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**Bedford, MA**

# **Modeling and Analysis of Gated, Pulsed RFI and Its Effect on GPS Receivers**

## **Analysis of Average Cycle Slip Rate and Average Bit Error Probability**

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**April 2014**

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## **Abstract**

We develop two new models for gated, pulsed radio frequency interference (RFI), one for cycle slip performance, and one for bit error performance. We compare the results of these two models to an existing, approximate model that is currently present in ITU-R Report M.2220, herein referred to as the dynamic duty cycle factor (DDCF) model. After deriving our new models, we show through case studies that the new time-varying models often predict much worse performance than the corresponding DDCF counterparts. We therefore conclude that further investigation (either by simulation or laboratory measurement) is warranted to verify the new models presented herein.

Keywords: pulsed RFI, cycle slips, bit errors, duty cycle.

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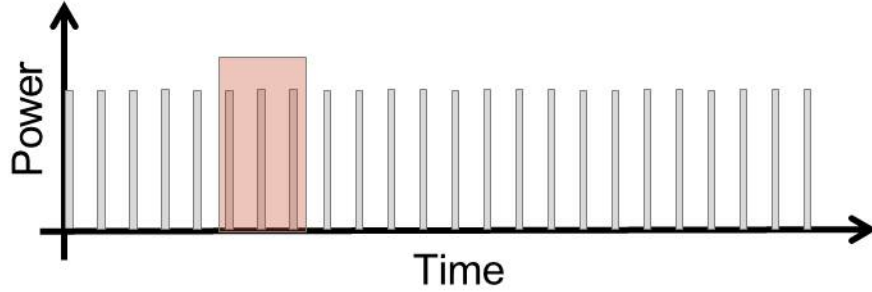


Figure 1: Graphical depiction of pulsed RFI model. The red box depicts a window of time over which  $C/N_0$  is averaged to produce an estimate of the degraded  $C/N_0$ .

## 1 Introduction

*Pulsed Radio Frequency Interference* (pulsed RFI) is a phenomenon that can degrade the performance of GPS receivers [1]—[9]. When strong pulses that lie within the GPS band are present at the input of a GPS receiver, the input often saturates, which has the effect of masking any useful data during the time periods that the pulses are present. The effect of the pulses on system performance is a function of the receiver type (e.g., blanking vs. non-blanking, fast vs. slow), and, hence, a significant amount of effort has been put into modeling the effect of pulsed RFI on various receiver types.

Recommendation ITU-R M.2030 [1] provides a set of models for calculating the degradation effects of pulsed RFI by computing an equivalent continuous wave (CW) noise interference that lowers the nominal carrier-to-noise density  $C/N_0$ . A nominal depiction of pulsed RFI interference that is considered in M.2030 is shown in Fig. 1. Here, the pulses are assumed to occur with a regular pulse repetition frequency (PRF). The post-correlator effective degradation on  $C/N_0$  for a saturating receiver as a function of the pulse duty cycle PDC is [1]:

$$\Delta C/N_0 = 20 \log(1 - PDC) \quad (1)$$

$$PDC \triangleq PW \times PRF \quad (2)$$

where  $PW$  represents the pulse width (sec) and  $PRF$  is the pulse repetition frequency. Eqn. 1 represents a theoretical model of how to estimate  $C/N_0$  degradation, while the red box in Fig. 1 shows how one might compute this degradation via measurement. Instantaneous  $C/N_0$  values would be obtained inside the window and the results would be averaged to obtain a crude estimate of  $C/N_0$  when the pulsed RFI is present. If the pulsed RFI is persistent, it is easy to see from the figure that the average value of  $C/N_0$  is roughly independent of the placement of the window and, therefore, the estimate of  $C/N_0$  is an approximately time-invariant quantity. Therefore, it seems reasonable to use this model of degraded  $C/N_0$  as an indicator for the four basic GPS receiver functions (namely acquisition, carrier tracking, code tracking, and data demodulation).

NASA has proposed a plan to introduce a spaceborne scatterometer for its Solid Moisture Active Passive (SMAP) Program that will be operating in the L2 frequency band [10]. GPS receivers operating in this band are subject to corruption, and NASA is obliged to show that their scatterometer

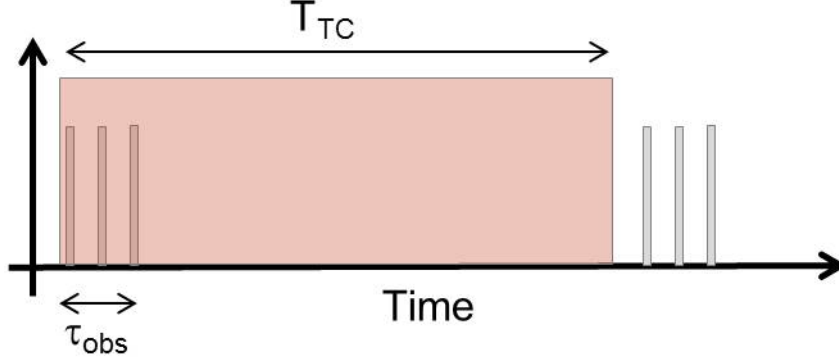


Figure 2: Pulsed RFI waveform with gating. The pulses are present for  $\tau_{obs}$  seconds and are absent for the remaining  $T_{TC} - \tau_{obs}$  seconds.

will not significantly degrade the performance of a GNSS receiver as compared to ambient operation. The scatterometer system uses a mechanical rotating antenna, which can illuminate a victim GPS receiver for a short time ( $\tau_{obs} = 300$  msec) during each rotation period ( $T_{TC} = 4.1$  sec). A graphical depiction of this new type of pulsed RFI waveform, where the pulsing is *gated* to model the fact that the high frequency pulses will only be present at the receiver for 300 msec, is shown in Fig. 2. There is a single sentence in ITU-R Report M.2220 which states that the corresponding degradation in  $C/N_0$  should now be modeled as

$$\Delta C/N_0 = 20 \log(1 - PDC_{LIM}) \quad (3)$$

$$PDC_{LIM} \triangleq PDC \frac{\tau_{obs}}{T_{TC}}. \quad (4)$$

The degradation model of Eqn. 3 and 4, herein referred to as the *Dynamic Duty Cycle Factor (DDCF) model*, states that one can view the gated pulsed RFI waveform as having a composite duty cycle that is the product of the duty cycle of the pulsed waveform ( $PDC$ ) and the duty cycle of the gating waveform ( $\tau_{obs}/T_{TC}$ ). While such a model is clearly convenient, its applicability to modeling the real-world effects of such a pulsing waveform is questionable. Effectively, Eqn. 3 states that the effective degradation in  $C/N_0$  can be computed by averaging the  $C/N_0$  values in the long red window of Fig. 2. While the instantaneous degradation may be large during the  $\tau_{obs}$  seconds that the pulsing is present, the remaining  $T_{TC} - \tau_{obs}$  seconds of quiescent behavior have the potential to hide the negative contributions of the pulsing waveform when the long term average is taken. It is possible—and in fact, quite likely—that the negative effect of the pulses during their "on" time can impact receiver functionality far more than Eqn. 3 and 4 suggest.

In the remainder of this document, we develop new models for analyzing the impact of a gated, pulsed RFI source which take into account the time-varying nature of the overall pulsed waveform. In particular, we develop one model which allows us to characterize the average number of cycle slips per second as a function of the gated, pulsed RFI parameters, and another model which allows us to assess the probability of a bit error in data demodulation schemes. We compare the results of these models to the corresponding results obtained by using the DDCF degradation model of Eqn. 3 and 4, and we effectively show that the time-varying models often predict much worse performance than the DDCF counterparts. We present our results for a variety of gating duty



cycles, pulsed waveform duty cycles, and nominal  $C/N_0$  values to illustrate the large range of differences between the two models.

## 2 Probabilistic Model for Cycle Slips

Cycle slips are an important measure of GPS receiver performance [5]. They occur when the phase estimate of the carrier waveform shifts by  $360^\circ$ . This offset is important as it degrades the accuracy of a PNT solution. Moreover, when the incoming signal is encoded with data, cycle slips (now of  $180^\circ$ ) can cause *data bit inversions*, thus creating bit errors. Here we present a model for assessing the *average number of cycle slips per second* (ACS) as a means of quantifying cycle slip performance, which, as we shall see, is largely linked to  $C/N_0$ .

When  $C/N_0$  is constant, the number of cycle slips  $k$  in a a period of time  $t$  is Poisson distributed [4, 5]. Specifically, if we denote by  $Pr\{k(t)\}$  the probability of  $k$  cycle slips occurring in  $t$  seconds, we have

$$\Pr\{k(t)\} = \frac{\left(\frac{t}{\bar{T}_{CS}}\right)^k \exp\left(-\frac{t}{\bar{T}_{CS}}\right)}{k!} \quad (5)$$

$$\bar{T}_{CS} = \frac{\pi \tanh(\pi\theta_e \Psi)}{2B_L\theta_e} \left( I_0^2(\Psi) + 2 \sum_{n=1}^{\infty} (-1)^n \frac{I_n^2(\Psi)}{1 + \left(\frac{n}{\Psi\theta_e}\right)^2} \right) \Bigg|_{\Psi = \zeta \left(\frac{p}{2\pi\sigma_\phi}\right)^2} \quad (6)$$

where  $\bar{T}_{CS}$  is called the *mean time to cycle slip*, and the parameters in Eqn. 6 are as listed in the table below:

- $\theta_e$ : steady-state jerk angle.
- $B_L$ : one sided loop bandwidth.
- $\zeta$ : third order implementation loss.
- $p$ : period of equivalent S-curve of tracking loop.
- $\sigma_\phi$ : RMS phase error of tracking loop.
- $I_n()$ :  $n$ -th order modified Bessel function of the first kind.

For carrier tracking, the parameter  $\sigma_\phi$  above is given by

$$\sigma_\phi = \sqrt{\sigma_t^2 + \sigma_R^2 + \sigma_T^2} \quad (7)$$

$$\sigma_t = \sqrt{\frac{B_L}{(C/N_0)} \left(1 + \frac{1}{2T_L(C/N_0)}\right)}. \quad (8)$$

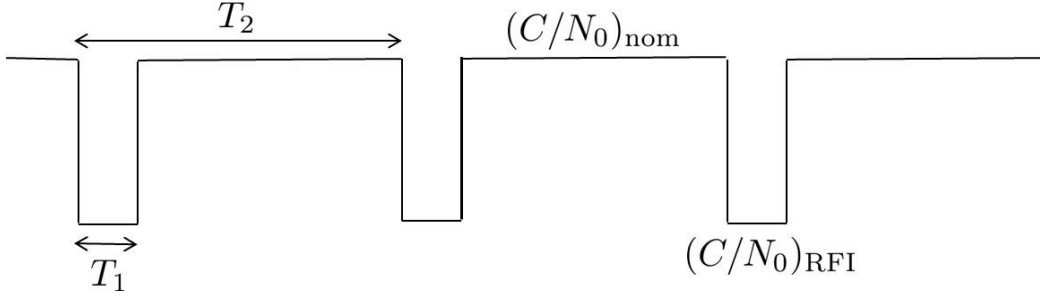


Figure 3: Graphical depiction of periodically varying  $C/N_0$  for a low frequency gating waveform.

In Eqn. 7 and 8,  $T_L$  refers to the coherent integration time, and  $\sigma_R$  and  $\sigma_T$  refer to the receiver and satellite RMS noise levels, respectively. It is clear from Eqn. 8 that the RMS phase error is highly a function of  $C/N_0$ . In particular, as  $C/N_0$  decreases,  $\sigma_\phi$  increases, and  $\bar{T}_{CS}$  decreases, meaning that cycle slips become more frequent.

By virtue of the Poisson distribution in Eqn. 5, the expected number of cycle slips in a given time  $E\{k(t)\}$  is

$$E\{k(t)\} = \frac{t}{\bar{T}_{CS}} \quad (9)$$

Observe that the expected number of cycle slips grows linearly with time. We define ACS, the average number of cycle slips per second, as the long-term average of the expected number of cycle slips:

$$ACS \triangleq \lim_{t \rightarrow \infty} \frac{E\{k(t)\}}{t} = \frac{1}{\bar{T}_{CS}}. \quad (10)$$

When  $C/N_0$  is constant, Eqn. 10 tell us that the ACS is simply the reciprocal of the mean-time-to-cycle slip, an intuitive result.

## 2.1 ACS for Periodically Varying $C/N_0$

We model the effect of gating on the high frequency pulsed waveform by considering  $C/N_0$  that is periodically varying, as depicted in Fig. 3. When pulsed RFI is not present (corresponding to the case when the rotating antenna is not illuminating the GPS receiver), then the value of  $C/N_0$  is some nominal value  $(C/N_0)_{\text{nom}}$ . When the pulsed RFI is present, the effective noise level increases, and  $C/N_0$  is lowered for  $T_1$  seconds to a value we denote as  $(C/N_0)_{\text{RFI}}$ . This process repeats periodically with period  $T_2$ . In terms of the NASA SMAP example discussed in the introduction,  $T_1 = \tau_{obs}$  and  $T_2 = T_{TC}$  (refer to Eqn. 4).

To derive an expression for the ACS in this new scenario, we first need to model the probability that  $k$  cycle slips occur in a given time  $t$ . Because  $C/N_0$  now varies with time, the simple Poisson model of Eqn. 5 does not directly apply. To begin, let us first find the probability mass function for the number of cycle slips  $k$  that occur in one period of the  $C/N_0$  waveform  $T_2$ . To aid us in our computation, it is helpful to partition the time length  $T_2$  into two separate regions: a region of length  $T_1$  where  $C/N_0 = (C/N_0)_{\text{RFI}}$  and a region of length  $T_2 - T_1$  where  $C/N_0 = (C/N_0)_{\text{nom}}$ .

Observe now that the following equality holds

$$\Pr\{k(T_2)\} = \sum_{m=0}^k \Pr\{m(T_1)\} \Pr\{(k-m)(T_2 - T_1)\} \quad (11)$$

The above relationship says the following: suppose that a total of cycle slips that occurs in time  $T_2$  is  $k$ , and that of these  $k$  cycle slips,  $m$  of them occur in the first  $T_1$  seconds. It follows then that  $k - m$  cycle slips occur in the remaining time  $T_2 - T_1$ . The probability that  $k$  cycle slips occurs in time  $T_2$  is therefore the sum over all possible combinations of having  $m$  cycle slips occur in time  $T_1$  and  $k - m$  cycle slips occur in  $T_2$ . Observe that because  $C/N_0$  is constant over each interval:

$$\Pr\{m(T_1)\} = \frac{\left(\frac{T_1}{\bar{T}_{CS_{RFI}}}\right)^m \exp\left(-\frac{T_1}{\bar{T}_{CS_{RFI}}}\right)}{m!} \quad (12)$$

$$\Pr\{(k-m)(T_2 - T_1)\} = \frac{\left(\frac{T_2 - T_1}{\bar{T}_{CS_{nom}}}\right)^{k-m} \exp\left(-\frac{T_2 - T_1}{\bar{T}_{CS_{nom}}}\right)}{(k-m)!} \quad (13)$$

Substituting Eqn. 12 and 13 into Eqn. 11 and evaluating the sum yields

$$\Pr\{k(T_2)\} = \frac{\mu^k \exp(-\mu)}{k!} \quad (14)$$

$$\mu = \frac{T_1}{\bar{T}_{CS_{RFI}}} + \frac{T_2 - T_1}{\bar{T}_{CS_{nom}}}. \quad (15)$$

The above result is likely familiar to those who know queuing theory, as it is simply a restatement that independent Poisson processes regenerate under addition. Those seeking a proof of this statement are referred the Appendix. While the probability mass function (PMF) for the number of cycle slips is not Poisson for an arbitrary time  $t$ , it *behaves* as a Poisson random variable when the time interval under consideration is the period length  $T_2$ . Moreover, it can be shown that for integer multiples  $n$  of the period length  $T_2$  (see the Appendix for a proof):

$$\Pr\{k(nT_2)\} = \frac{\mu^k \exp(-\mu)}{k!} \quad (16)$$

$$\mu = \frac{nT_1}{\bar{T}_{CS_{RFI}}} + \frac{n(T_2 - T_1)}{\bar{T}_{CS_{nom}}}. \quad (17)$$

A direct result of Eqn. 16 is that  $E\{k(nT_2)\} = \mu$  of Eqn. 17. We can use this result to compute the ACS for the time-varying  $C/N_0$  waveform. We are interested in computing

$$ACS = \lim_{t \rightarrow \infty} \frac{E\{k(t)\}}{t}. \quad (18)$$

To compute this limit, we first note that the  $E\{k(t)\}$  is monotonically increasing with  $t$ . Indeed, because the arrival times of individual cycle slips are modeled as independent, then whenever  $t_2 > t_1$ ,

$$E\{k(t_2)\} = E\{k(t_1)\} + E\{k(t_2 - t_1)\} \geq E\{k(t_1)\}. \quad (19)$$

In particular, whenever  $nT_2 \leq t \leq (n+1)T_2$ , it follows that  $E\{k(nT_2)\} \leq E\{k(t)\} \leq E\{k((n+1)T_2)\}$  and, moreover, that

$$\frac{E\{k(nT_2)\}}{(n+1)T_2} \leq \frac{E\{k(t)\}}{t} \leq \frac{E\{k((n+1)T_2)\}}{nT_2}. \quad (20)$$

By the squeeze theorem,

$$\lim_{n \rightarrow \infty} \frac{E\{k(nT_2)\}}{(n+1)T_2} \leq \lim_{t \rightarrow \infty} \frac{E\{k(t)\}}{t} \leq \lim_{n \rightarrow \infty} \frac{E\{k((n+1)T_2)\}}{nT_2}. \quad (21)$$

Utilizing Eqn. 17, we find that

$$\lim_{n \rightarrow \infty} \frac{E\{k(nT_2)\}}{(n+1)T_2} = \lim_{n \rightarrow \infty} \frac{E\{k((n+1)T_2)\}}{nT_2} = \frac{T_1}{\bar{T}_{CS_{RFI}}} + \frac{1 - \frac{T_1}{T_2}}{\bar{T}_{CS_{nom}}} \quad (22)$$

which, by virtue of the inequality chain in Eqn. 21, leads to

$$ACS = \lim_{t \rightarrow \infty} \frac{E\{k(t)\}}{t} = \frac{T_1}{\bar{T}_{CS_{RFI}}} + \frac{1 - \frac{T_1}{T_2}}{\bar{T}_{CS_{nom}}} \quad (23)$$

The result of Eqn. 23 has a somewhat intuitive appeal. If we define the variables

$$ACS_{nom} \triangleq \frac{1}{\bar{T}_{CS_{nom}}} \quad (24)$$

$$ACS_{RFI} \triangleq \frac{1}{\bar{T}_{CS_{RFI}}} \quad (25)$$

then Eqn. 23 is equivalent to

$$ACS_{\text{eff}} = \frac{T_1}{T_2} ACS_{RFI} + \left(1 - \frac{T_1}{T_2}\right) ACS_{nom}. \quad (26)$$

where the "eff" subscript is used to denote the effective ACS of the time-varying model. Eqn. 26 states that the effective ACS when  $C/N_0$  varies periodically is a weighted average of two *time-invariant* ACS's, one corresponding to the nominal  $C/N_0$  and one corresponding to the  $C/N_0$  during the pulsed interference.

## 2.2 Procedure for Comparing ACS: Time-varying Model vs. Average $C/N_0$ Model

We now describe the method for computing  $ACS_{\text{eff}}$  using our time-varying model and compare it to the  $ACS_{\text{DDCF}}$ , the ACS that is predicted by the DDCF model of Eqn. 3 and 4. After describing the general methods in both cases, we shall present results for a variety of case studies.

### 2.2.1 Computing ACS Using Time-varying Model (Eqn. 26)

As mentioned in the introduction, ITU-R Report M.2030 provides a model for computing the degradation in  $C/N_0$  when non-gated, pulsed RFI is present (repeated here for convenience):

$$\Delta C/N_0 = 20 \log(1 - PDC) \quad (27)$$

where  $PDC$  is the duty cycle of the high frequency, pulsed RFI signal. To compute ACS using the time-varying model of Eqn. 26, we use the following procedure:

- For a nominal value of  $(C/N_0)_{\text{nom}}$  (when the pulsing is turned off by the gating waveform), compute  $\bar{T}_{CS_{\text{nom}}}$  using Eqn. 6.
- Use the computed value of  $\bar{T}_{CS_{\text{nom}}}$  to compute  $ACS_{\text{nom}}$  via Eqn. 10.
- For a given value of  $PDC$ , compute  $(C/N_0)_{\text{RFI}} = (C/N_0)_{\text{nom}} + \Delta C/N_0$ , where  $\Delta C/N_0$  is computed via Eqn. 27.
- Use  $(C/N_0)_{\text{RFI}}$  to compute  $\bar{T}_{CS_{\text{RFI}}}$  via Eqn. 6 and then  $ACS_{\text{RFI}}$  via Eqn. 10.
- For given values of  $T_1 = \tau_{\text{obs}}$  and  $T_2 = T_{TC}$ , compute  $ACS_{\text{eff}}$  via Eqn. 26.

### 2.2.2 Computing ACS Using the DDCF Model (Eqn. 3 and 4)

Also mentioned in the introduction, ITU-R Report M.2220 [2] and ITU-R Recommendation M.2030 [1] provide a model for computing degradation in  $C/N_0$  when gated, pulsed RFI is present (repeated here for convenience):

$$\Delta C/N_0 = 20 \log(1 - PDC_{LIM}) \quad (28)$$

$$PDC_{LIM} \triangleq PDC \frac{\tau_{\text{obs}}}{T_{TC}}. \quad (29)$$

where  $PDC$  is the duty cycle of the high frequency pulsed RFI waveform,  $T_{TC}$  is the period of the gating waveform, and  $\tau_{\text{obs}}$  is the amount of time during one gating period for which the pulsed RFI is present. To compute ACS using the DDCF model (denoted  $ACS_{\text{DDCF}}$ ), we use the following procedure:

- For given values of  $PDC$ ,  $\tau_{\text{obs}}$  and  $T_{TC}$ , compute  $(C/N_0)_{\text{DDCF}} = (C/N_0)_{\text{nom}} - \Delta C/N_0$ , where  $\Delta C/N_0$  is computed via Eqn. 28 and 29.
- Using the calculated value of  $(C/N_0)_{\text{DDCF}}$ , compute  $\bar{T}_{CS_{\text{DDCF}}}$  via Eqn. 6.
- Using the calculated value of  $\bar{T}_{CS_{\text{DDCF}}}$ , compute  $ACS_{\text{DDCF}}$  via Eqn. 10.

Parameter	Value
$\theta_e$	$0.7^\circ$
$B_L$	10Hz
$\zeta$	-1.0dB
$p$	$2\pi$ , Pilot Tone $\pi$ , with Data
$T_L$	20msec
$\sigma_R$	$2.9^\circ$
$\sigma_T$	$2.2^\circ$

Table 1: Parameter values used in ACS calculations that were used to generate Fig. 4 through Fig. 7.

### 2.2.3 Results

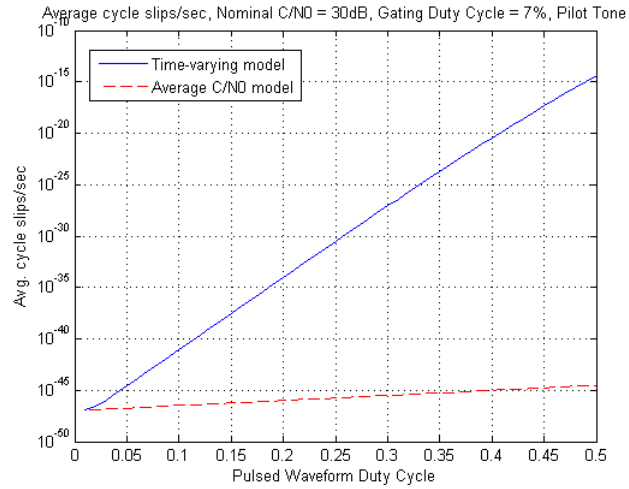
Figures 4 through 7 show the results of computing the ACS via the two different models for a variety of scenarios. In each plot, the horizontal axis represents the duty cycle of the pulsed waveform from 1% to 50%, and the vertical axis shows the corresponding ACS (the gating duty cycle is held constant for each plot). The dashed red waveform indicates the ACS predicted via the DDCF model,  $ACS_{DDCF}$ , whereas the blue waveform represents the ACS predicted by the time-varying model,  $ACS_{eff}$ . The nominal value of  $C/N_0$  is shown in the caption for each plot, and the values for the various parameters that are inputs to Eqn. 6 for computing  $\bar{T}_{CS}$  are shown in Table 1.

Two different signal scenarios are depicted in the figures. Fig. 4 and Fig. 5 show ACS results for the case where the signal being received is a pilot tone (no data modulation). Such a scenario is characteristic of L2C receivers. In this case, a cycle slip occurs when a full  $2\pi$  radians of phase error has accumulated, which accounts for the choice of  $p = 2\pi$  in Table 1. By contrast, Fig. 6 and Fig. 7 depict ACS results when the received signal has modulated data. In this case, a cycle slip can occur only after  $\pi$  radians of phase error (due to ambiguity of the data bit multiplication of  $\pm 1$ ).

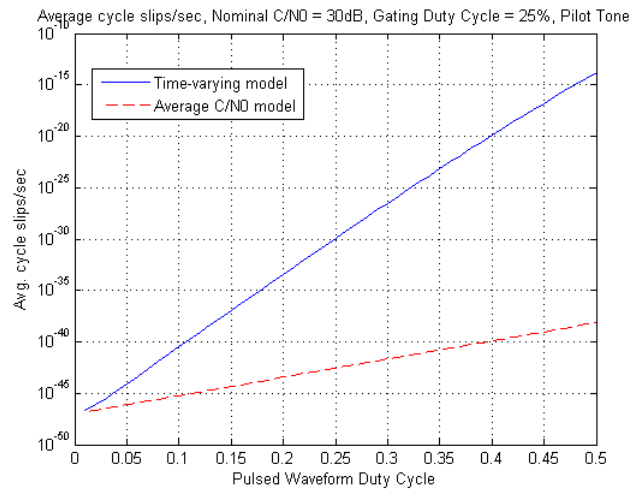
Some comments are in order. First, observe that whenever the gating duty cycle is 100%, the two models produce the exact same result. Such is to be expected since the original (non-gated) pulsed RFI waveform is obtained whenever gating is 100%. Also, observe that whenever PDC is small, the two models also effectively produce the same results (when PDC is zero, there is no pulsed RFI, and the two models should again produce identical results). As PDC increases from zero, the two models produce strikingly different results, with the time-varying model producing significantly higher estimates  $ACS_{eff}$  than that of ACS than the DDCF model  $ACS_{DDCF}$ . This effect is most notable for small gating duty cycles (e.g., 7%), but is also pronounced for moderate gating duty cycles (e.g., 25%) as well.

Overall, the depicted results indicate that the DDCF model may be overly optimistic in predicting ACS. While some simulated and/or measured data should be obtained to verify the time-varying model developed here, at the very least, the results suggest that one should use extreme caution in applying the DDCF model for PDC values that are more than a few percent.

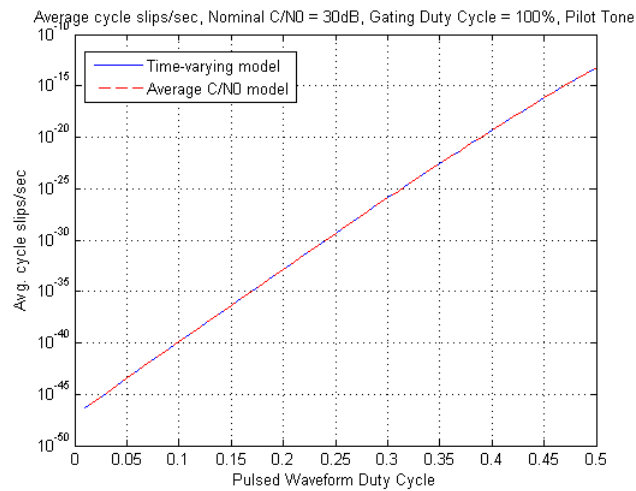
Figure 4:  $ACS_{\text{eff}}$  vs.  $ACS_{\text{DDCF}}$  for a pilot tone,  $(C/N_0)_{\text{nom}} = 30\text{dB}$



(a) Pilot Tone, Gating Duty Cycle = 7%

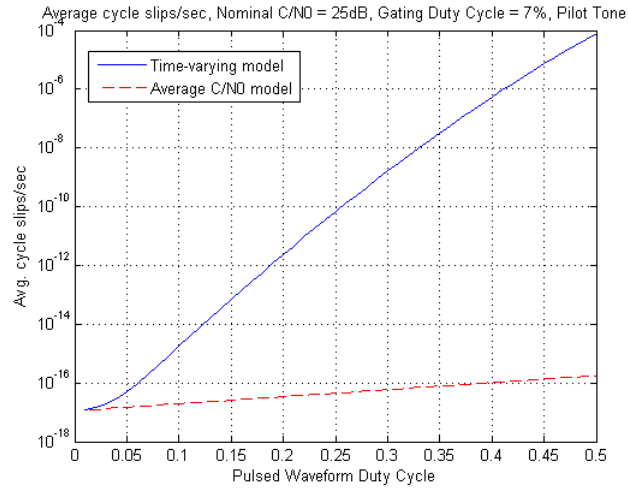


(b) Pilot Tone, Gating Duty Cycle = 25%

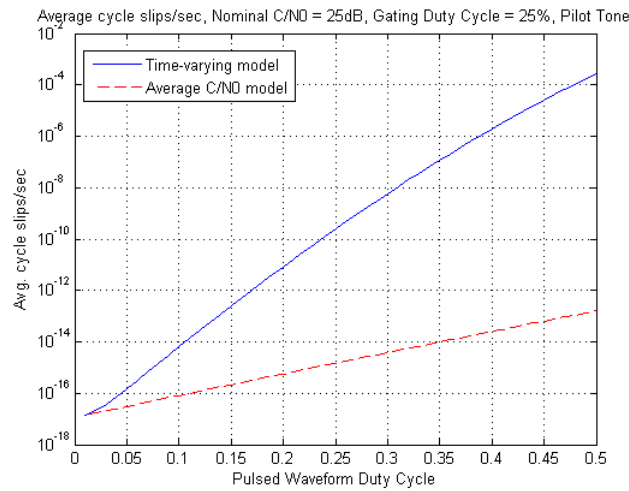


(c) Pilot Tone, Gating Duty Cycle = 100%

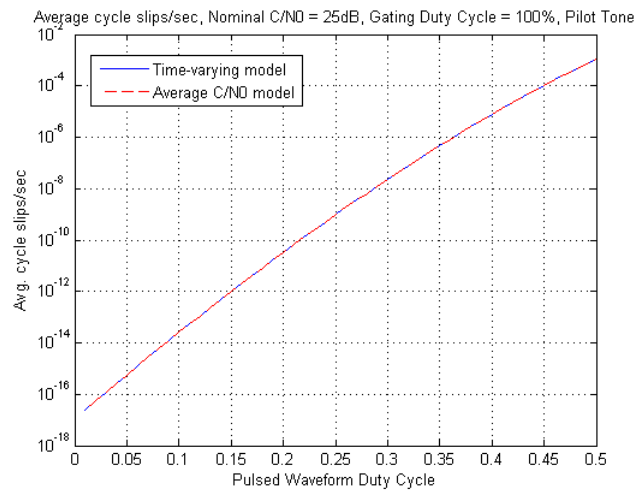
Figure 5:  $ACS_{\text{eff}}$  vs.  $ACS_{\text{DDCF}}$  for a pilot tone,  $(C/N_0)_{\text{nom}} = 25\text{dB}$



(a) Pilot Tone, Gating Duty Cycle = 7%



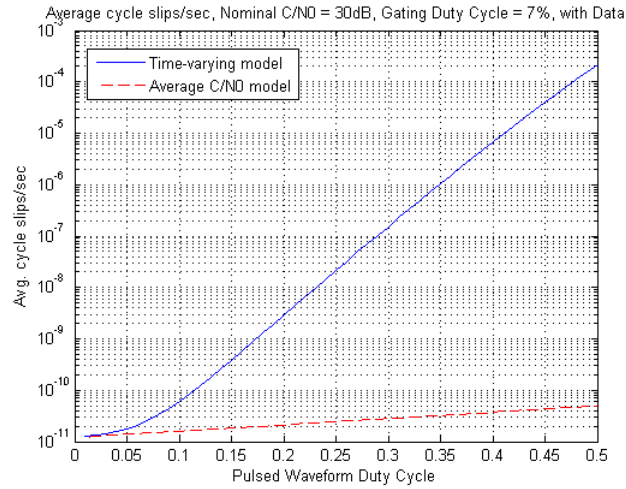
(b) Pilot Tone, Gating Duty Cycle = 25%



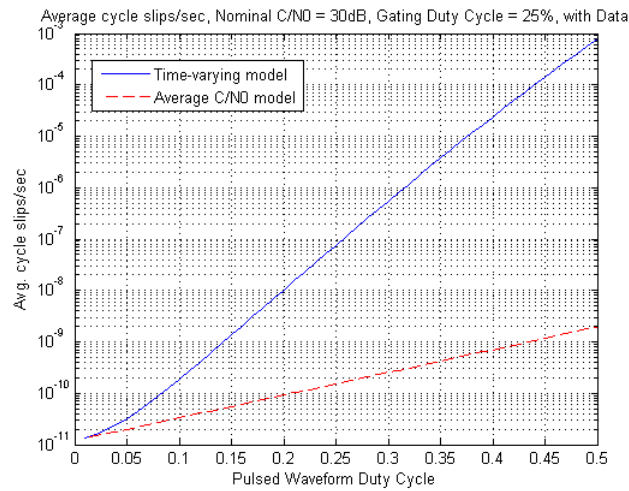
(c) Pilot Tone, Gating Duty Cycle = 100%



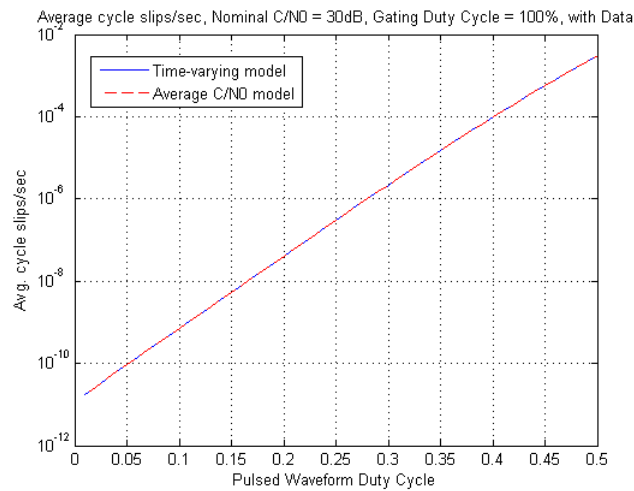
Figure 6:  $ACS_{\text{eff}}$  vs.  $ACS_{\text{DDCF}}$  for a data-modulated signal,  $(C/N_0)_{\text{nom}} = 30\text{dB}$



(a) Data-modulated, Gating Duty Cycle = 7%

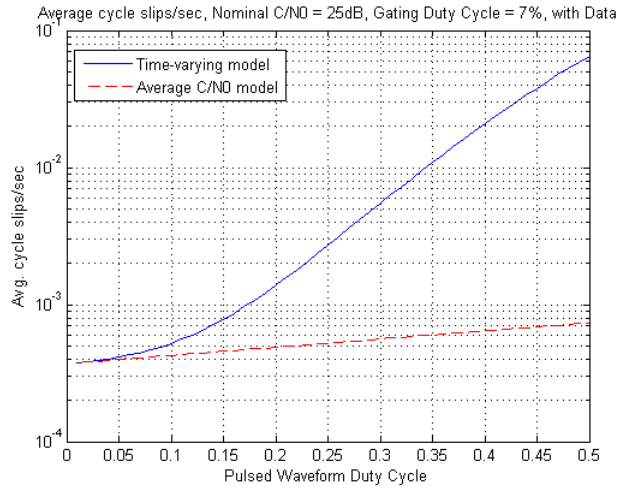


(b) Data-modulated, Gating Duty Cycle = 25%

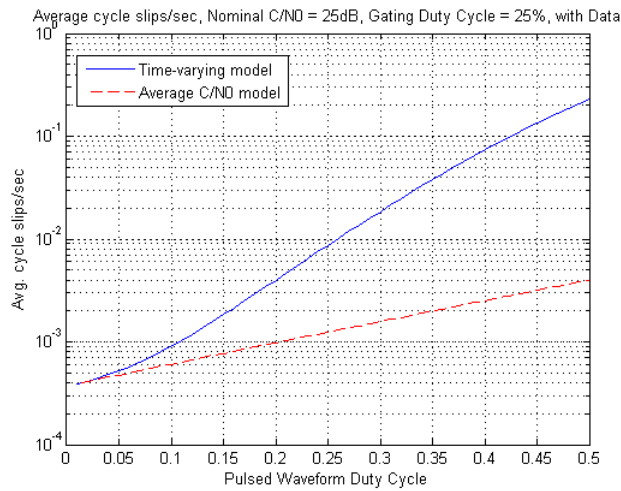


(c) Data-modulated, Gating Duty Cycle = 100%

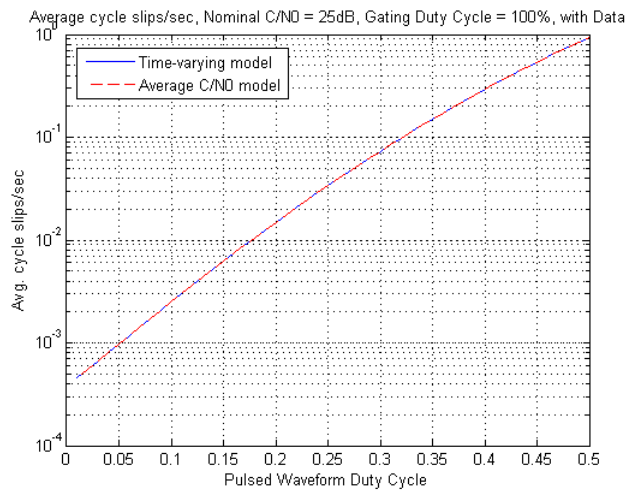
Figure 7:  $ACS_{\text{eff}}$  vs.  $ACS_{\text{DDCF}}$  for a data-modulated signal,  $(C/N_0)_{\text{nom}} = 25\text{dB}$



(a) Data-modulated, Gating Duty Cycle = 7%



(b) Data-modulated, Gating Duty Cycle = 25%



(c) Data-modulated, Gating Duty Cycle = 100%

### 3 Bit Error Probability for L1 C/A and P(Y) Codes

We now turn our attention to modeling the effect of gated, pulsed RFI on *data bit errors*. The cycle slip analysis of the previous section provides us some insight into how time-varying  $C/N_0$  will affect the probability of bit errors (cycle slips will cause data bit inversions on data modulated channels). Here we perform a direct analysis of data bit error probability for a specific scenario, namely L1 C/A (or P(Y) code) which uses a (32,26) Hamming code to decode the data. In this particular case, for constant  $C/N_0$ , the probability of a bit error conditioned on knowledge of carrier phase error  $\phi_e$  is given by [5]

$$P_{B|\phi_e} = \frac{1}{2} \operatorname{erfc} \left( \frac{(C/N_0) \cos^2 \phi_e}{R_B} \right) \quad (30)$$

where  $R_B$  is the bit transmission rate. The carrier phase error  $\phi_e$  is itself a random variable with PDF [5].

$$p(\phi_e) = \frac{\exp\left(\frac{\cos(\phi_e - \theta_e)}{\sigma_\phi^2}\right)}{2\pi I_0\left(\frac{1}{\sigma_\phi^2}\right)}, \quad |\phi_e| \leq \pi. \quad (31)$$

where, again,  $\sigma_\phi^2$  is computed via Eqn. 7 and 8. The average probability of bit error is thus

$$P_B = \int_{-\pi}^{\pi} P_{B|\phi_e} p(\phi_e) d\phi_e. \quad (32)$$

When  $C/N_0$  is piecewise constant, as in Fig. 3, we can compute the average probability of error using Bayes' Rule. Suppose we denote by  $P_B^{\text{nom}}$  the average probability of a bit error when  $C/N_0 = (C/N_0)_{\text{nom}}$ , and  $P_B^{\text{RFI}}$  as the average probability of a bit error when  $C/N_0 = (C/N_0)_{\text{RFI}}$ . Then the effective average probability of a bit error  $P_B^{\text{eff}}$  is given by

$$\begin{aligned} P_B^{\text{eff}} &= P_B^{\text{RFI}} P\{C/N_0 = (C/N_0)_{\text{RFI}}\} + P_B^{\text{nom}} P\{C/N_0 = (C/N_0)_{\text{nom}}\} \\ &= \boxed{P_B^{\text{RFI}} \frac{T_1}{T_2} + P_B^{\text{nom}} \left(1 - \frac{T_1}{T_2}\right)}. \end{aligned} \quad (33)$$

#### 3.1 Procedure for Comparing Average Bit Error Probability: Time-varying Model vs. Average $C/N_0$ model

The procedure we use to compare the average bit error probability obtained via the time-varying model to that obtained by the DDCF model is very similar to the procedure for comparing ACS for the two models. We describe the exact procedures here and then present results for a variety of case studies.

##### 3.1.1 Computing Average Probability of Bit Error for Time-varying Model (Eqn. 34)

- For a nominal value of  $(C/N_0)_{\text{nom}}$  (when the pulsing is turned off by the gating waveform), compute  $\sigma_\phi$  via Eqn. 7 and 8.

Parameter	Value
$\theta_e$	0.35°
$B_L$	10Hz
$T_L$	10msec
$\sigma_R$	2.9°
$\sigma_T$	2.2°
$R_B$	50 bits/sec

Table 2: Parameter values used in  $P_B$  calculations that were used to generate Fig. 8 and Fig. 9.

- Substitute the computed value of  $\sigma_\phi$  into Eqn. 30 and use this to compute  $P_B^{\text{eff}}$  via Eqn. 32.
- For a given value of  $PDC$ , compute  $(C/N_0)_{\text{RFI}} = (C/N_0)_{\text{nom}} + \Delta C/N_0$ , where  $\Delta C/N_0$  is computed via Eqn. 27.
- Use  $(C/N_0)_{\text{RFI}}$  to compute the corresponding value of  $\sigma_\phi$  via Eqn. 7 and 8, and use this value to compute  $P_B^{\text{RFI}}$  via Eqn. 32.
- For given values of  $T_1 = \tau_{\text{obs}}$  and  $T_2 = T_{TC}$ , compute  $P_B^{\text{eff}}$  via Eqn. 34.

### 3.1.2 Computing Average Bit Error Probability Using DDCF Model (Eqn. 3 and 4)

The procedure for computing average bit error probability using the DDCF model, denoted by  $P_B^{\text{DDCF}}$ , is again very similar to the procedure for computing ACS using the DDCF model:

- For given values of  $PDC$ ,  $\tau_{\text{obs}}$  and  $T_{TC}$ , compute  $(C/N_0)_{\text{DDCF}} = (C/N_0)_{\text{nom}} + \Delta C/N_0$ , where  $\Delta C/N_0$  is computed via Eqn. 28 and 29.
- Using the calculated value of  $(C/N_0)_{\text{DDCF}}$ , compute the corresponding value of  $\sigma_\phi$  via Eqn. 7 and 8.
- Substitute  $\sigma_\phi$  into Eqn. 30, and use this expression to compute  $P_B^{\text{DDCF}}$  via Eqn. 34.

## 3.2 Results

Figures 8 and 9 depict the results of computing the average bit error probability for two different scenarios. As before, the horizontal axis in each plot represents the duty cycle of the pulsed waveform from 1% to 50%, and the vertical axis shows the corresponding average bit error probability. The dashed red waveform depicts  $P_B^{\text{DDCF}}$  while the blue waveform depicts  $P_B^{\text{eff}}$  for the time-varying model. The nominal values of  $C/N_0$  are shown in the caption for each plot, and the values for various parameters that inputs to the relevant equations are shown in Table 2.

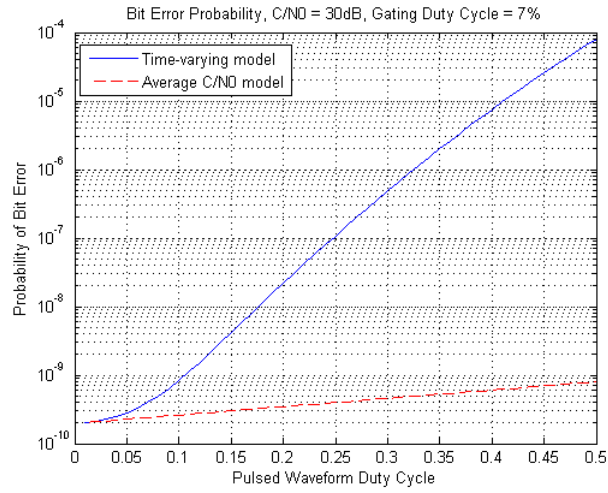
The plots in Fig. 8 and Fig. 9 depict the same qualitative behavior as their ACS counterparts Fig. 4 through Fig. 7; the two models coincide for gating duty cycles of 100% and for pulse duty

cycles of 0%. Furthermore, for low gating duty cycles,  $P_B^{\text{eff}}$  quickly grows to be much larger than  $P_B^{\text{DDCF}}$  as PDC increases. Therefore, the same general conclusions and warnings apply to the average probability of bit error as for ACS: the DDCF model exhibits overly optimistic behavior, so extreme caution should be used when applying it for situations where the PDC values are more than a few percent.

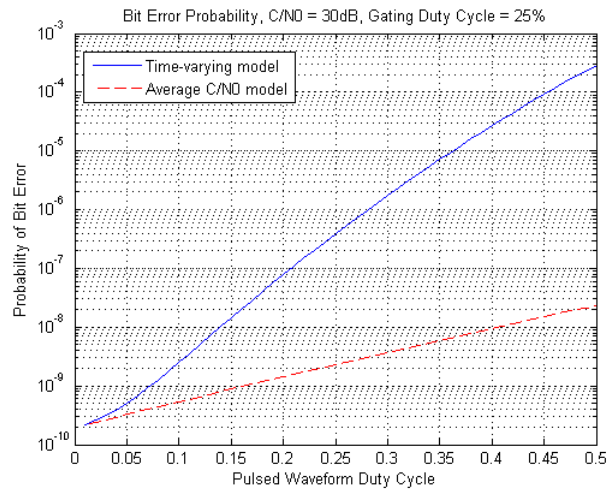
## 4 Summary

Our goal in this document has been to evaluate the validity of the DDCF model that is currently present in ITU-R Recommendation M.2030 [1]. Using the average number of cycle slips per second and the average probability of a bit error as high-level performance indicators, we developed models which explicitly take into account the time-varying nature of the  $C/N_0$  profile and compared their performance to the approximate DDCF model. Our results indicate that in several realistic scenarios, the newly developed time-varying models predict much worse performance than the corresponding DDCF model and, therefore, the applicability of the DDCF model should be questioned. Further analysis, either via simulation or laboratory measurement, is warranted to assess the validity of the time-varying models presented herein.

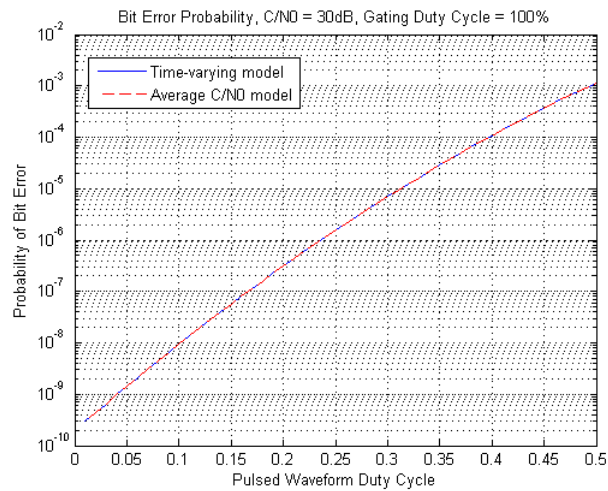
Figure 8:  $P_B^{\text{eff}}$  vs.  $P_B^{\text{DDCF}}$ ,  $(C/N_0)_{\text{nom}} = 30\text{dB}$



(a) Gating Duty Cycle = 7%

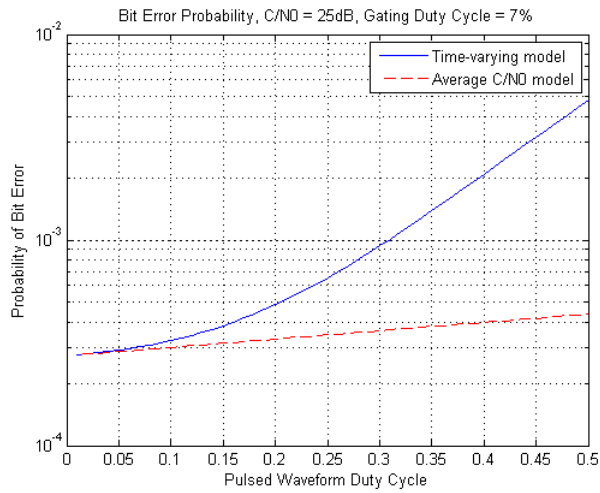


(b) Gating Duty Cycle = 25%

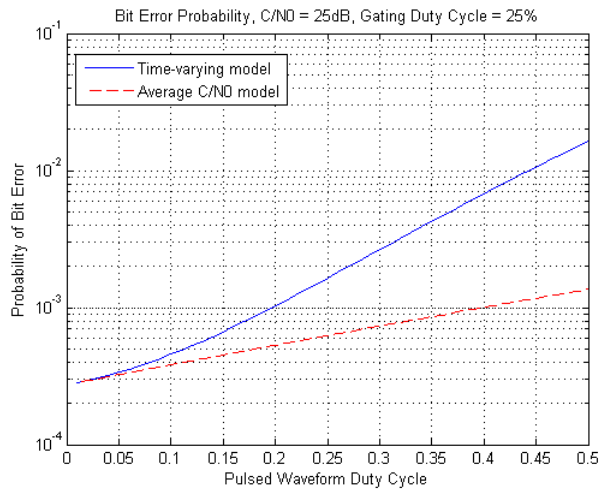


(c) Gating Duty Cycle = 100%

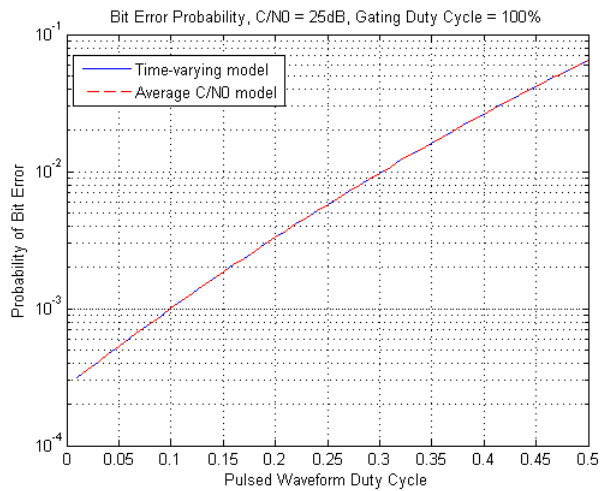
Figure 9:  $P_B^{\text{eff}}$  vs.  $P_B^{\text{DDCF}}$ ,  $(C/N_0)_{\text{nom}} = 25\text{dB}$



(a) Gating Duty Cycle = 7%



(b) Gating Duty Cycle = 25%



(c) Gating Duty Cycle = 100%

## 5 References

- [1] Anon., "Evaluated method for pulsed interference from relevant radio sources other than in the radionavigation-satellite service to the radionavigation-satellite service systems and networks operating in the 1 164-1 215 MHz, 1 215- 1 300 MHz, and 1 559 - 1 610 MHz frequency bands," *Recommendation ITU-R M.2030*, ITU, Geneva, 2012.
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# Appendix A

## A.1 Proof of Eqn. 12 and 13

To simplify notation, we make the following definitions:

$$\alpha \triangleq \frac{T_1}{\bar{T}_{CS_{RFI}}} \quad (34)$$

$$\beta \triangleq \frac{T_2 - T_1}{\bar{T}_{CS_{nom}}}. \quad (35)$$

With the above substitutions, the sum of Eqn. 11 becomes

$$\sum_{m=0}^k \frac{\alpha^m e^{-\alpha} \beta^{k-m} e^{-\beta}}{m! (k-m)!} = \beta^k e^{-(\alpha+\beta)} \sum_{m=0}^k \frac{\left(\frac{\alpha}{\beta}\right)^m}{m! (k-m)!} \quad (36)$$

$$= \frac{\beta^k e^{-(\alpha+\beta)}}{k!} \sum_{m=0}^k \frac{\left(\frac{\alpha}{\beta}\right)^m k!}{m! (k-m)!} \quad (37)$$

$$= \frac{\beta^k e^{-(\alpha+\beta)}}{k!} \sum_{m=0}^k \binom{k}{m} \left(\frac{\alpha}{\beta}\right)^m \quad (38)$$

$$= \frac{\beta^k e^{-(\alpha+\beta)}}{k!} \left(1 + \frac{\alpha}{\beta}\right)^k \quad (39)$$

$$= \frac{(\alpha + \beta)^k e^{-(\alpha+\beta)}}{k!} \quad (40)$$

Observe that the binomial theorem was applied in Eqn. 38 to obtain Eqn. 39. Noting that  $\mu = \alpha + \beta$  completes the proof.

Implicit in the above derivation is the assumption that we consider a time interval of length  $T_2$  which starts on the falling edge of the  $C/N_0$  waveform of Fig. 3 and ends at the next falling edge. This assumption, however, is not critical and the result holds for an *arbitrary* interval of length  $T_2$  as we now show. Suppose we consider an interval of length  $T_2$  which starts  $t$  units of time before a falling edge of the  $C/N_0$  waveform. Then the probability that  $k$  cycle slips occur in an interval that ends  $T_2$  seconds after this starting time can be expressed as

$$\Pr\{k(T_2)\} = \sum_{m=0}^k \left( \sum_{n=0}^m \frac{\alpha^n e^{-\alpha} \beta^{n-m} e^{-\beta}}{n! (n-m)!} \right) \frac{\gamma^{k-m} e^{-\gamma}}{(k-m)!} \quad (41)$$

$$\alpha \triangleq \frac{t}{\bar{T}_{CS_{nom}}} \quad (42)$$

$$\beta \triangleq \frac{T_1}{\bar{T}_{CS_{RFI}}} \quad (43)$$

$$\gamma \triangleq \frac{T_2 - T_1 - t}{\bar{T}_{CS_{nom}}}. \quad (44)$$

Observe now that

$$\Pr\{k(T_2)\} = \sum_{m=0}^k \left( \sum_{n=0}^m \frac{\alpha^n e^{-\alpha}}{n!} \frac{\beta^{n-m} e^{-\beta}}{(n-m)!} \right) \frac{\gamma^{k-m} e^{-\gamma}}{(k-m)!} \quad (45)$$

$$= \sum_{m=0}^k \frac{(\alpha + \beta)^m e^{-(\alpha+\beta)}}{m!} \frac{\gamma^{k-m} e^{-\gamma}}{(k-m)!} \quad (46)$$

$$= \frac{(\alpha + \beta + \gamma)^k e^{-(\alpha+\beta+\gamma)}}{k!}. \quad (47)$$

In the above, we have applied the first part of this proof twice in evaluating the sums. Noting that  $\alpha + \beta + \gamma = \mu$ , we see that Eqn. 47 is equivalent to Eqn. 14. A similar set of sums can be written for the situation that the initial starting point for the length  $T_2$  interval lies between the falling edge of the  $C/N_0$  waveform and the rising edge, and the result is exactly the same. Hence, Eqn. 14 and 15 represent the PMF for the number of cycle slips that occur in an arbitrary time interval of length  $T_2$ .

## A.2 Proof of Eqn. 16 and 17

The proof of this statement is very similar to the last proof of Eqn. 14 and 15 and, hence, many of the details are omitted for brevity. Note that we have already proved the base case  $n = 1$ . Assuming the statement is true for some arbitrary value of  $n$ , then the probability that  $k$  cycle slips occur in time  $(n + 1)T_2$  can be expressed as

$$\Pr\{k((n + 1)T_2)\} = \sum_{m=0}^k \Pr\{m(nT_2)\} \Pr\{(k - m)(T_2)\} \quad (48)$$

Stated in words, the probability that  $k$  cycle slips occur in time  $(n + 1)T_2$  is the probability that  $m$  cycle slips occur over the first  $nT_2$  seconds followed by  $k - m$  cycle slips occur in the following  $T_2$  seconds, summed over all possible values of  $m$ . We have already established that the two quantities in the sum in Eqn. 48 are Poisson random variables. Hence, the same computation may be carried out as in the Proof of Eqn. 14 and 15 to show the general result.

# Distribution List

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