. Also, it should be noted that if the target bar modulations decrease with increasing frequency, and those target modulations can be measured, then this would be an additional normalization factor to apply to the imaging system's measured CTF.

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Appendix

Computed CTFs

Computer Print-Outs of N-Bar CTFs Used to Generate Proposed Spec CTFs in Table 1

Nomenclature:

bestphaseCTF - used for final spec CTF; centers pixel on bar image peak and valley for maximum modulation (see Section 3.1, step (e)).

PeakCTF - maximum modulation from pixels centered at integer multiples of pixel pitch.

$T(w)$ = equation 4, 5, 6, or 7, which are curve fits to: (FBI IQS spec MTF) divided by $\text{sinc}(y)$.

5-Bar Target - implies there are 5 target bars at each target frequency (cy/mm) evaluated.

Target frequency sequence type - defines the user-selected bar frequency values.

5-BAR TARGET @ 500 PPI

```
T(w) case (first term): 4.8597D-07*f**7
tgt frequency sequence type: 1      tgt form: BAR
#tgt bars = 5          tgt mod = 1.0000000000000000
integral inc.in computing each BarSinImg value = 1.000000E-04  cy/mm
# pts in integral to compute each BarSinImg value = 400001
max frEquation of input MTF = 40.0001  cy/mm
pixelwidth (mm) = 5.080000E-02      pixel pitch (mm) = 5.080000E-02
# points across pixel for pixel integration = 401
pixel integration increment = 1.270000E-04
```

cy/mm	PeakCTF	bestphaseCTF
0.50000000	0.97679330	0.97679330
1.00000000	0.94837350	0.94837720
1.50000000	0.91613640	0.91629111
2.00000000	0.87563520	0.87564660
2.50000000	0.83397450	0.83399051
3.00000000	0.79415291	0.79486203
3.50000000	0.75908440	0.75976770
4.00000000	0.71254193	0.72042240
4.50000000	0.67889170	0.68077892
5.00000000	0.64079840	0.64088684
5.50000000	0.59598500	0.60127440
6.00000000	0.54082632	0.56083160
6.50000000	0.46290740	0.48141790
7.00000000	0.44677844	0.48604760
7.50000000	0.42327590	0.44946280
8.00000000	0.39748110	0.41310470
8.50000000	0.36846140	0.37629732
9.00000000	0.33657401	0.33944490
9.50000000	0.30310210	0.30353140
10.00000000	0.26907180	0.26907180

15-BAR TARGET @ 500 PPI

T(w) case (first term): 4.8597D-07*f**7
tgt frequency sequence type: 1 tgt form: BAR
#tgt bars = 15 tgt mod = 1.0000000000000000
integral inc.in computing each BarSinImg value = 1.000000E-04 cy/mm
pts in integral to compute each BarSinImg value = 400001
max frEquation of input MTF = 40.0001 cy/mm
pixelwidth (mm) = 5.080000E-02 pixel pitch (mm) = 5.080000E-02
points across pixel for pixel integration = 401
pixel integration increment = 1.270000E-04

cy/mm	PeakCTF	bestphaseCTF
0.50000000	0.97703284	0.97703284
1.00000000	0.94693982	0.94694380
1.50000000	0.91353881	0.91369820
2.00000000	0.87199562	0.87200784
2.50000000	0.82957100	0.82958820
3.00000000	0.78903001	0.78974200
3.50000000	0.75308910	0.75379100
4.00000000	0.70610490	0.71400433
4.50000000	0.67195153	0.67384120
5.00000000	0.63345640	0.63355034
5.50000000	0.58820600	0.59353670
6.00000000	0.53290070	0.55290470
6.50000000	0.45529990	0.47380100
7.00000000	0.43885880	0.47769320
7.50000000	0.41498780	0.44084750
8.00000000	0.38921110	0.40460040
8.50000000	0.36079174	0.36844462
9.00000000	0.32971152	0.33244830
9.50000000	0.29542250	0.29579880
10.00000000	0.25924620	0.25924620

50-BAR TARGET @ 500 PPI

T(w) case (first term): 4.8597D-07*f**7
tgt frequency sequence type: 1 tgt form: BAR
#tgt bars = 50 tgt mod = 1.000000000000000
integral inc.in computing each BarSinImg value = 1.000000E-04 cy/mm
pts in integral to compute each BarSinImg value = 400001
max frEquation of input MTF = 40.0001 cy/mm
pixelwidth (mm) = 5.080000E-02 pixel pitch (mm) = 5.080000E-02
points across pixel for pixel integration = 401
pixel integration increment = 1.270000E-04

cy/mm	PeakCTF	bestphaseCTF
0.50000000	0.98126300	0.98126300
1.00000000	0.94846254	0.94846640
1.50000000	0.91395604	0.91411490
2.00000000	0.87170910	0.87172140
2.50000000	0.82882070	0.82883781
3.00000000	0.78790470	0.78861860
3.50000000	0.75167270	0.75237691
4.00000000	0.70440262	0.71232080
4.50000000	0.67005930	0.67195220
5.00000000	0.63141750	0.63151110
5.50000000	0.58603680	0.59137060
6.00000000	0.53066414	0.55151730
6.50000000	0.46432414	0.49905410
7.00000000	0.43664741	0.47538253
7.50000000	0.41273100	0.43851410
8.00000000	0.38694770	0.40228200
8.50000000	0.35864913	0.36626520
9.00000000	0.32747760	0.33020030
9.50000000	0.29352520	0.29389880
10.00000000	0.25693010	0.25693010

50 CYCLE SINE WAVE TARGET @ 500 PPI

T(w) case (first term): 4.8597D-07*f**7
tgt frequency sequence type: 2 tgt form: SINE
#tgt bars = 50 tgt mod = 1.000000000000000
integral inc.in computing each BarSinImg value = 1.000000E-04 cy/mm
pts in integral to compute each BarSinImg value = 400001
max frEquation of input MTF = 40.0001 cy/mm
pixelwidth (mm) = 5.080000E-02 pixel pitch (mm) = 5.080000E-02
points across pixel for pixel integration = 401
pixel integration increment = 1.270000E-04

cy/mm	PeakMTF	bestphaseMTF
1.00000000	0.90816210	0.90821391
2.00000000	0.79893451	0.79903161
3.00000000	0.69179350	0.69499970
4.00000000	0.58297440	0.59965870
5.00000000	0.51372690	0.51386642
6.00000000	0.41670331	0.43773480
7.00000000	0.33931073	0.37199431
8.00000000	0.30096500	0.31289473
9.00000000	0.25462052	0.25667250
10.00000000	0.20003460	0.20003460

5-BAR TARGET @ 1000 PPI

T(w) case (first term): -2.99661568D-8*F**5 tgt frequency sequence type:
 1 tgt form: BAR
 #tgt bars = 5 tgt mod = 1.000000000000000
 integral inc.in computing each BarSinImg value = 1.000000E-04 cy/mm
 # pts in integral to compute each BarSinImg value = 800001
 max frEquation of input MTF = 80.0001 cy/mm
 pixelwidth (mm) = 2.540000E-02 pixel pitch (mm) = 2.540000E-02
 # points across pixel for pixel integration = 401
 pixel integration increment = 6.350000E-05

cy/mm	PeakCTF	bestphaseCTF
0.50000000	0.97750970	0.97750991
1.00000000	0.95400960	0.95401114
1.50000000	0.93073450	0.93073570
2.00000000	0.90736390	0.90736824
2.50000000	0.88434980	0.88435852
3.00000000	0.86046230	0.86063790
3.50000000	0.83804310	0.83830273
4.00000000	0.81657850	0.81659954
4.50000000	0.79321503	0.79402680
5.00000000	0.77085030	0.77088140
5.50000000	0.74581590	0.74780273
6.00000000	0.72356490	0.72466410
6.50000000	0.70161320	0.70162590
7.00000000	0.67917484	0.67984990
7.50000000	0.65614241	0.65970870
8.00000000	0.63364110	0.64026212
8.50000000	0.61787920	0.62146550
9.00000000	0.60188090	0.60327830
9.50000000	0.58564031	0.58586690
10.00000000	0.56913950	0.56919062
10.50000000	0.55219800	0.55318641
11.00000000	0.53418870	0.53781092
11.50000000	0.51410690	0.52297250
12.00000000	0.49102530	0.50762522
12.50000000	0.46431261	0.48273330
13.00000000	0.42704750	0.44431722
13.50000000	0.42130103	0.46553412
14.00000000	0.41307610	0.45137882
14.50000000	0.40473670	0.43725323
15.00000000	0.39624590	0.42317184
15.50000000	0.38754743	0.40915940
16.00000000	0.37857404	0.39526300
16.50000000	0.36928410	0.38159910
17.00000000	0.35964372	0.36824340
17.50000000	0.34961660	0.35517620
18.00000000	0.33916560	0.34236764
18.50000000	0.32826894	0.32978913
19.00000000	0.31692810	0.31740531
19.50000000	0.30516240	0.30518963
20.00000000	0.29239472	0.29239472

15-BAR TARGET @ 1000 PPI

T(w) case (first term): -2.99661568D-8*F**5
tgt frequency sequence type: 1 tgt form: BAR
#tgt bars = 15 tgt mod = 1.000000000000000
integral inc.in computing each BarSinImg value = 1.000000E-04 cy/mm
pts in integral to compute each BarSinImg value = 800001
max frEquation of input MTF = 80.0001 cy/mm
pixelwidth (mm) = 2.540000E-02 pixel pitch (mm) = 2.540000E-02
points across pixel for pixel integration = 401
pixel integration increment = 6.350000E-05

cy/mm	PeakCTF	bestphaseCTF
0.50000000	0.97774374	0.97774392
1.00000000	0.95255273	0.95255450
1.50000000	0.92808120	0.92808234
2.00000000	0.90367323	0.90367811
2.50000000	0.87971723	0.87972690
3.00000000	0.85494834	0.85513270
3.50000000	0.83171212	0.83198370
4.00000000	0.80951330	0.80953620
4.50000000	0.78541320	0.78623360
5.00000000	0.76240640	0.76244032
5.50000000	0.73672430	0.73876351
6.00000000	0.71396541	0.71506673
6.50000000	0.69158240	0.69159320
7.00000000	0.66863930	0.66933690
7.50000000	0.64505141	0.64868990
8.00000000	0.62224274	0.62884420
8.50000000	0.60611402	0.60967022
9.00000000	0.58975911	0.59112620
9.50000000	0.57315313	0.57336622
10.00000000	0.55632940	0.55638740
10.50000000	0.53909710	0.54011370
11.00000000	0.52080154	0.52446820
11.50000000	0.50047051	0.50937122
12.00000000	0.47721084	0.49375233
12.50000000	0.45050784	0.46888460
13.00000000	0.41387933	0.43104530
13.50000000	0.40785700	0.45118070
14.00000000	0.39947770	0.43693290
14.50000000	0.39101650	0.42274883
15.00000000	0.38243311	0.40864550
15.50000000	0.37366540	0.39464414
16.00000000	0.36456620	0.38070890
16.50000000	0.35512140	0.36699384
17.00000000	0.34536024	0.35361140
17.50000000	0.33520520	0.34050872
18.00000000	0.32462360	0.32765520
18.50000000	0.31373930	0.31515643
19.00000000	0.30268010	0.30310881
19.50000000	0.29139900	0.29141830
20.00000000	0.27907571	0.27907571

50-BAR TARGET @ 1000 PPI

T(w) case (first term): -2.99661568D-8*F**5
 tgt frequency sequence type: 1 tgt form: BAR
 #tgt bars = 50 tgt mod = 1.000000000000000
 integral inc.in computing each BarSinImg value = 1.000000E-04 cy/mm
 # pts in integral to compute each BarSinImg value = 800001
 max frEquation of input MTF = 80.0001 cy/mm
 pixelwidth (mm) = 2.540000E-02 pixel pitch (mm) = 2.540000E-02
 # points across pixel for pixel integration = 401
 pixel integration increment = 6.350000E-05

cy/mm	PeakCTF	bestphaseCTF
0.50000000	0.98197472	0.98197490
1.00000000	0.95407820	0.95407990
1.50000000	0.92849890	0.92850010
2.00000000	0.90339250	0.90339740
2.50000000	0.87891352	0.87892330
3.00000000	0.85371650	0.85390220
3.50000000	0.83011150	0.83038514
4.00000000	0.80759763	0.80762080
4.50000000	0.78319823	0.78402360
5.00000000	0.75994170	0.75997580
5.50000000	0.73401890	0.73606914
6.00000000	0.71106112	0.71216700
6.50000000	0.68848431	0.68849492
7.00000000	0.66537950	0.66607950
7.50000000	0.64164102	0.64528860
8.00000000	0.61869070	0.62529772
8.50000000	0.60243880	0.60599600
9.00000000	0.58596820	0.58733410
9.50000000	0.56925340	0.56946610
10.00000000	0.55232830	0.55238664
10.50000000	0.53500550	0.53602220
11.00000000	0.51663661	0.52029640
11.50000000	0.49624370	0.50512051
12.00000000	0.47296091	0.48998230
12.50000000	0.44643214	0.47165050
13.00000000	0.41675290	0.44792620
13.50000000	0.40366550	0.44673090
14.00000000	0.39522720	0.43244910
14.50000000	0.38673520	0.41825400
15.00000000	0.37812500	0.40414774
15.50000000	0.36931714	0.39013590
16.00000000	0.36022050	0.37622830
16.50000000	0.35078150	0.36254671
17.00000000	0.34098420	0.34915664
17.50000000	0.33082351	0.33607363
18.00000000	0.32031953	0.32331700
18.50000000	0.30950270	0.31090320
19.00000000	0.29838440	0.29880714
19.50000000	0.28701233	0.28703063
20.00000000	0.27539940	0.27539940

50 CYCLE SINE WAVE TARGET @ 1000 PPI

T(w) case (first term): -2.99661568D-8*F**5
 Target frequency sequence type: 1 tgt form: SINE
 #tgt bars = 50 tgt mod = 1.0000000000000000
 integral inc.in computing each BarSinImg value = 1.000000E-04 cy/mm
 # pts in integral to compute each BarSinImg value = 800001
 max frEquation of input MTF = 80.0001 cy/mm
 pixelwidth (mm) = 2.540000E-02 pixel pitch (mm) = 2.540000E-02
 # points across pixel for pixel integration = 401
 pixel integration increment = 6.350000E-05

cy/mm	PeakMTF	bestphaseMTF
0.50000000	0.96847522	0.96848170
1.00000000	0.92852234	0.92856341
1.50000000	0.89223100	0.89225083
2.00000000	0.85773890	0.85781264
2.50000000	0.82478433	0.82487094
3.00000000	0.79162722	0.79327172
3.50000000	0.76113950	0.76292520
4.00000000	0.73365200	0.73376584
4.50000000	0.70256330	0.70573920
5.00000000	0.67867183	0.67879670
5.50000000	0.64637392	0.65289402
6.00000000	0.62459892	0.62798870
6.50000000	0.60400611	0.60404092
7.00000000	0.57883220	0.58101210
7.50000000	0.54896580	0.55886660
8.00000000	0.52103820	0.53756830
8.50000000	0.50848102	0.51708430
9.00000000	0.49408240	0.49738180
9.50000000	0.47791761	0.47843120
10.00000000	0.46006971	0.46020300
10.50000000	0.44062793	0.44266820
11.00000000	0.41969910	0.42580060
11.50000000	0.39740580	0.40957440
12.00000000	0.37388831	0.39358240
12.50000000	0.34930911	0.37433950
13.00000000	0.32384523	0.35116913
13.50000000	0.31353271	0.35061680
14.00000000	0.30688430	0.33725312
14.50000000	0.29993173	0.32439932
15.00000000	0.29270310	0.31203890
15.50000000	0.28523200	0.30015224
16.00000000	0.27754440	0.28871881
16.50000000	0.26967042	0.27772143
17.00000000	0.26163520	0.26714340
17.50000000	0.25346922	0.25696951
18.00000000	0.24520080	0.24718783
18.50000000	0.23686010	0.23778551
19.00000000	0.22843930	0.22872020
19.50000000	0.21995791	0.21997001
20.00000000	0.21141960	0.21141960

GLOSSARY

ANSI	American National Standards Institute
avg	average
CCD	Charge-Coupled Device
COTS	Commercial Off-The-Shelf
CTF	Contrast Transfer Function
cy/mm	cycles per millimeter
FBI	Federal Bureau of Investigation
IAFIS	Integrated Automated Fingerprint Identification System
I/O	Input / Output
IQS	Image Quality Specification
ISO	International Organization of Standardization
mm	millimeter
MTF	Modulation Transfer Function
NBS	National Bureau of Standards
ppi	pixels per inch
sinemtf	MITRE's computer program to compute the sine wave MTF or square wave CTF
tgt	target
USAF	United States Air Force

