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New Advanced RAIM with Improved Availability for Detecting Constellation-wide Faults, Using Two Independent Constellations

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ABSTRACT

Recently there has been much interest in the feasibility of providing a robust LPV-200 service worldwide circa 2030 relying on RAIM. However, conventional RAIM methods that were developed for the en route through nonprecision approach phases of flight and assume up to a single fault are not adequate for LPV-200 applications. For these applications, constellation-wide faults may need to be considered. Among threats of this class, faults caused by erroneous earth orientation parameters (EOPs) have been identified to be of particular concern. Inter-constellation comparison (ICC) methods were previously proposed to detect constellation-wide faults. However, these methods do not provide as high an LPV-200 availability as desired. This paper presents a new method that can detect constellation-wide faults using two independent constellations and examines its LPV-200 availability. This method has been shown to be very effective in detecting the EOP fault and provides significantly higher LPV-200 availability than the ICC methods.

INTRODUCTION

Receiver Autonomous Integrity Monitoring (RAIM) was initially developed in the early years of GPS-based navigation to ensure integrity for en route through nonprecision approach (NPA) phases of flight assuming up to a single fault at any one time [1–4]. Now, however, RAIM is being investigated to determine if it can provide integrity for LPV-200, an instrument approach procedure with vertical guidance. For this application, the possible occurrence of simultaneous multiple satellite faults needs to be considered in the new RAIM design. Specifically, a type of constellation-wide consistent fault (causing the range errors for all satellites to be mutually consistent) due to an erroneous Earth Orientation Parameter (EOP) has been identified to be of particular concern. A few studies have been reported in the literature proposing ways to handle constellation-wide satellite faults. However, these methods do not provide as high an LPV-200 availability as desired. The purpose of this paper is to present a new RAIM method and examine the availability of LPV-200 that this method provides in the presence of constellation-wide faults, such as an EOP fault.

Recent Interest in RAIM Use for LPV-200 Through the 2020s

For the last several years, a working group sponsored by the FAA, called the GNSS Evolutionary Architecture Study (GEAS) Panel [5, 6], has been exploring GNSS-based architectures capable of providing robust LPV-200 service worldwide through the 2020s. LPV-200 is an instrument approach procedure in which guidance is provided down to a minimum decision height (DH) of 200 ft above the runway threshold. One of the architectures considered by GEAS relies on RAIM. It is based on the expectation that a modernized GPS will provide a greatly improved navigation capability for civil aviation in the 2030 time frame; RAIM performance will benefit from multiple-frequency civil signals and possibly a larger number of satellites.

The RAIM requirements for LPV-200 were initially described in the GEAS Phase II Report [6] and have been recently refined and presented to the Navigation Systems Panel of the International Civil Aviation Organization (ICAO) [7]. The GEAS developed a RAIM algorithm that satisfies those requirements and named it “Advanced RAIM (ARAIM).” This ARAIM algorithm differs from the earlier versions of RAIM in the following respects:

- Explicit consideration of individual satellite range error bias terms that may appear random but could affect users in the same way repeatedly in a non-faulted condition, such as antenna biases or nominal signal deformations. (In this paper, these nominal biases are called fault-free biases to distinguish them from the fault biases caused by a fault condition).

- LPV-200 availability determined on the basis of the following:

  - ✓ accuracy (a 95% bound on the vertical position error (VPE) and a 10^{-7} bound on fault-free vertical error)
  - ✓ Effective Monitor Threshold (EMT), which targets fault conditions having a probability of occurrence greater than 10^{-5}
  - ✓ Protection Levels (PLs).

In its Phase II Report [6], GEAS considered the combined use of GPS and new satellite navigation systems, such as Galileo developed by Europe and the modernized GLONASS with Code Division Multiple Access (CDMA) developed by Russia. As will be shown, ARAIM, using multiple core constellations, not only will provide improved availability under a single fault assumption but also can be designed to detect the presence of constellation-wide consistent satellite faults that are impossible to detect using only one constellation.

Need for Consideration of Multiple Satellite Faults for LPV-200

The single-fault assumption of conventional RAIM methods used for en route through NPA phases of flight was acceptable because the alert limits (maximum operationally acceptable magnitudes of the position errors (PEs)) are relatively large for these phases of flight, and the probability of multiple faults occurring simultaneously and causing a PE larger than any of these alert limits was considered negligible. In contrast, alert limits are tight for the newly desired LPV-200 service. Therefore, even when the ranging errors of individual satellites are not large, they could potentially result in Hazardously Misleading Information (HMI), i.e., the PE would exceed the alert limit for the operation without being detected. For this reason, an application of RAIM for LPV-200 requires that consideration be given to the possibility of HMI involving multiple simultaneous satellite faults. None of the conventional RAIM methods can ensure that HMI caused by simultaneous faults can always be detected with a high enough probability to meet the integrity requirement.

Different Classes of Multiple Satellite Faults

Multiple satellite faults may occur independently and simultaneously, or they may occur as a result of a common-mode fault, a fault affecting multiple satellites from a single fault source. If a common-mode fault affects all the satellites in the constellation, it is called a constellation-wide fault, which is the type of fault that this paper is concerned with. Consistent faults are faults affecting multiple satellites in a mutually consistent manner such that they are the most problematic for RAIM/ARAIM, as will be further discussed later.

EOP Identified To Be Of Major Concern

Upon completing its Phase II Report, GEAS formed the ARAIM Working Group (WG) and tasked it to specifically develop threat models and methods to mitigate of constellation-wide faults. Among possible threats of this class, the EOP fault, which occurs in the generation of EOPs and EOP Predictions (EOPPs), was identified to be of particular concern [8]. EOPs are used to transform between ground-based coordinates (terrestrial reference frame) and “inertial” coordinates (celestial reference frame). The EOP fault is a consistent fault; it is the only consistent fault specifically identified as a potential integrity-failure mode in the current GPS Standard Positioning Service (SPS) Performance Standard [9]. As a result, the GEAS ARAIM WG focused on the characterization of that fault and identified several potential mitigations, including an inter-constellation comparison (ICC) method to be discussed later [8].

Two Groups of RAIM Methods

All the RAIM (or ARAIM) methods may be divided into two groups, based on the domain in which the consistency check is performed: one in the range domain and the other in the position domain. The former is represented by the Chi-square method, matured by R. G. Brown [2] and the latter is represented by the Solution Separation (SS) method, proposed by Mats Brenner [4]. Both methods have been used extensively. While the ARAIM algorithm, developed by GEAS, is a modified version of the SS method, the ARAIM algorithm, presented in this paper, is a modified Chi-square method. The author was the first to develop a method to address the EOP fault, as well as another consistent fault of a more general nature, and presented the algorithm in June 2011 [10] to the WG-C ARAIM Technical Subgroup of the EU-US Cooperation on Satellite Navigation. At a subsequent meeting, J. Blanch presented to the same group a modified SS method that can handle the same types of fault [11]. This paper analyzes the performance of the modified Chi-square method for these types of fault and two other types of constellation-wide fault.

Overview of the Report

The next section discusses previously published studies of RAIM addressing multiple satellite faults relevant to the new ARAIM presented in this paper. The following section describes the proposed new ARAIM algorithm methodology for constellation-wide consistent faults. The subsequent section describes four different types of constellation-wide faults, examined in this paper. The next section derives the formulas for the Vertical Slope (VSlope) and Horizontal Slope (HSlope) for each of those types of constellation-wide faults. A Slope is a computed quantity corresponding to the ratio of the magnitudes of the (vertical or horizontal) PE and the test statistic, solely due to a fault, and is the most critical parameter determining the PL for the Chi-square method. Based on those Slope formulas, the Vertical Protection Level (VPL) and Horizontal Protection Level (HPL) are calculated and availability results are presented. The paper concludes with a summary/conclusion. An appendix is added at the end that briefly describes the conventional Chi-square method to facilitate the discussion of the new ARAIM method in the main text.

PREVIOUS STUDIES OF RAIM DETECTING MULTIPLE SATELLITE FAULTS

This section reviews two groups of previous RAIM studies addressing multiple satellite faults. The first group evaluated how much PLs increase, when the presence of multiple satellite faults is assumed. The second group of studies developed RAIM algorithms that can handle constellation-wide faults, using dual constellations.

Study of Protection Level Increase When Assuming Multiple Simultaneous Faults

R. G. Brown [12] published a study in 1997 on the impact of multiple simultaneous satellite faults, showing for the Chi-square method how much the maximum Slope (and thus the PL) increases, when it is assumed that two satellites are simultaneously faulty. He derived the formula by solving a generalized eigenvalue problem, formulated using a Lagrange equation. J. Angus published a paper in 2006 [13] to study the same problem. Interestingly, while J. Angus started with a slightly different formulation for derivation of the maximum Slopes, he arrived at the same generalized eigenvalue problem as R. G. Brown; he showed that, as the number of assumed simultaneous satellite faults increases from 1 to 2 and 2 to 3, the maximum Slope, and thus the PL, rapidly increase.

The formulas derived in these papers apply for any number of multiple satellite faults. However, if the presence of more than a few simultaneous faults is assumed, the maximum Slopes, and thus PLs, would quickly become too large to provide meaningful service. Certainly, the studies are not for constellation-wide faults, which this paper addresses.

Inter-constellation Comparison (ICC) Methods

Two ICC RAIM methods have been developed to detect multiple satellite faults using two independent constellations when it is assumed that a fault occurs only in one of the two constellations at a time. One is the Optimally Weighted Average Solution (OWAS) method, published by Y. Lee in 2005 [14] and identified as a possible method to mitigate the EOP fault by the GEAS ARAIM WG; the other is the Novel Integrity-Optimized RAIM (*NIO*RAIM), published by P. Hwang and R. G. Brown in 2008 [15]. In these methods, the fault can be a combination of any types of faults, including a single fault, multiple independent faults, or a constellation-wide consistent fault. Both methods detect the presence of a fault or faults in one of the constellations on the basis of the two position solutions, each obtained using the visible satellites from the respective constellations. Both methods trade off accuracy for higher availability of integrity.

In the OWAS method, the navigation solution is not a full-set solution (using all visible satellites) but a weighted average of the two respective solutions from the two constellations, where the weight is optimally chosen to minimize PL (thus maximizing the availability of integrity function) while meeting the accuracy requirements.

The original concept of *NIO*RAIM was published by P. Hwang and R. G. Brown in 2006, in which the conventional Chi-square method was modified to improve RAIM availability under the single-fault assumption [16]. *NIO*RAIM applies weights to individual satellite range measurements so that the Slopes may become nearly equal to one another and thus both the maximum Slope and the PL may be reduced. In order to determine the optimal weights, *NIO*RAIM uses a numerical ad hoc iterative search method in offline processing to generate a multi-dimensional look-up table, which is then used to determine the PL via interpolation in online processing. Subsequently, the *NIO*RAIM concept was applied by the same authors to handle constellation-wide faults, using two independent constellations [15]. Here again, for some pre-specified false detection and missed detection probabilities, *NIO*RAIM creates a look-up table off-line, which is then used to calculate the weights and the PL via interpolation in online processing. A comparison of VPLs, calculated for the OWAS method, with those tabulated for *NIO*RAIM in [15] shows that VPL for *NIO*RAIM is slightly smaller (on the order of 3-7 % reduction in VPL). Unlike OWAS, however, *NIO*RAIM does not allow selection of weights that would also ensure meeting the accuracy requirement at the same time. While the accuracy requirement is typically not a dominant criterion, it is, nevertheless, an important criterion that must be met for LPV-200.

In the ICC methods, the navigation solution is obtained on the basis of two subset solutions where the subsets of satellites are from the respective constellations. Therefore, the availability of LPV-200 with the ICC methods depends on the user-to-satellite geometry from the two constellations separately, and thus the level of availability with the ICC methods does not quite meet the FAA's goal in providing worldwide LPV-200 service. The availability goal stated in the GEAS Phase I Report [5] is 99.5% over a very high fraction (e.g., close to 99%) of the globe between 70 deg N and 70 deg S. When using two constellations, this level of availability/coverage is sought with reduced constellations; in the Phase II Report [6], LPV-200 service availability was evaluated assuming a reduced number of satellites operating in each of three dual constellations (18+18, 21+21, and 21+24 satellites). There are two reasons for this assumption. First, a reduced constellation may be available years before the respective constellations are fully populated. Second, there may be strong fluctuations in the size of an otherwise mature constellation. For these reasons, the FAA has been supporting research for an ARAIM solution that can handle multiple faults with such high availability/coverage even with the reduced constellations.

NEW ARAIM METHODOLOGY FOR CONSTELLATION-WIDE CONSISTENT FAULTS

This section describes the methodology of the newly proposed method for detecting constellation-wide faults. The method is developed by modifying the conventional Chi-square method summarized in the Appendix. The new method focuses on an ability to detect constellation-wide consistent faults, and this paper describes how to calculate the PLs if the presence of such a fault is assumed. On the other hand, the ICC methods do not require any particular assumptions regarding the types of fault in their calculation of the PLs.

Effect of a Constellation-wide Consistent Fault When a Single Constellation Is Used

As shown in the Appendix, a linearized measurement equation for RAIM using a single constellation can be expressed as

$$\Delta r = G\Delta x + \varepsilon + \Delta r_{FB} \tag{1}$$

Ordinarily RAIM detects a fault from the lack of consistency in the range measurements caused by the fault. In the presence of a constellation-wide consistent fault, all the satellites in the constellation are affected, and the fault biases in Δr_{FB} are perfectly consistent with each other such that the fault escapes RAIM fault detection.

More specifically, if use of a single constellation is assumed, the type of constellation-wide consistent fault that we are concerned about would cause a user PE in all directions in a local reference frame (East, North, and Up):

$$\Delta x_{CF} = \begin{bmatrix} b_E & b_N & b_U & 0 \end{bmatrix}^T \tag{2}$$

In this case, the fault range bias vector causing this PE is exactly

$$\Delta r_{FB} = \mathrm{G}\,\Delta x_{CF} \tag{3}$$

This can be seen immediately because Δr_{FB} given in (3) indeed causes an error Δx_{CF} in the least squares estimate of Δx:

$$\Delta\tilde{x} = (G^T W G)^{-1} G^T W(\Delta r_{FB}) = (G^T W G)^{-1} G^T W(G\Delta x_{CF}) = \Delta x_{CF} \tag{4}$$

On the other hand, one can see that a consistent fault causing Δx_{CF} of arbitrarily large magnitude is invisible to RAIM as follows. If one inserts (3) into (1),

$$\Delta r = G(\Delta x + \Delta x_{CF}) + \varepsilon \tag{5}$$

This is statistically equivalent to the apparently "non-faulted" model,

$$\Delta r = G\Delta X + \varepsilon \tag{6}$$

$$\text{where } \Delta X = \Delta x + \Delta x_{CF} \tag{7}$$

It follows that the least squares estimator of ΔX in (6) will be estimating $\Delta X = \Delta x + \Delta x_{CF}$ and thus will be unable to separate Δx from Δx_{CF}.

Therefore, in the presence of a constellation-wide consistent fault, the test statistic is not inflated by the fault bias, making the fault completely invisible to RAIM.

Mathematical Formulas In the Presence of Constellation-wide Consistent Fault When a Dual Constellation Is Used

The new method combines the satellite range measurements from two constellations. In this case, the unknown, Δx, is a 5-dimensional vector with the last two elements representing the user clock offset to the respective constellation clocks. The user PE in (2) caused by a constellation-wide consistent fault is now expressed as

$$\Delta x_{CF} = \begin{bmatrix} b_E & b_N & b_U & 0 & 0 \end{bmatrix}^T \tag{8}$$

As stated earlier, the new method assumes that a fault affects only one of the two constellations at a time. If it is assumed that the consistent fault occurs in the first constellation, the linearized measurement equation can be expressed as:

$$\begin{bmatrix} \Delta r_1 \\ \Delta r_2 \end{bmatrix} = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \Delta x + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} + \begin{bmatrix} G_1 \, \Delta x_{CF} \\ \underline{0} \end{bmatrix} \tag{9}$$

Equivalently,

$$\Delta r = G\Delta x + \varepsilon + L_1 \, \Delta \mathrm{x}_{CF} \tag{10}$$

where

$$\Delta r \equiv \begin{bmatrix} \Delta r_1 \\ \Delta r_2 \end{bmatrix}, \quad G \equiv \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, \quad \varepsilon \equiv \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}, L_1 \equiv \begin{bmatrix} G_1 \\ \underline{0} \end{bmatrix} \tag{11}$$

The subscript, 1 or 2, indicates the constellation (e.g., GPS or Galileo) to which the parameter belongs. G_1 and G_2 are ($N_1 \times 5$) and ($N_2 \times 5$) matrices, respectively, where N_1 and N_2 are the numbers of satellites visible from the respective constellations. The last two columns of G correspond to the user clock references to the respective constellations and can be expressed as

$$G_{i,j+3} = \begin{cases} 1, & \text{if satellite } i \text{ belongs to constellation } j \ (j = 1, 2) \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

For the above Eq. (10), the least squares estimate of Δx is obtained as

$$\Delta x_0 = S_0 \Delta r \tag{13}$$

where

$$S_0 = \left(G^T W G\right)^{-1} G^T W \tag{14}$$

$$W^{-1} = E[\varepsilon \varepsilon^T] \tag{15}$$

From Eq. (10), the PE and parity vector caused by the fault can be derived as:

Position error:

$$\Delta \tilde{x} = S_0 (L_1 \Delta x_{CF}) = K \ \Delta x_{CF} \qquad \textit{where} \ \ \mathrm{K} = (G^T W G)^{-1} G^T W L_1 \tag{16}$$

Horizontal and vertical position errors (HPE and VPE) are given by

$$HPE = \begin{bmatrix} \Delta x_E \\ \Delta x_N \end{bmatrix} = K_H \ \Delta x_{CF} \tag{17a}$$

where K_H is a submatrix of K, consisting of its first two rows.

$$VPE = \Delta x_U = K_V \ \Delta x_{CF} \tag{17b}$$

where K_V is a submatrix of K, consisting of its third row.

Parity vector:

$$P = Q_P^T W^{1/2} (L_1 \Delta x_{CF}) = M \ \Delta x_{CF} \qquad \textit{where} \ \ \mathrm{M} = Q_P^T W^{1/2} L_1 \tag{18}$$

where Q_P is derived from the QR factorization of $W^{1/2} G$:

$$W^{1/2} G = QR, \quad \mathrm{Q} = [\mathrm{Q_X} \ \mathrm{Q_P}], \quad \mathrm{R} = [\mathrm{R_x} \ \ 0]^{\mathrm{T}}$$

Eqs. (16) through (18) will be used later to calculate the Slopes for different types of constellation-wide consistent fault.

Key Parameters for the New ARAIM Method

Key parameters for the new ARAIM method are: test statistic, fault detection threshold, and PL. The PL is derived from two other parameters: maximum Slope and Pbias. The HPL/VPL formulas are slightly modified for the new method in order to explicitly consider the nominal range biases present in a non-faulted condition. That is,

$$\text{HPL} = (\text{Maximum HSlope}) \times \text{Pbias} + B_{0H} \tag{19a}$$
$$\text{VPL} = (\text{Maximum VSlope}) \times \text{Pbias} + B_{0V} \tag{19b}$$

The maximum Slope formulas are derived later for each type of constellation-wide faults. The test statistic, detection threshold, and Pbias are derived in the same manner as described in the Appendix for the conventional Chi-square method.

The B_{0H} and B_{0V} terms represent the maximum position offsets in the all-in-view solution due to fault-free biases on the individual satellites. For this calculation, it is assumed that the magnitudes of fault-free range biases are bounded ($b_{ff,bound}$), but their signs are chosen to have the worst impact on the PL calculation [5–7]. For example, B_{0V} can be derived as

$$B_{0V} = \left(\sum_{j=1}^{N} \left| S_0(3, j) \right| \right) b_{ff,bound} \tag{20}$$

The formula for B_{0H} is similarly derived in [17].

FOUR DIFFERENT TYPES OF CONSTELLATION-WIDE FAULTS

The performance of the new ARAIM algorithm is examined in this paper for each of four different types of constellation-wide faults described below. Constellation-wide faults may be either consistent or inconsistent. The first is the EOP fault, a consistent fault. The second and the third are consistent faults postulated by extending the EOP fault. The fourth belongs to the class of inconsistent faults.

EOP Fault

The results of U.S. Naval Observatory (USNO) analysis [8] indicates that, when the user position is estimated using only one constellation, the EOP fault affects the PE primarily in the direction of longitude; PE would be significantly smaller in the direction of latitude, and negligible in the vertical direction. In our mathematical formulation of the EOP fault, it is assumed that the EOP fault may cause errors of any magnitude in the direction of both longitude and latitude. For the EOP fault, Δx_{CF} in Eq. (8) becomes

$$\Delta x_{CF} = \Delta x_{EOP} = \begin{bmatrix} b_E & b_N & 0 & 0 & 0 \end{bmatrix}^T \tag{21}$$

Unconstrained Consistent Fault

This type of fault is postulated to affect the user position in both horizontal and vertical directions. That is, when the user position is estimated using one constellation, the PE caused by the fault can be expressed as Eq. (8):

$$\Delta x_{CF} = \begin{bmatrix} b_E & b_N & b_U & 0 & 0 \end{bmatrix}^T$$

Since the three components of Δx_{CF} can take any values, VPL and HPL are calculated for the worst combination of these values that would maximize the respective PLs.

The analysis of this unconstrained consistent fault reveals that the size of b_U relative to the sizes of the other two elements (b_E and b_N) has a dominant effect on VPL and HPL for the new method using a dual constellation. This observation led to consideration of the fault type discussed next in which the relative size of b_U is constrained.

Constrained Consistent Fault

This type of fault, when the user position is estimated using only one constellation, is assumed to cause a PE, Δx_{CF}, in which the size of its vertical element, b_U, relative to the sizes of the other two elements is bounded,. It will be shown later that if the relative size of b_U is bounded by a small value (e.g., 1), the availability of LPV-200 using a dual constellation is much higher than when it is unconstrained. It is noted that a bound of zero corresponds to the case of an EOP fault.

Constellation-wide Inconsistent Faults

One of the constellation-wide faults mentioned informally as a potential candidate for study [18] is a fault caused by an erroneous Earth's gravitational constant. This fault is evaluated in this paper as an example of a constellation-wide inconsistent fault.

In general, when a single constellation is used, a range error vector caused by a constellation-wide fault (either consistent or inconsistent) can be expressed as a sum of two orthogonal vectors:

$$\Delta r_{ICF} = \Delta r_1 + \Delta r_2 = G \Delta x_{ICF} + \Delta r_2 \tag{22}$$

Δr_1 is a vector lying within the space spanned by the matrix G; Δr_2 is another vector orthogonal to Δr_1. If Δr_2 = 0, the fault is completely undetectable by RAIM. If $\Delta r_2 \neq 0$, the fault is inconsistent and thus visible to RAIM. How well RAIM can detect the fault depends on the magnitude of Δr_2 relative to that of Δr_1. This ability also becomes stronger if two independent constellations are used.

AVAILABILITY EVALUATION OF THE NEW ARAIM METHOD

The LPV-200 service availability of the new ARAIM method is evaluated in this section. First, the parameter values assumed in the analysis are discussed. This is followed by four subsections in which the maximum VSlope and HSlope formulas are derived in the presence of each of the four types of constellation-wide faults described above, and their VPL and HPL are evaluated. The last subsection shows availability results based on the VPL and HPL, which are the most dominant and most important requirements for the LPV-200 service.

Parameter Values Assumed for LPV-200 Service Availability Analysis

The LPV-200 service availability of the new ARAIM algorithm is evaluated under the set of assumptions listed in Table 1. Note that reduced constellations were purposely chosen as in the GEAS Phase II Report [6]. The variance of the nth satellite range error is determined from

$$\sigma_n^2 = URA_n^2 + \sigma_{n,DF\_air}^2 + \sigma_{n,tropo}^2$$

where the three terms respectively represent the variances of three error components: the clock/ephemeris error, the dual-frequency airborne error, and the tropospheric delay estimation error.

The analysis assumes the same airborne error models and GPS and Galileo satellite orbital positions, as in that report [6].

Onset fault probability and Pr{HMI} are used to calculate P_{md}, which is, in turn, used to calculate Pbias. The onset probability is given on a *per hour* basis. It is assumed that the mean time for the GPS Control Segment to inform the user is within an hour of the fault onset [6]. (The current GPS SPS PS [9] assures a maximum of 6 hours, but the 2001 GPS SPS PS [19] noted that typical times are on the order of 15 to 45 minutes or less under favorable conditions, e.g., most ground assets working. A mean duration of 1 hour was selected by the GEAS.) This assumption results in an independent integrity failure rate of 10^{-5} per approach per satellite. Therefore, for the parameters in the table, P_{md} is calculated, as follows:

<u>Singe fault</u>
P_{md} = Pr{HMI-single} / Prior fault probability = $8.7 \times 10^{-8} / (10^{-5} \times N)$
where $8.7 \times 10^{-8} = 10^{-7} - 1.3 \times 10^{-8}$ where 1.3×10^{-8} is the probability of any multiple faults assumed in the GEAS Phase I Report and N is the number of satellites in view of the user.

Constellation-wide fault

P_{md} = Pr{HMI-constel} / Prior fault probability = 1.3×10^{-8} / (2.3×10^{-5})

Table 1. Parameters Assumed in Analysis of LPV-200 Service Availability

Vertical Alert Limit (VAL)	35 m
Horizontal Alert Limit (HAL)	40 m
Constellation size (No. of operating SVs)	21 GPS + 24 Galileo 21 GPS + 21 Galileo
URA	0.5 m
Maximum fault-free bias	0.75 m
Time interval	5 min
Time Duration	10 days
Continuity risk	8×10^{-6} in any 15-sec interval evenly divided between V and H
Fault onset probability	Single fault: 10^{-5} /hr /SV Constellation-wide fault 2.3×10^{-5} /hr/constellation (~ one fault every 5 years per constel.)
Integrity risk Pr{HMI}	2×10^{-7}/approach evenly divided between V and H Single fault 8.7×10^{-8}/approach for H and V, respectively Constellation-wide fault 1.3×10^{-8}/approach for H and V, respectively

Note: H: Horizontal, V: Vertical

Evaluation of Protection Levels Assuming the Presence of an EOP Fault

To derive the formulas for HSlope and VSlope, one can express Δx_{EOP} as

$$\Delta x_{EOP} = \begin{bmatrix} b_E & b_N & 0 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 & r & 0 & 0 & 0 \end{bmatrix}^T b_E \tag{23}$$

where

$$r = b_N / b_E \tag{24}$$

Substituting Δx_{EOP} above for Δx_{CF} in (16) through (18), one can derive the expression for the Slope (HSlope or VSlope):

$$Slope = \frac{|PE|}{|P|} = \sqrt{\frac{(c_0 + c_1 r + c_2 r^2)}{(d_0 + d_1 r + d_2 r^2)}} \tag{25}$$

where PE is either HPE or VPE, and P is the parity vector; c's for HPE and VPE are determined from matrices K_H in (17a) and K_V in (17b), respectively; d's are determined from matrix M defined in (18).

Figure 1 shows a typical variation of HSlope as a function of r in (24). Each curve assumes one or the other constellation is affected by the EOP fault.
One can easily solve analytically for the value of r that gives the maximum Slope by differentiating (25) with respect to r and solving the resulting quadratic equation of r. Another way to calculate the maximum Slope is to use the Lagrangian multiplier method described in the next section.

Once the maximum Slope is obtained for the case in which the first constellation is affected by the fault, the derivation can be repeated for the case in which the second constellation is affected. In this case, L_1 in (16) through (18) is replaced with L_2 given by

$$L_2 = \begin{bmatrix} 0 \\ G_2 \end{bmatrix} \tag{26}$$

On the basis of the larger of the two maximum Slopes between the two cases above, each of HPL and VPL is calculated.

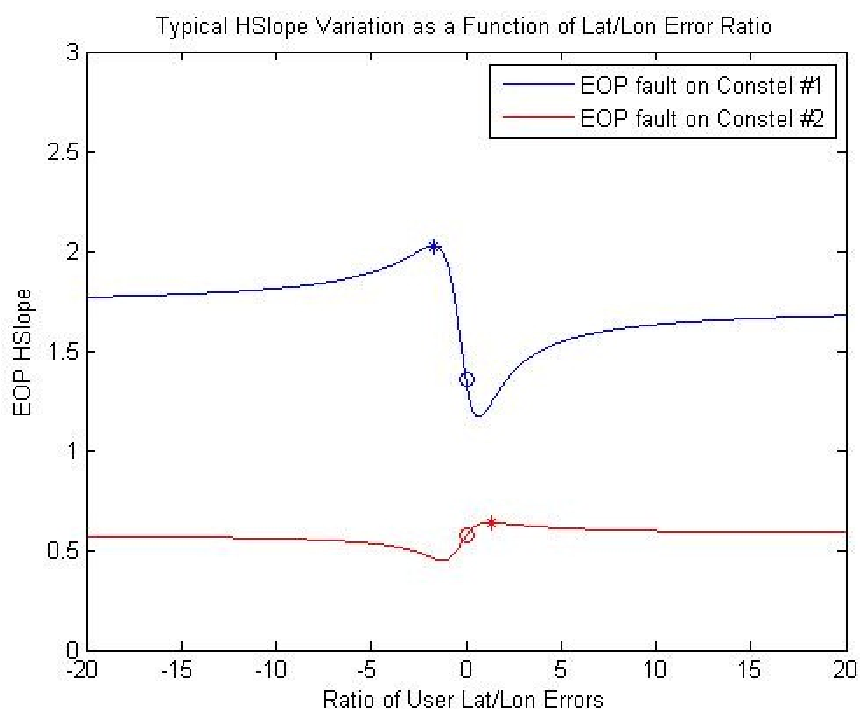

Figure 1. Typical HSlope Variation for the New Method Assuming an EOP Fault as a Function of the Ratio of b_N / b_E

In Figure 2, a typical variation of HPL assuming an EOP fault, obtained using the formula in (19a) for the new ARAIM is compared with those for two existing RAIM methods: OWAS and Chi-Square RAIM using a dual constellation under a single fault assumption. It is shown that most of the time, the HPLs with OWAS and the new method assuming an EOP fault are in a similar range; however, occasionally the OWAS HPL spikes up and exceeds the HAL of 40 m for LPV-200. The spiking of the OWAS HPL occurs when the user-to-satellite geometry from either one of the two constellations is bad. On the other hand, the HPL for the new method remains relatively small because the method uses the satellites from both constellations together. Therefore, the new method can provide a significantly improved availability over the ICC methods, one of which is OWAS.

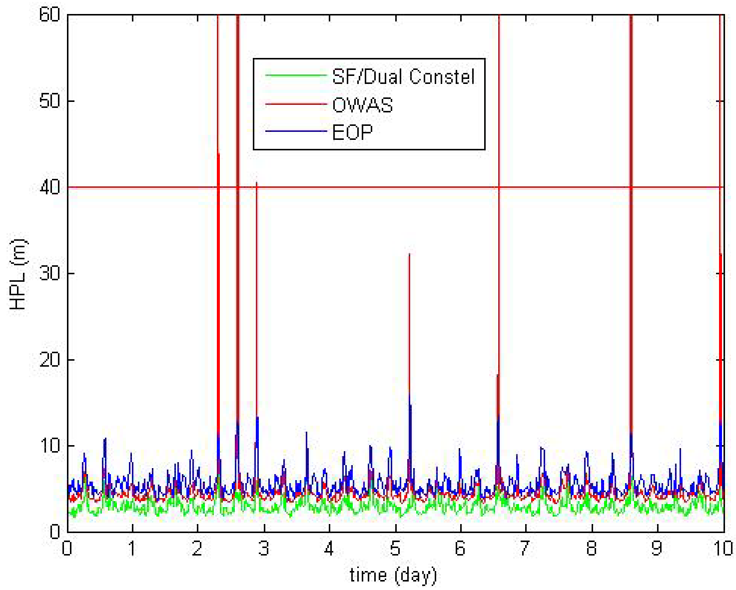

Figure 2. Typical HPL Variation for the New Method Assuming an EOP Fault Compared with HPL for OWAS

Availability Analysis Assuming an Unconstrained Consistent Fault

For derivation of the formulas for HSlope and VSlope, Δx_{CF} is expressed as

$$\Delta x_{CF} = [b_E \quad b_N \quad b_U \quad 0 \quad 0]^T = [1 \quad r_N \quad r_U \quad 0 \quad 0]^T b_E \tag{27}$$

$$where \quad r_N = \frac{b_N}{b_E}, \quad r_U = \frac{b_U}{b_E} \tag{28}$$

As was the case for the EOP fault, HSlope and VSlope depend not on the absolute sizes of the elements of Δx_{CF} but on the ratios of those elements. As was stated earlier, the first three components of Δx_{CF} can take any values, and thus there is no constraint on their relative sizes.

From (17a) and (18), the magnitudes of the HPE and the parity vector P can be expressed as

$$|HPE|^2 = \Delta x_{CF}^T \left(K_H^T K_H\right) \Delta \mathrm{x}_{CF} \tag{29}$$

$$|P|^2 = \Delta x_{CF}^T (M^T M) \Delta x_{CF} \tag{30}$$

The maximum Slope for the case of unconstrained consistent faults can be obtained using the Lagrangian multiplier method [12, 13]. With a constraint, $|P|^2 = P^T P = 1$, define a function f as follows.

$$\begin{aligned} f &= |HPE|^2 - \lambda\left(P^T P - 1\right) \\ &= \Delta x_{CF}^T \left(K_H^T K_H\right) \Delta \mathrm{x}_{CF} - \lambda\left\{\Delta x_{CF}^T (M^T M)\, \Delta \mathrm{x}_{CF} - 1\right\} \end{aligned} \tag{31}$$

Differentiating f with respect to Δx_{CF} and setting the result to zero gives a generalized eigenvalue equation of the following form:

$$\left(K_H^T K_H\right) \Delta \mathrm{x}_{CF} = \lambda\, (M^T M)\, \Delta \mathrm{x}_{CF} \tag{32}$$

The maximum HSlope is therefore given by the square-root of the maximum eigenvalue in (32):

$$\max \mathrm{H}Slope = \max_j \left\{\sqrt{\lambda_j}\right\} \tag{33}$$

for λ_j's from (32)

The maximum VSlope can be derived in a similar manner:

$$\max VSlope = \max_j \left\{\sqrt{\lambda_j}\right\} \tag{34}$$

$$\text{for } \lambda_j\text{'s from } \left(K_V^T K_V\right) \Delta \mathrm{x}_{CF} = \lambda\, (M^T M)\, \Delta \mathrm{x}_{CF} \tag{35}$$

where K_V was defined in (17b).

Once the maximum HSlope and VSlope are calculated according to the above formulas, the calculation is repeated assuming that the fault occurs on the other constellation, with L1 in K in (16) replaced with L2 defined in (26). On the basis of the larger between the two, each of the HPL and VPL is calculated for the given geometry.

Figure 3 shows the worldwide map of LPV-200 service availability of vertical integrity (between 70 deg S and 70 deg N) for the new method assuming an unconstrained consistent fault. The availability for this type of fault is significantly lower than the availability result obtained under the assumption of an EOP fault.

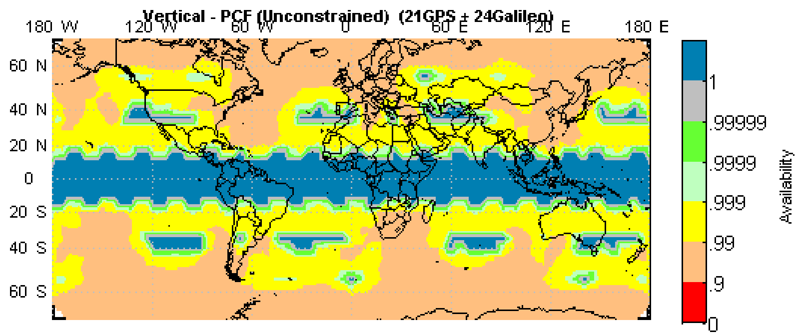

Figure 3. LPV-200 Availability of Vertical Integrity for the New Method Assuming an Unconstrained Consistent Fault (21 GPS and 24 Galileo)

Availability Analysis Assuming a Constrained Consistent Fault

As mentioned earlier, analysis of the impact of an unconstrained consistent fault reveals that r_U is the dominant factor, and if it is assumed that the size of r_U is bounded, a higher availability can be obtained than that obtained with a consistent fault with no constraint. This section evaluates the VSlope/HSlope as a function of the r_U bound; b_E and b_N can take any values but r_U is bounded by some fixed magnitude.

As in (27), Δx_{CF} may be expressed as

$$\Delta x_{CF} = \begin{bmatrix}1 & r_N & r_U & 0 & 0\end{bmatrix}^{\mathrm{T}} b_E$$

(If b_N is larger than b_E, Δx_{CF} is expressed as $\Delta x_{CF} = \begin{bmatrix}r_E & 1 & r_U & 0 & 0\end{bmatrix}^{\mathrm{T}} b_N$)

The maximum Slope is obtained as a function of the r_U bound as follows.

Step 1: Express VSlope (HSlope) as a function of r_N and r_U.
First express the vertical user PE and the parity vector magnitudes as a function of r_N and r_U as follows:

$$VPE = (k_{31} + k_{32} r_N + k_{33} r_U) b_E \tag{36}$$

where $k_{\mathrm{i,j}}$'s are the elements of matrix K defined in (16)

$$|VPE| = |k_{31} + K_{32} r_N + K_{33} r_U|\, b_E \tag{37}$$

Defining M_j as the j[th] column vector of M,

$$|P|^2 = (M_1 + M_2 r_N + M_3 r_U)^T (M_1 + M_2 r_N + M_3 r_U) b_E^2 \tag{38}$$

Then

$$VSlope = \frac{|VPE|}{|P|} = \frac{|k_{31} + K_{32} r_N + K_{33} r_U|}{\sqrt{(M_1 + M_2 r_N + M_3 r_U)^T (M_1 + M_2 r_N + M_3 r_U)}} \tag{39}$$

The expression for HSlope is derived in a similar manner.

Step 2: For a selected magnitude bound B for r_U, set r_U = +B and r_U = –B

Step 3: For the selected r_U bound, each of $|PE|^2$ and $|P|^2$ can be expressed as a quadratic function of r_N. From these expressions, the maximum VSlope (HSlope) can be derived. Then, choose the larger of the maximum VSlope (HSlope) between one for r_U = +B and the other for r_U = –B.

Step 4: Repeat the steps above assuming that the consistent fault occurs in the other constellation. Then choose the larger of the two Slopes as the final maximum VSlope (HSlope).

The worldwide map of LPV-200 service availability of vertical integrity for the new method assuming a consistent fault with the $|r_U|$ bound of 1 is shown in Figure 4. Comparison of this figure and Figure 3 shows that, if it can be assumed that r_U is bounded by 1, a significant improvement in availability can be achieved.

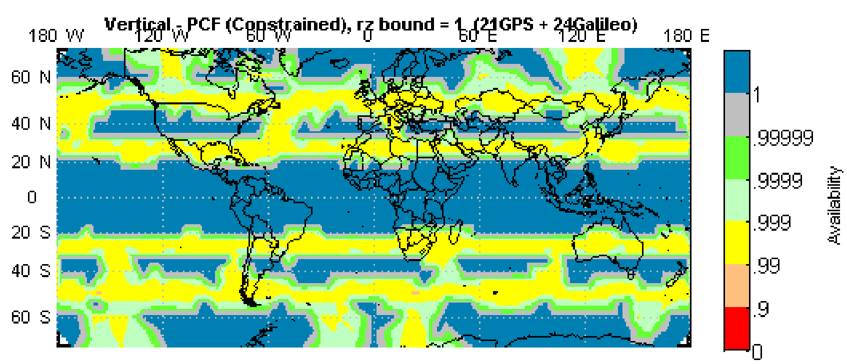

Figure 4. LPV-200 Availability of Vertical Integrity for the New Method Assuming a Consistent Fault Constrained with $|r_U|$ Bound = 1 (21 GPS and 24 Galileo)

A typical VPL variation under different fault assumptions is shown in Figure 5. It is shown that the large VPL obtained under the assumption of an unconstrained consistent fault (denoted as 'CF (No Constraint') is mitigated when it is assumed that $|r_U|$ is bounded by 1.

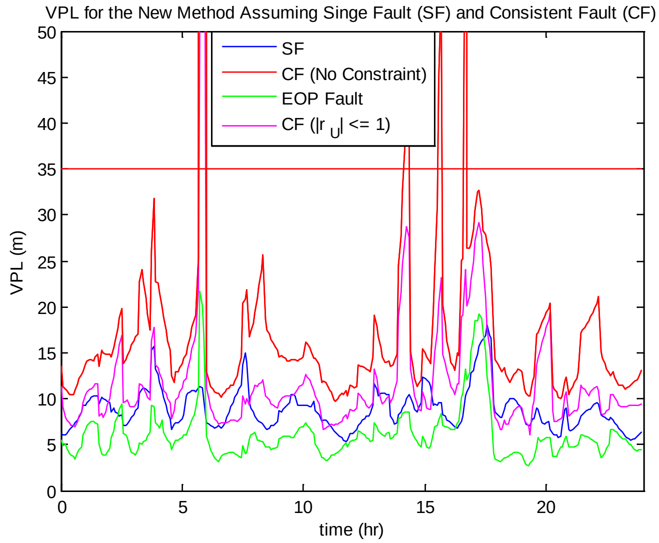

Figure 5. Typical VPL Variation under Different Fault Assumptions

Availability Analysis Assuming a Constellation-wide Inconsistent Fault Caused by Erroneous Earth's Gravitational Constant

In case a fault is caused by an erroneous Earth's gravitational constant affecting the first constellation, a linearized measurement equation can be expressed as

$$\Delta r = G\Delta x + \varepsilon + L_G \tag{40a}$$

where

$$\Delta r = \begin{bmatrix} \Delta r_1 \\ \Delta r_2 \end{bmatrix}, \quad G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, \quad L_G = \begin{bmatrix} \Delta r_G \\ \underline{0} \end{bmatrix} \tag{40b}$$

and Δr_G is the range bias for the satellites in the constellation affected by the fault.

The PE vector and the parity vector caused by the fault, when using a dual constellation, are given by

$$\text{PE: } \Delta\tilde{x} = (G^T W G)^{-1} G^T W L_G \tag{41}$$

$$\text{Parity vector: } P = Q_P^T W^{1/2} L_G \tag{42}$$

The range bias Δr_G is computed in the following steps for each user-to-satellite geometry.

1. Satellite positions are computed with correct and perturbed Earth's gravitational constant. The difference represents the satellite position error caused by the fault.
2. Projection of the satellite position error onto the line of sight from the user to the satellite is that satellite's range bias in Δr_G caused by the erroneous Earth's gravitational constant.

As Δr_G varies as a function of the amount of the perturbation in the Earth's gravitational constant and the elapsed time since the fault occurred, PE and the parity vector vary proportionately. Therefore, VSlope and HSlope can be calculated using (41) and (42) as a function of the user-to-satellite geometry.

Acceptable integrity performance and VPL/HPL can be more readily obtained in the inconsistent fault case than in the unconstrained consistent fault case, for the following reasons. First, since this fault is not a consistent fault, the residual component (Δr_2 in (22)) of Δr_G makes the fault visible to RAIM even when using a single constellation. Second, while the worst combination of values is purposely chosen for the ratios, r_N and r_U, in the calculation of VPL and HPL in the case of the unconstrained consistent fault, the ratios are simply given as a function of Δr_G in the case of an inconsistent fault.

Figure 6 shows the worldwide map of LPV-200 service availability of vertical integrity assuming the presence of a fault caused by an erroneous Earth's gravitational constant when 21 GPS and 24 Galileo satellites are operational. It is shown that availability is higher than 99.9% almost everywhere in the world (between 70 deg S and 70 deg N).

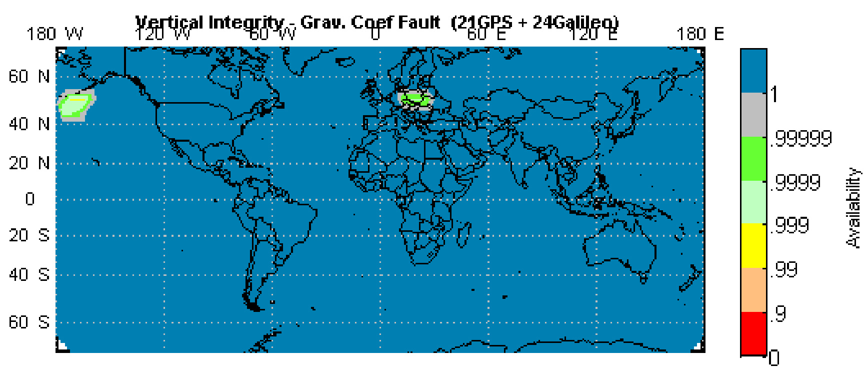

Figure 6. LPV-200 Availability of Vertical Integrity for the New Method Assuming Presence of an Erroneous Earth's Gravitational Constant (21 GPS and 24 Galileo)

LPV-200 Service Availability/Coverage Results

For the set of parameters shown in Table 1, Table 2 shows the percentage of the globe between 70 deg S and 70 deg N that has 99% and 99.5% ARAIM availability of LPV-200 as an average over a period of 10 days with the new ARAIM method. The table shows that even with a 99.5% availability goal, a global coverage of 100 percent can be provided even for the

21GPS+21Galileo constellation under three assumptions: a single fault, EOP fault, and an inconsistent fault caused by an erroneous Earth’s gravitational constant. For the case of an unconstrained consistent fault, the global coverage is inadequate even for the 21GPS+24Galileo constellation. However, in case of a consistent fault constrained with $|r_U| \leq 1$, then a global coverage of 99.9 percent can be obtained with 99% availability for the 21GPS+24Galileo constellation.

Table 2. LPV-200 Service Availability of Both Vertical and Horizontal Integrity for the Newly Proposed Method

Case			Constellation	99%	99.5%
Single Fault			**21GPS+24Gal**	**1.000**	**1.000**
			21GPS+21Gal	**1.000**	**1.000**
Constellation-wide Consistent Fault	**Unconstrained Consistent Fault**		**21GPS+24Gal**	**0.695**	**0.619**
			21GPS+21Gal	**0.483**	**0.386**
	Constrained Consistent Fault	r_U **= 0 (EOP fault)**	**21GPS+24Gal**	**1.000**	**1.000**
			21GPS+21Gal	**1.000**	**1.000**
		$\|r_U\| \leq 1$	**21GPS+24Gal**	**0.999**	**0.961**
			21GPS+21Gal	**0.859**	**0.737**
Constellation-wide Inconsistent Fault	**Erroneous Earth’s Gravitational Constant**		**21GPS+24Gal**	**1.000**	**1.000**
			21GPS+21Gal	**1.000**	**1.000**

SUMMARY/CONCLUSION

Recently the GNSS community has been increasingly interested in the feasibility of providing robust LPV-200 service worldwide circa 2030 relying on RAIM. However, for LPV-200 applications, conventional RAIM methods are not adequate because these methods, which were developed in the early years of GPS-based navigation for the en route through nonprecision approach (NPA) phases of flight, rely on a single-fault assumption. For LPV-200 applications, multiple satellite faults cannot be ignored, and constellation-wide consistent faults (causing the range errors for all satellites to be mutually consistent) are problematic for RAIM. Among possible threats in this class, a fault that occurs in the generation of EOPs and EOP Predictions (EOPPs) has been identified to be of particular concern. Such a fault is considered credible because it is explicitly listed as a potential integrity failure mode in the current GPS Standard Positioning Service Performance Standard. Inter-constellation comparison (ICC) methods were previously proposed to detect consistent faults using two independent constellations. However, those methods do not provide as high availability of LPV-200 as desired by the FAA.

In search of a more effective ARAIM, particularly for the EOP fault, a modified Chi-square algorithm has been developed. The new method, using two independent constellations, has been analyzed for four different types of constellation-wide faults.

- EOP fault, which affects only horizontal positioning when a single constellation is used.

- Unconstrained consistent fault, assumed to affect both horizontal and vertical positioning when a single constellation is used.
- Constrained consistent fault, which is assumed to affect both horizontal and vertical positioning when using only a single constellation, but with a constraint that the ratio of the size of the vertical position error is bounded relative to the size of the horizontal position error due to the fault.
- A fault caused by use of an erroneous Earth's gravitational constant value, which is a constellation-wide inconsistent fault.

The analysis shows that the new ARAIM algorithm gives a very high availability of LPV-200 under the assumptions of the EOP fault and an inconsistent fault caused by an erroneous Earth's gravitational constant value. On the other hand, under the assumption of an unconstrained consistent fault, availability is reduced significantly. However, in case the fault is constrained, that is, if the relative size of the vertical error to the horizontal error caused by the fault when a single constellation is used is bounded (e.g., a ratio no larger than 1), the significant degradation can be mitigated. A question we need to answer is: What is the full range of fault conditions that we need to consider? Once we know what other faults are credible, and we know the characteristics of the range errors produced from such faults, we can determine the integrity performance and availability that can be achieved with the new ARAIM method proposed in this paper. This will in turn determine the size of the dual constellation we will need in order to provide an LPV-200 service relying on ARAIM.

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APPENDIX

PROTECTION LEVEL (PL) FORMULAS FOR THE CONVENTIONAL CHI-SQUARE METHOD

This Appendix briefly describes the VPL and HPL formulas for the Chi-square method, which was developed in the early years of GNSS-based navigation under a single-fault assumption. The method is described in detail in many published papers [2, 3, 20]. (The following derivation accommodates the case in which the error variances are not equal and therefore is slightly different from the original derivations, which assumed equal error variances for all range measurements.)

A linearized measurement equation is given as follows.

$$\Delta r = G\Delta x + \varepsilon + \Delta r_{FB} \tag{A-1}$$

where

Δr: Pseudorange measurements to the visible satellites in the linearized equation

G: (N×4) linearized connection matrix where N is the number of satellites in view and 4 is the number of unknowns when using a single constellation; the first three elements of the k^{th} row of G are the direction cosines of the vector from the user to the k^{th} satellite; the last column corresponds to the user clock offset with respect to the GNSS system clock and has all 1’s.

Δx: Its first three components are the user position offsets in a local reference frame and its last component is the user clock offset from the GNSS time ε: Nominal error vector (N×1) in a fault-free condition

Δr_{FB}: Fault range bias vector (N×1)
In case of a single fault on the k[th] satellite,
$\Delta r_{FB} = \lfloor 0 \quad 0 \quad .. \quad r_{FB,k} \quad 0 \quad .. \quad 0 \rfloor$ where $r_{FB,k}$ is the fault bias error on satellite k.

Navigation Solution

For the above measurement equation, the navigation solution is obtained as

$$\Delta x_0 = S_0 \Delta r \tag{A-2}$$

where the projection matrix S_0 is expressed as

$$S_0 = \left(G^T W G\right)^{-1} G^T W \tag{A-3}$$

$$W^{-1} = E[\varepsilon \varepsilon^T]$$

There are three key parameters for a RAIM method: test statistic, fault detection, and PL.

Test Statistic

For the Chi-square method, the test statistic is the magnitude of the parity vector obtained from the mapping of the range error vector, Δr, into the parity space. The square of the test statistic has a (N–m) degrees of freedom chi-square distribution where N is the number of satellites in view and m is the number of unknowns [2].

Fault detection threshold

The fault detection threshold for the above test statistic is set such that the test statistic would exceed the threshold with no more than a prespecified maximum false detection probability (P_{fd}) in the absence of any fault. The square of this threshold is derived from an (N–m) degrees of freedom central chi-square distribution. If the test statistic exceeds the threshold, a detection flag is raised; otherwise, the PL is computed.

Protection Level (PL)

PL is the limit that RAIM guarantees PE will not exceed without being detected by the fault detection function, in accordance with the missed detection (P_{md}) and P_{fd} requirements. For the original Chi-square method, PL is the product of two terms: Pbias and Maximum Slope.

Pbias

Pbias is the parity vector magnitude caused by a fault that will be misdetected with the specified P_{md} to meet the Pr{HMI} requirement. For a selected fault detection threshold, Pbias is calculated from a non-central Chi-square distribution as described in [2].

Maximum Slope

Slope is the ratio of the PE to the test statistic solely due to the presence of a fault [2]. This is calculated as follows.

The PE is given by

$$\Delta\tilde{x} = (G^T W G)^{-1} G^T W \Delta r_{FB} \tag{A-4}$$

The parity vector P given by

$$P = Q_P^T W^{1/2} \Delta r_{FB} \tag{A-5}$$

where Q_P is derived from the QR factorization of the weighted matrix G:

$$W^{1/2} G = QR, \quad \mathrm{Q} = [\mathrm{Q_X}, \mathrm{Q_P}] \tag{A-6}$$

Due to a single fault on the k^{th} satellite alone,

$$\text{PE: } \Delta\tilde{x} = K\, \Delta r_{FB} \quad \text{where } K = (G^T W G)^{-1} G^T W \tag{A-7}$$

$$\text{Parity vector: } P = M\, \Delta r_{FB} \quad \text{where } M = Q_P^T W^{1/2} \tag{A-8}$$

The VPE due to a single fault on the k^{th} satellite is then obtained as

$$VPE = K(3,k)\, r_{FB,k} \tag{A-9}$$

Likewise, the parity vector P that results from a single fault on the k^{th} satellite, is given by

$$P = M(:,k)\, r_{FB,k} \tag{A-10}$$

where *M(:,k)* is the k^{th} column vector of matrix *M*.

$$|P|^2 = P^T P = M(:,k)^T M(:,k)\, r_{FB,k}^2 \tag{A-11}$$

For the Chi-square method, VPL and HPL are calculated from the VSlope and HSlope, respectively, where Slope is the ratio of the PE magnitude to the parity vector magnitude in the presence of a particular fault.

From (A-7) and (A-9), VSlope in the presence of a fault on satellite k is given by

$$VSlope(k) = \frac{\left|K(3,k)\right|}{\sqrt{M(:,k)^T M(:,k)}} \tag{A-12}$$

VPL is then obtained with the following formula:

$$VPL = \max_k \{VSlope(k)\} \times Pbias \tag{A-13}$$

The expression for *HSlope(k)* and HPL can be similarly obtained.